Calculus III: Homework Solutions

Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 11.1

Solutions to Even-Numbered Textbook Exercises:

11.1 # 16 (textbook):

$$a_n = \frac{(-1)^{n+1}}{n^2},\ n = 1, 2, \ldots$$

11.1 # 24 (textbook):

$$\lim_{n\to\infty} \frac{n+(-1)^n}{n} = \lim_{n\to\infty} 1 + \frac{(-1)^n}{n} = 1 \Rightarrow \text{converges}$$

11.1 # 26 (textbook):

$$\lim_{n\to\infty} \frac{2n+1}{1-3\sqrt{n}} = \lim_{n\to\infty} \frac{2\sqrt{n} + \left(\frac{1}{\sqrt{n}}\right)}{\left(\frac{1}{\sqrt{n}} - 3\right)} = -\infty \Rightarrow \text{diverges}$$

11.1 # 40 (textbook):

$$\lim_{n\to\infty} n\pi \cos(n\pi) = \lim_{n\to\infty} (n\pi)(-1)^n \text{ does not exist} \Rightarrow \text{diverges}$$

11.1 # 56 (textbook):

$$\lim_{n\to\infty} [\ln n - \ln(n+1)] = \lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right) = \ln\left(\lim_{n\to\infty} \frac{n}{n+1}\right) = \ln 1 = 0 \Rightarrow \text{converges}$$

Solutions to Supplemental Exercises:

11.1 # 20 (supplemental):

$$\lim_{n\to\infty} \frac{n!}{(n+2)!} = \lim_{n\to\infty} \frac{1\cdot 2\cdot 3\cdot\cdots\cdot n}{1\cdot 2\cdot 3\cdot\cdots\cdot n\,(n+1)\,(n+2)} = \lim_{n\to\infty} \frac{1}{(n+2)\,(n+1)} = 0.\ \text{Convergent}$$

11.1 # 21 (supplemental):

The series converges, since

$$a_n = \frac{1+2+3+\cdots+n}{n^2} = \frac{n\,(n+1)\,/2}{n^2} \text{ [sum of the first } n \text{ positive integers]} = \frac{n+1}{2n} = \frac{1+1/n}{2} \to \frac{1}{2}$$

as $n \to \infty$.

11.1 # 22 (supplemental):

$a_n = \cos(n\pi/2)$ is not monotonic. The first few terms are $0, -1, 0, 1, 0, -1, 0, 1, \ldots$. In fact, the sequence consists of the terms $0, -1, 0, 1$ repeated over and over again in that order. The sequence is bounded since $|a_n| \leq 1$ for all $n \geq 1$.