Calculus III: Homework Solutions

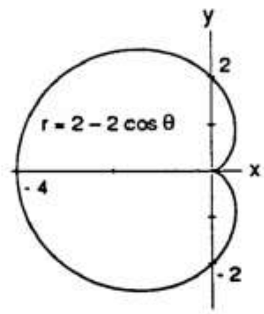
Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 10.6

Solutions to Even-Numbered Textbook Exercises:

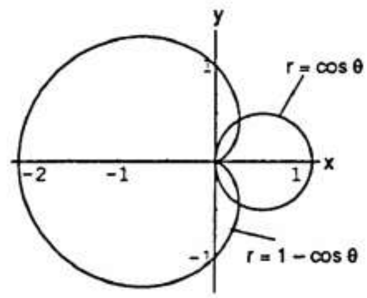
10.6 # 2 (textbook):

$2-2\cos(-\theta)=2-2\cos\theta=\mathrm{r} \Rightarrow$ symmetric about the x-axis; $2-2\cos(-\theta)\neq-\mathrm{r}$ and $2-2\cos(\pi-\theta)$ $=2+2\cos\theta\neq\mathrm{r} \Rightarrow$ not symmetric about the y-axis; therefore not symmetric about the origin



10.6 # 34 (textbook):

$\cos\theta=1-\cos\theta \Rightarrow 2\cos\theta=1 \Rightarrow \cos\theta=\frac{1}{2}$ $\Rightarrow \theta=\frac{\pi}{3},-\frac{\pi}{3} \Rightarrow \mathrm{r}=\frac{1}{2}$; points of intersection are $\left(\frac{1}{2},\frac{\pi}{3}\right)$ and $\left(\frac{1}{2},-\frac{\pi}{3}\right)$. The point $(0,0)$ is found by graphing.



Solutions to Supplemental Exercises:

10.6 # 15 (supplemental):

$$r=1/\theta \quad\Rightarrow\quad x=r\cos\theta=(\cos\theta)/\theta,\ y=r\sin\theta=(\sin\theta)/\theta \quad\Rightarrow$$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{\sin\theta\left(-1/\theta^2\right)+(1/\theta)\cos\theta}{\cos\theta\left(-1/\theta^2\right)-(1/\theta)\sin\theta}\cdot\frac{\theta^2}{\theta^2}=\frac{-\sin\theta+\theta\cos\theta}{-\cos\theta-\theta\sin\theta}=-\pi \text{ when } \theta=\pi$$

10.6 # 16 (supplemental):

$$\frac{dy}{d\theta}=e^\theta\sin\theta+e^\theta\cos\theta=e^\theta(\sin\theta+\cos\theta)=0 \quad\Rightarrow\quad \sin\theta=-\cos\theta \quad\Rightarrow\quad \tan\theta=-1 \quad\Rightarrow$$

$\theta=-\frac{1}{4}\pi+n\pi$ (n any integer) $\Rightarrow$ horizontal tangents at $\left(e^{\pi(n-1/4)},\pi\left(n-\frac{1}{4}\right)\right)$.

$$\frac{dx}{d\theta}=e^\theta\cos\theta-e^\theta\sin\theta=e^\theta(\cos\theta-\sin\theta)=0 \quad\Rightarrow\quad \sin\theta=\cos\theta \quad\Rightarrow\quad \tan\theta=1 \quad\Rightarrow\quad \theta=\frac{1}{4}\pi+n\pi$$

(n any integer) $\Rightarrow$ vertical tangents at $\left(e^{\pi(n+1/4)},\pi\left(n+\frac{1}{4}\right)\right)$.

10.6 # 17 (supplemental):

$dr/d\theta = (1/r)\cos 2\theta$ (by differentiating implicitly), so

$\dfrac{dy}{d\theta} = \dfrac{1}{r}\cos 2\theta \sin\theta + r\cos\theta = \dfrac{1}{r}\left(\cos 2\theta \sin\theta + r^2\cos\theta\right) = \dfrac{1}{r}\left(\cos 2\theta \sin\theta + \sin 2\theta \cos\theta\right) = \dfrac{1}{r}\sin 3\theta$. This is 0 when $\sin 3\theta = 0 \quad\Rightarrow\quad \theta = 0, \frac{\pi}{3}$ or $\frac{4\pi}{3}$ (restricting θ to the domain of the lemniscate), so there are horizontal tangents at $\left(\sqrt[4]{\frac{3}{4}}, \frac{\pi}{3}\right)$, $\left(\sqrt[4]{\frac{3}{4}}, \frac{4\pi}{3}\right)$ and $(0, 0)$. Similarly, $dx/d\theta = (1/r)\cos 3\theta = 0$ when $\theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$, so there are vertical tangents at $\left(\sqrt[4]{\frac{3}{4}}, \frac{\pi}{6}\right)$ and $\left(\sqrt[4]{\frac{3}{4}}, \frac{7\pi}{6}\right)$ [and $(0, 0)$].