Calculus III: Homework Solutions

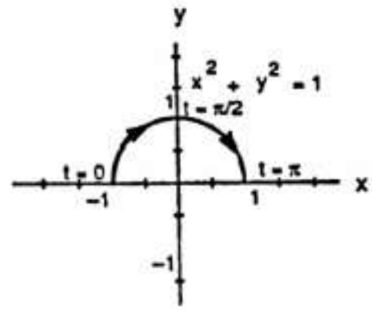
Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 3.5

Solutions Even-Numbered Textbook Exercises:

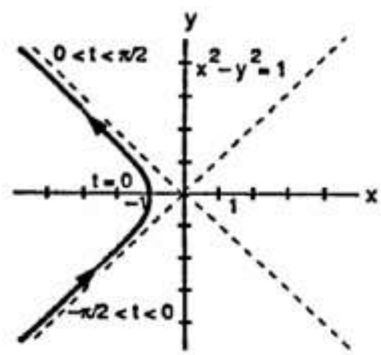
3.5 # 68 (textbook):

$x = \cos(\pi - t),\ y = \sin(\pi - t),\ 0 \le t \le \pi$
$\Rightarrow\ \cos^2(\pi - t) + \sin^2(\pi - t) = 1$
$\Rightarrow\ x^2 + y^2 = 1,\ y \ge 0$



3.5 # 78 (textbook):

$x = -\sec t,\ y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$
$\Rightarrow\ \sec^2 t - \tan^2 t = 1\ \Rightarrow\ x^2 - y^2 = 1$



3.5 # 90 (textbook):

$t = 3\ \Rightarrow\ x = -\sqrt{3+1} = -2,\ y = \sqrt{3(3)} = 3;\ \frac{dx}{dt} = -\frac{1}{2}(t+1)^{-1/2},\ \frac{dy}{dt} = \frac{3}{2}(3t)^{-1/2}\ \Rightarrow\ \frac{dy}{dx} = \frac{\left(\frac{3}{2}\right)(3t)^{-1/2}}{\left(-\frac{1}{2}\right)(t+1)^{-1/2}}$
$= -\frac{3\sqrt{t+1}}{\sqrt{3t}} = \left.\frac{dy}{dx}\right|_{t=3} = \frac{-3\sqrt{3+1}}{\sqrt{3(3)}} = -2$; tangent line is $y - 3 = -2[x - (-2)]$ or $y = -2x - 1$;
$\frac{dy'}{dt} = \frac{\sqrt{3t}\left[-\frac{3}{2}(t+1)^{-1/2}\right] + 3\sqrt{t+1}\left[\frac{3}{2}(3t)^{-1/2}\right]}{3t} = \frac{3}{2t\sqrt{3t}\sqrt{t+1}}\ \Rightarrow\ \frac{d^2y}{dx^2} = \frac{\left(\frac{3}{2t\sqrt{3t}\sqrt{t+1}}\right)}{\left(\frac{-1}{2\sqrt{t+1}}\right)} = -\frac{3}{t\sqrt{3t}}$
$\Rightarrow\ \left.\frac{d^2y}{dx^2}\right|_{t=3} = -\frac{1}{3}$

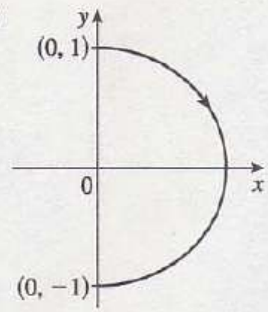
Solutions to Supplemental Exercises:

3.5 # 1 (supplemental):

(a) $x = \sin\theta, y = \cos\theta, 0 \le \theta \le \pi.$

$x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1, 0 \le x \le 1.$

(b)



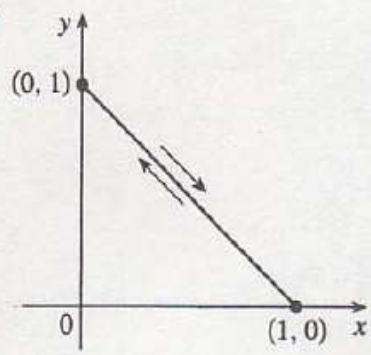
3.5 # 2 (supplemental):

(a) $x = \sin^2\theta, y = \cos^2\theta.$

$x + y = \sin^2\theta + \cos^2\theta = 1, 0 \le x \le 1.$

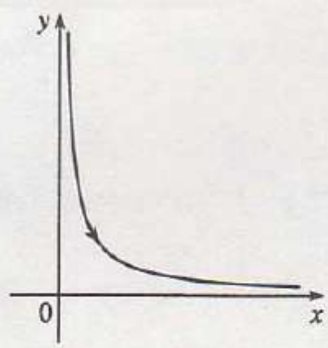
Note that the curve is at $(0, 1)$ whenever $\theta = \pi n$ and is at $(1, 0)$ whenever $\theta = \frac{\pi}{2}n$ for every integer n.

(b)



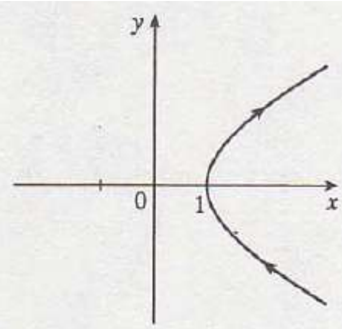
3.5 # 3 (supplemental):

(a) $x = e^t, y = e^{-t}, y = 1/x, x > 0$ (b)



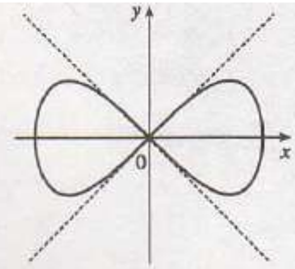
3.5 # 4 (supplemental):

(a) $x = \cosh t, y = \sinh t, x^2 - y^2 = \cosh^2 t - \sinh^2 t = 1, x \ge 1$ (b)



3.5 # 5 (supplemental):

$x = \cos t$, $y = \sin t \cos t$. $\frac{dx}{dt} = -\sin t$,
$\frac{dy}{dt} = -\sin^2 t + \cos^2 t = \cos 2t$. $(x, y) = (0, 0) \quad \Leftrightarrow \quad \cos t = 0 \quad \Leftrightarrow$
t is an odd multiple of $\frac{\pi}{2}$. When $t = \frac{\pi}{2}$, $\frac{dx}{dt} = -1$ and $\frac{dy}{dt} = -1$, so
$\frac{dy}{dx} = 1$. When $t = \frac{3\pi}{2}$, $\frac{dx}{dt} = 1$ and $\frac{dy}{dt} = -1$. So $\frac{dy}{dx} = -1$. Thus,
$y = x$ and $y = -x$ are both tangent to the curve at $(0, 0)$.



3.5 # 6 (supplemental):

The line with parametric equations $x = -7t$, $y = 12t - 5$ is $y = 12\left(-\frac{1}{7}x\right) - 5$, which has slope $-\frac{12}{7}$. The curve $x = t^3 + 4t$, $y = 6t^2$ has slope $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{12t}{3t^2 + 4}$. This equals $-\frac{12}{7} \quad \Leftrightarrow \quad 3t^2 + 4 = -7t \quad \Leftrightarrow$
$(3t + 4)(t + 1) = 0 \quad \Leftrightarrow \quad t = -1$ or $t = -\frac{4}{3} \quad \Leftrightarrow \quad (x, y) = (-5, 6)$ or $\left(-\frac{208}{27}, \frac{32}{3}\right)$.

3.5 # 7 (supplemental):

$$A = \int_0^1 (y - 1)\,dx = \int_{\pi/2}^0 (e^t - 1)(-\sin t)\,dt = \int_0^{\pi/2} (e^t \sin t - \sin t)\,dt \overset{98}{=} \left[\tfrac{1}{2}e^t(\sin t - \cos t) + \cos t\right]_0^{\pi/2}$$
$$= \tfrac{1}{2}\left(e^{\pi/2} - 1\right)$$

3.5 # 8 (supplemental):

By symmetry of the ellipse about the x- and y-axes,

$$A = 4\int_0^a y\,dx = 4\int_{\pi/2}^0 b\sin\theta\,(-a\sin\theta)\,d\theta = 4ab\int_0^{\pi/2}\sin^2\theta\,d\theta = 4ab\int_0^{\pi/2}\tfrac{1}{2}(1 - \cos 2\theta)\,d\theta$$
$$= 2ab\left[\theta - \tfrac{1}{2}\sin 2\theta\right]_0^{\pi/2} = 2ab\left(\tfrac{\pi}{2}\right) = \pi ab$$