

5. Let's examine how inverse functions for sine, cosine, and tangent might work: suggest possible values for the following:

Suggestions for the inverse sine of $\frac{1}{2}$:

(Hint: which angle(s) have a sine of $\frac{1}{2}$?)

Suggestions for the inverse cosine of $\frac{\sqrt{2}}{2}$:

Suggestions for the inverse tangent of $(\sqrt{3})$:

6. We have (correctly) noted that, as it stands right now, these inverse "functions" we have been informally using are not actually functions. Perhaps we can fix this problem and 'make' these into functions by requiring that there be only one answer. Nominate one of your answers as "the" answer.

7. Let's examine how inverse functions for sine, cosine, and tangent might work for negative values. Keeping in mind what we discussed for part (6), suggest possible values for the following:

Suggestions for the inverse sine of $-\frac{1}{2}$:

Suggestions for the inverse cosine of $-\frac{\sqrt{2}}{2}$:

Suggestions for the inverse tangent of $(-\sqrt{3})$:

8. Now that we have seen how these “functions” behave for positive and negative trigonometric values, devise a list of quadrants that we want our resulting angles to be in for each of sine, cosine, and tangent inverse (taking into account that we want these to be functions).

9. With our new modifications in mind, suppose I wrote the following values corresponding to the inverse sine of $-1/2$:

$$\frac{7\pi}{6}, \frac{19\pi}{6}$$

Should this be considered correct? Why or why not? Do any modifications need to be made?

10. How can we bound our quadrants with interval notation to ensure that the previous example is not correct (i.e. to ensure that we do not violate the definition of a function)? Write the final version of the restricted range in the table below, along with the domain of each inverse function.

	Domain	Range
$\sin^{-1}(x)$, $\arcsin x$		
$\cos^{-1}(x)$, $\arccos x$		
$\tan^{-1}(x)$, $\arctan x$		

11. With our new, restricted ranges of the inverse trigonometric functions, evaluate the following:

$$\sin^{-1}\left(\frac{1}{2}\right)$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\tan^{-1}\left(\sqrt{3}\right)$$

$$\sin^{-1}\left(-\frac{1}{2}\right)$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$\tan^{-1}\left(-\sqrt{3}\right)$$

Note: the answers you have written above are the correct ones (assuming that they have been checked!). Your original suggestions for answers in parts 5 and 7, while instrumental in figuring out the restricted range, may not be the actual correct answers. This does not mean that we need to go back to change them, but it would probably be a good idea to make a note by those parts that reminds you that the answers there are not the final, correct ones (to avoid confusion!).

12. Evaluate the following:

a. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

b. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

c. $\arctan(\sqrt{3})$

d. $\cos^{-1}(0)$

e. $\arctan(-1)$

f. $\sin^{-1}(-1)$

g. $\cos^{-1}(1)$

h. $\arcsin\left(\sin \frac{3\pi}{2}\right)$

i. $\cos\left(\cos^{-1} \frac{8}{6}\right)$

j. $\cos\left(\tan^{-1} \frac{5}{12}\right)$

Homework: textbook section 7.7 #s 1-17 odd, 25-35 odd