

Mathematics 1613: Trigonometry Quiz #14

41: Using any identities that we have proven to this point, prove that another valid formula for $\cos 2\theta$ is $2 \cos^2 \theta - 1$. Use this formula to find another for $\cos \frac{\theta}{2}$.

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \quad \text{Since } \sin^2 \theta + \cos^2 \theta = 1, \text{ then } \sin^2 \theta = 1 - \cos^2 \theta \\ &= \cos^2 \theta - (1 - \cos^2 \theta) = \boxed{2\cos^2 \theta - 1} \end{aligned}$$

Plug in $\theta/2$:

$$\cos 2\frac{\theta}{2} = 2\cos^2 \frac{\theta}{2} - 1 \implies \frac{\cos \theta + 1}{2} = \cos^2 \frac{\theta}{2}$$

$$\implies \cos \frac{\theta}{2} = \pm \sqrt{\frac{\cos \theta + 1}{2}}$$

42: Use the above formula for $\cos \frac{\theta}{2}$ to find one for $\sin \frac{\theta}{2}$.

$$\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} = 1$$

$$\sin^2 \frac{\theta}{2} + \left(\pm \sqrt{\frac{\cos \theta + 1}{2}} \right)^2 = 1$$

$$\sin^2 \frac{\theta}{2} \mp \frac{\cos \theta + 1}{2} = 1$$

$$\sin^2 \frac{\theta}{2} = \frac{1 \mp \cos \theta}{2} \implies \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

43: Evaluate the following:

$$\cos \frac{\pi}{8} = \pm \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\csc \frac{11\pi}{12} = \frac{1}{\sin \frac{11\pi}{12}} = \frac{2}{\sqrt{2 - \sqrt{3}}}$$

$$\sin \frac{11\pi}{12} = \pm \sqrt{\frac{1 - \cos \frac{11\pi}{6}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

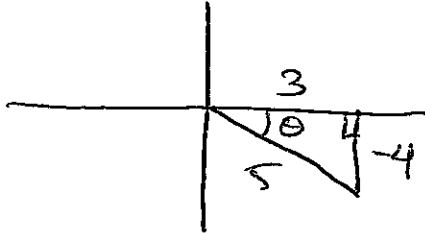
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44: Given that $\sin \theta = -\frac{4}{5}$ and $\frac{3\pi}{2} \leq \theta < 2\pi$, find $\sin \frac{\theta}{2}$.

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \rightarrow \text{need to find } \cos \theta!$$

$$\sin \theta = -\frac{4}{5}:$$

$$\text{and } \frac{3\pi}{2} \leq \theta < 2\pi$$



$$\Rightarrow \cos \theta = \frac{3}{5}$$

$$\Rightarrow \pm \sqrt{\frac{1 - \frac{3}{5}}{2}} = \pm \sqrt{\frac{1}{5}} = \pm \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}$$

+ or -?

$$\frac{3\pi}{2} \leq \theta < 2\pi \Rightarrow \frac{3\pi}{4} \leq \frac{\theta}{2} < \pi$$

$$\Rightarrow \frac{\theta}{2} \text{ is in QII}$$

$$\Rightarrow \text{sine is +}$$