International Physics Online Olympiad

The Olympiad: Round 1

Week 1: March 23 to March 30, 2014

Thanks to the following individuals.

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Instructions

- There is no allotted time to finish all three problems, but you should attempt to complete the contest in about 90 minutes. This is not mandatory.
- Submit your answers via e-Mail. Mailing or faxing submissions are **not** options. You may want to learn a bit of LaTeX before submitting, but this is not a requirement. If you do not know how to format your answers and do not want to follow the syntax instructions for the form, you may scan your solutions and e-Mail them to us.
- Submit by 8:59 AM EST on to receive a score.
- You may use any techniques to solve the problems. Some problems may require calculus.
- Calculators are permitted. However, graphing aids, internet resources, textbooks, and all other aids are not permitted.
- You **must** submit full solutions to receive full credit. Some questions will ask you to prove particular statements. Others will ask you to find certain quantities. However, for all problems, you must show all your work. A correct answer with no work will receive 0 points.

- **Scoring**: Each question will be graded out of 7 points based on completeness and correctness. The maximum score for this round will be 42.
- Your score for Round 1 will be added to 2.5 times your score in this round, and then 5 will be subtracted from the sum. The result will be your index. This will be used to determine your final position in the contest.
- Have fun and try as many problems as you have time for! We encourage you to submit any answers you may have for this round.

Clarifications

Clarifications/errata (if any) will be updated in this PDF as the contest progress. To ask for a clarification, send us an e-Mail at

`iPhOOContest@gmail.com`.

Week 1

1. The evil Dr. Doom seeks to destroy his enemy, the Intergalactic Federation, and has devised a plan to despin the Federation's space station. The hoop-shaped space station of mass M and radius R rotates once every T hours to maintatin artificial gravity equal to that on IPhOO. Dr. Doom plans to mount two thruster rockets, one rocket on opposite sides of the space station, to stop its rotation. Dr. Doom must accomplish his crime within a time t to avoid getting caught. How much force should each rocket deliver in order to despin the Federation's space station in t? Express your answer in terms of M, R, T, t, and/or constants, as necessary.

2. Odysseus on his ten year return voyage to Ithaca sailed between two monsters. On one side, the creature Charybdis periodically sucked the oceans such that a whirlpool formed. On the opposing side the creature Scylla would lunge down from above and devour one sailor in each of her many mouths. Odysseus opted to sail near Scylla skirting Charybdis by 500 m. At this distance, when the maximum drop in water level of the ocean was 0.2 m from between when Charybdis was draining the oceans and when he was not. At Charybdis' mouth the funnel of the whirlpool is 25 m wide. Assume that the oceans are perfectly calm and that there are no intermolecular attractions between water molecules.
   (a) How deep is Charybdis under water?
   (b) The boat with a crew of 40 men weighs 5,000 kg. Each crew member displaces 3 kg of water at a velocity of 5 m/s every stroke every second. If at some point Odysseus is traveling radially away from Charybdis, what is the closest his ship can be without being sucked in? Assume that Odysseus' vessel has an extremely shallow draft (low friction).

3. Consider a very simple model for an open self-replicative system such as a cell, or an economy. A system $\mathbf{S}$ is comprised of two kinds of mass: one kind is $\mathbf{S}_R$ that is capable of of taking raw material that comes from outside the system, and converting it into components of the system, and the other is $\mathbf{S}_O$ that takes care of other things. Think of $\mathbf{S}_R$ like factories and $\mathbf{S}_O$ like street sweepers. One part is creating new things and the other is doing maintenance on what already exists. The catch is, the material in $\mathbf{S}_R$ must not only make the rest of the system, but also itself! Suppose that the materials in $\mathbf{S}_R$ and the materials in $\mathbf{S}_O$ cost the same amount of energy for $\mathbf{S}_R$ to make per unit amount. Suppose the material in $\mathbf{S}_R$ can convert raw material from the environment into system mass at the rate $\gamma_R = 3$ kg $\mathbf{S}$/hr/kg $\mathbf{S}_R$. If the system doubles in size once every 2 hrs, what fraction of the material in $\mathbf{S}$ is devoted to $\mathbf{S}_O$?

   Assumptions: The fact that the system continuously doubles in size in a fixed time means that this system is in exponential growth, i.e. $\dot{\mathbf{S}} = \lambda \mathbf{S}$.