excellent work!

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Mathematics 1613: Trigonometry Exam #1

This question involves deriving the sine and cosine of some basic angles.

(1) *Derive* the values for the sine and cosine of 30° and 60°. Justify and explain your steps.

$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$

an equilateral triangle ... at 60°

sides = 1

$\cos 60° = 1/2$
$\cos 30° = \sqrt{3}/2$
$\sin 60° = \sqrt{3}/2$
$\sin 30° = 1/2$

Pythagorean theorem

$x^2 + 1/2^2 = 1^2$

$x^2 + 1/4 = 1$ ✓ $\sqrt{x^2} = \sqrt{3/4}$ $x = \sqrt{3}/2$

(2) *Derive* the values for the sine and cosine of 45°. Justify and explain your steps.

start with isosceles Δ →

sides 1, x

$1^2 + 1^2 = x^2$

$2 = x^2$

$\sqrt{2} = x$

Pythagorean's → for x

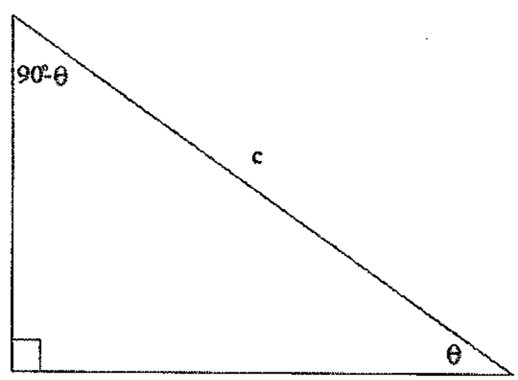
$\sin 45° = \sqrt{2}/2$
$\cos 45° = \sqrt{2}/2$

good!

$x^2 + (\sqrt{2}/2)^2 = 1^2$ ✓

$x^2 + 1/2 = 1$ $\sqrt{x^2} = \sqrt{1/2}$ $x = \sqrt{2}/2$

2. Given the following right triangle,



use the right triangle definitions of sine and cosine to find the indicated values:

a. $\sin\theta = \frac{a}{c}$ ✓

b. $\sin(90° - \theta)$ b/c ✓

c. $\cos\theta = b/c$ ✓

d. $\cos(90° - \theta)$ a/c ✓

e. Based on this information, formulate a conjecture about the relationships between these values.

$\sin\theta = \cos(90-\theta)$ ✓ good!

$\cos\theta = \sin(90-\theta)$

✓

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3. This question involves radian measure.

(1) Explain what it means for an angle to be measured in radians.

It is a measure of the arc length of [illegible] angle [illegible] on the unit circle.

(2) Explain why converting from degrees to radians involves multiplying by $\frac{\pi}{180°}$:

circumference of a circle is $2\pi r$
r=1 on the unit circle, so circ. $= 2\pi$

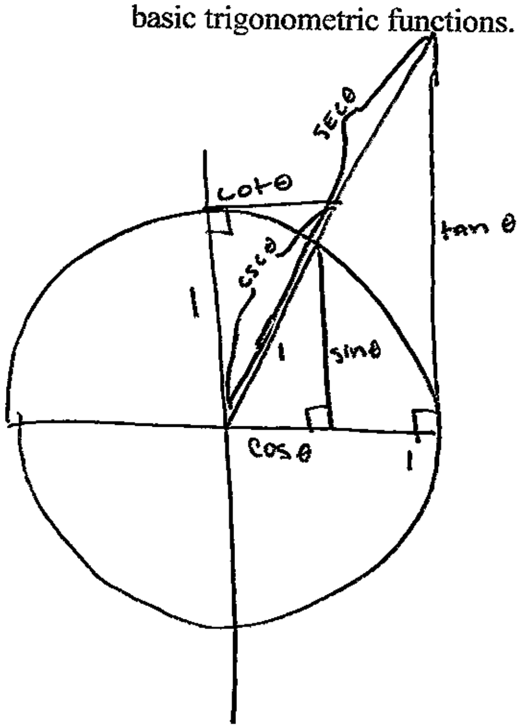
Because $\frac{\pi}{180°}$ is the radian measure of 1°:

$2\pi = 360°$

$\frac{2\pi}{360°} = \frac{360°}{360°}$

$\frac{\pi}{180} = 1°$

4. Given the following angle on the unit circle, construct and label the lengths corresponding to the six basic trigonometric functions. (You do *not* need to justify, simply label.



Using the diagram(s) above, derive the identities $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$ (that is, explain why they are true). They are true because of the pythagorean theorem $a^2 + b^2 = c^2$

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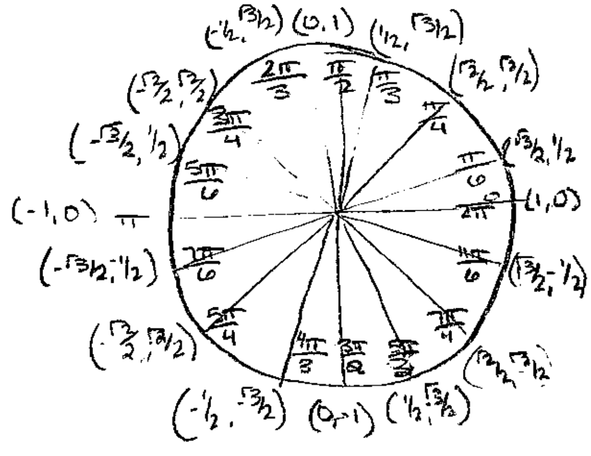
5. Find the following trigonometric values. Justify your answers by showing your work.

(1) $\cos\left(-\frac{7\pi}{6}\right)$ $\cos\left(\frac{11\pi}{6}\right) = -\sqrt{3}/2$ ✓

$\sqrt{3}/2$



(2) $\sin\left(\frac{27\pi}{3}\right)$ $\sin(9\pi) = 0$ ✓

(3) $\csc\left(\frac{5\pi}{2}\right)$ $\csc\left(2\pi + \frac{\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = \frac{1}{1} = 1$ ✓

(4) $\sec\left(\frac{11\pi}{3}\right)$ $\sec\left(3\pi + \frac{2\pi}{3}\right) = \frac{1}{\cos\left(3\pi + \frac{2\pi}{3}\right)} = \frac{1}{1/2} = 2$ ✓

(5) $\cot\left(-\frac{23\pi}{6}\right)$ $- \cot\left(3\pi + \frac{5\pi}{6}\right) = \frac{\cos}{\sin}\left(3\pi + \frac{5\pi}{6}\right) = -\left(\frac{\sqrt{3}/2}{-1/2}\right) = -(\sqrt{3}/2)(-2/1) = \sqrt{3}$ ✓

(6) $\cos\left(-\frac{7\pi}{4}\right)$ $\cos\left(\frac{7\pi}{4}\right) = \cos\left(\pi + \frac{3\pi}{4}\right) = \sqrt{2}/2$ ✓

(7) $\tan\left(-\frac{25\pi}{2}\right)$ $- \frac{\sin}{\cos}\left(12\pi + \frac{\pi}{2}\right) = \frac{-1}{0}$ = undefined ✓

6. Using the even/odd properties of cosine and sine, prove that tangent is odd.

$f(-x) = f(x)$
even
cos

$f(-x) = -f(x)$
odd
sin

$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\tan\theta$ ✓

good!

✓

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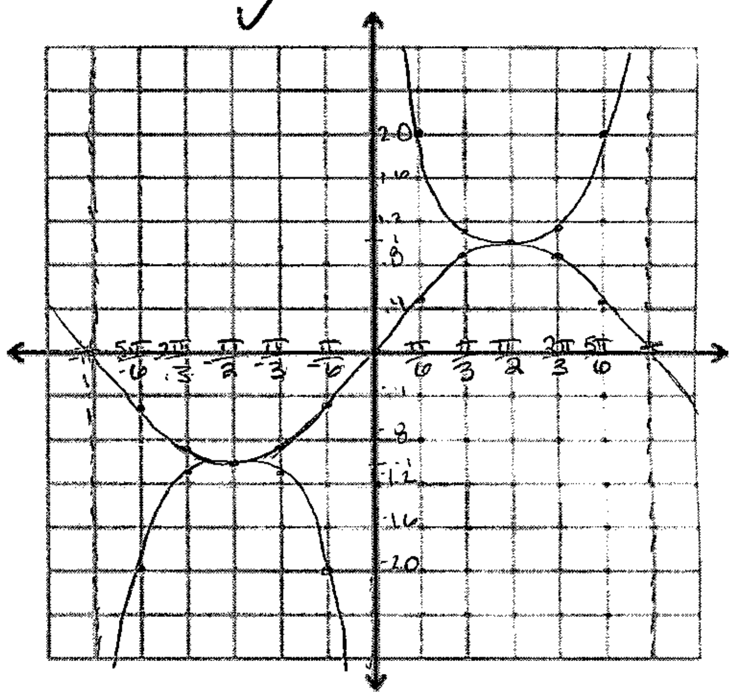
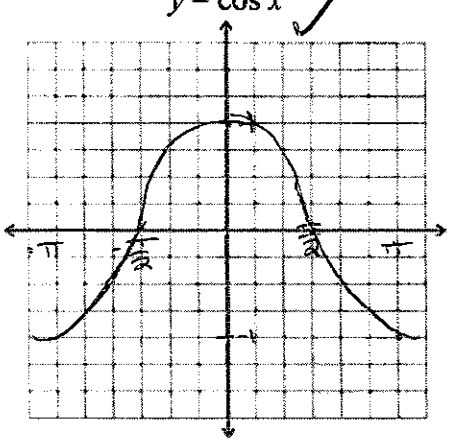
7. Use the given table to graph the functions $y = \csc x$ and $y = \sin x$ *on the same coordinate plane*. Use increments of .4 on the y-axis and $\frac{\pi}{6}$ on the x-axis. You may need to use the decimal approximations $\frac{\sqrt{3}}{2} \approx .87$ and $\frac{2}{\sqrt{3}} \approx 1.15$. If desired, you may use any relevant properties of these functions (though please mention and explain your work).

x	$y = \sin x$	$y = \csc x$
$-\pi$	0	undefined
$-5\pi/6$	$-1/2$	-2
$-2\pi/3$	$-\sqrt{3}/2 = -.87$	$\frac{1}{\sqrt{3}/2} = -1.15$
$-\pi/2$	-1	-1
$-\pi/3$	$-.87$	-1.15
$-\pi/6$	$-1/2$	-2
0	0	undefined
$\pi/6$	$1/2$	2
$\pi/3$	.87	1.15
$\pi/2$	1	1
$2\pi/3$	.87	1.15
$5\pi/6$	$1/2$	2
π	0	undefined

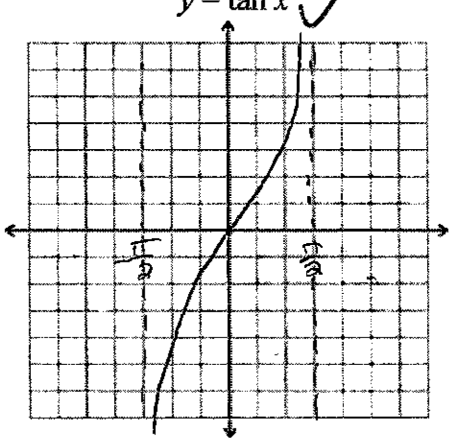


8. Provide *quick, accurate sketches* of the graphs for the given trigonometric functions. You may use any scale you wish, so long as it is consistent and clearly indicated.

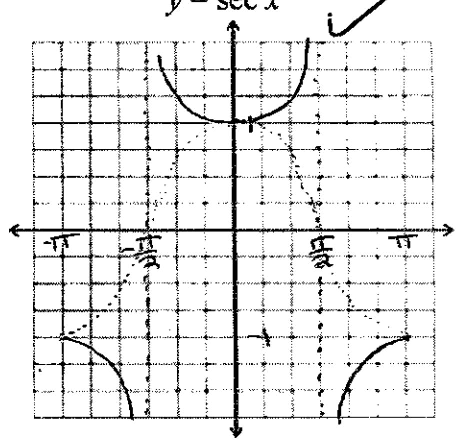
$y = \cos x$



$y = \tan x$



$y = \sec x$



good!

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9. Using the above graphs as a guide, fill in the following tables:

$\mathbb{R}$ = All real numbers

Function	Domain	Range
$y = \sin x$	$\mathbb{R}$	$[-1, 1]$
$y = \cos x$	$\mathbb{R}$	$[-1, 1]$
$y = \tan x$	All real numbers except odd multiples of $\frac{\pi}{2}$	$\mathbb{R}$
$y = \csc x$	All real numbers except multiples of π	$(-\infty, -1] \cup [1, \infty)$
$y = \sec x$	All real numbers except odd multiples of $\frac{\pi}{2}$	$(-\infty, -1] \cup [1, \infty)$
$y = \cot x$	All real numbers except multiples of π.	$\mathbb{R}$

10. Define the term *period* of a function, and state the period of each of the basic trigonometric functions:

Period The shortest repetitive length of a function

Sine 2π

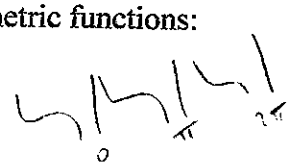
Cosine 2π

Tangent π

Cosecant 2π

Secant 2π

Cotangent π



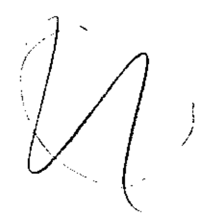
11. Solve the following trigonometric equations:

(1) $2\cos\theta + 1 = 0$

$\cos\theta = -1/2$

$\frac{2\pi}{3} + 2\pi k$

$\frac{4\pi}{3} + 2\pi k$



(2) $\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

$\frac{\pi}{3} + \pi k$

$\frac{2\pi}{3} + \pi k$

$\frac{4\pi}{3}\ \pi k$

$\frac{5\pi}{3} + \pi k$

12. Is $\frac{7\pi}{4}$ a solution to $\sin 4x = 0$? Justify your assertion without actually solving the equation.

$\sin\left(\frac{7\pi}{4}\right)(4)$

$\sin \frac{28\pi}{4} = \sin(7\pi)$

$\sin 0$

$\frac{\pi}{4} + \frac{1}{2}\pi k \quad \frac{\pi}{2} + \frac{1}{2}\pi k$

$\boxed{\text{Yes}}$

it makes $\sin = 7\pi$, which $\sin 4x = 0$ will hit, $\frac{\pi}{2} + \frac{1}{2}\pi k$

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13. Find the following values:

(1) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ $\frac{\pi}{3}$ ✓

(2) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$ ✓

(3) $\arctan\left(-\frac{\sqrt{3}}{3}\right)$ $\frac{-\pi}{6}$ ✓

(4) $\arccos(0)$ $\frac{\pi}{2}$ ✓

(5) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$ $\cancel{-\frac{5\pi}{3}}$ $-\frac{\pi}{3}$ ✓

$\sin^{-1}(-\sqrt{3}/2)$

14. Explain why $\arcsin\left(-\frac{1}{2}\right) \neq \frac{11\pi}{6}$ even though $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$.

$\frac{11\pi}{6}$ is A positive angle To rolate $\frac{11\pi}{6}$ radians would mean to cross All 4 Quadrants Arcsin($-\frac{1}{2}$) calls for a negative angle.

Because Arcsin($-\frac{1}{2}$) is A negative inverse function, we must go in a negative direction to satisfy it. $-\frac{\pi}{6}$ is the proper answer to arcsin($-\frac{1}{2}$). ✓

15. Fill in the following table:

Function	Domain	Range
arcsin x	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
arccos x	$[-1, 1]$	$[0, \pi]$
arctan x	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$