<u>Modern Algebra I: Group Theory</u>

Classwork 3-12

Recall our (informal) definition of quotient group: A *quotient group* of a group G is a partition of G into (disjoint) subsets that forms a group under subset multiplication.

Task 1: After selecting a candidate for the identity, how do you figure out what the other subsets need to be?

(1) Suppose you want to use the subset $\{I, FR\}$ as the identity. Figure out which element would have to be paired with R. Use this strategy to fill out the rest of the table.

	$\{I, FR\}$			
$\{I, FR\}$	$\{I, FR\}$			
$\{R\ ?\}$				

Does this partition form a group under subset multiplication?

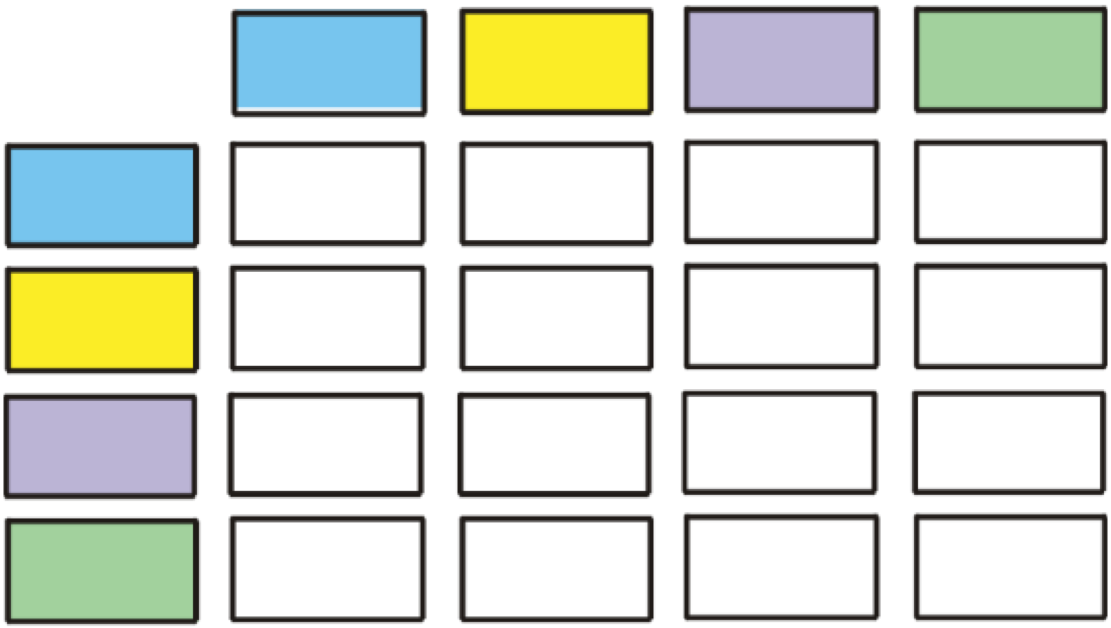
(2) Suppose you want to use the subset $\{I, R^2\}$ as the identity. Figure out which element would have to be paired with FR^2. Use this strategy to fill out the rest of the table.

	$\{I, R^2\}$			
$\{I, R^2\}$	$\{I, R^2\}$			
$\{FR^2\ ?\}$				

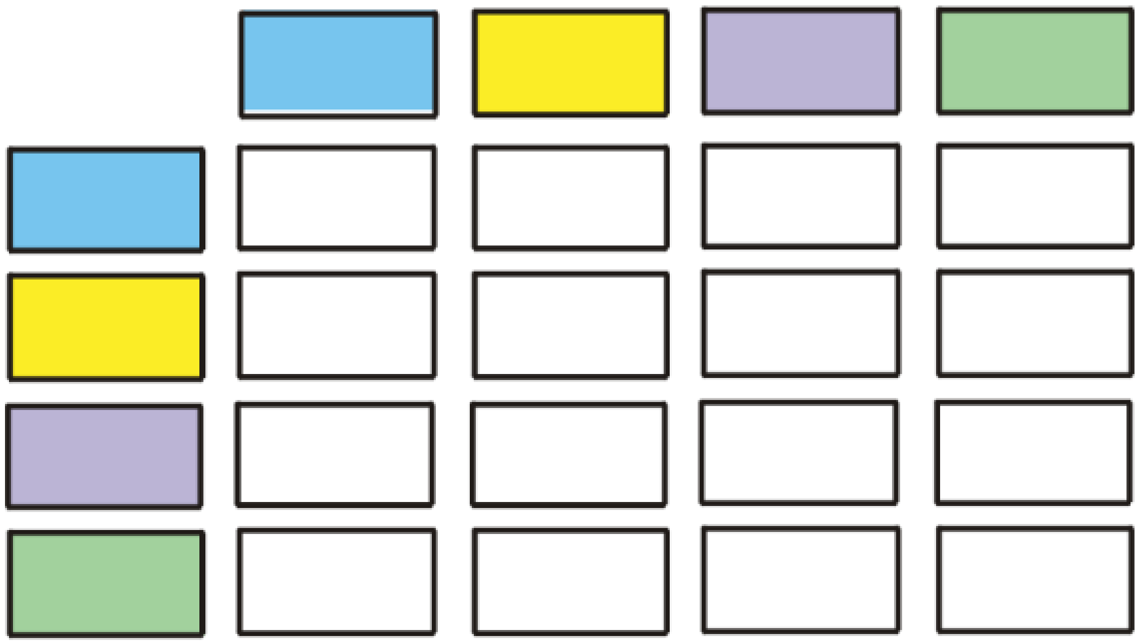
Does this partition form a group under subset multiplication?

Tasks 2-4 examine the conjecture that the identity element of a quotient group must be a subgroup:

Task 2: Does the identity element of a quotient group need to be a closed subset? Why? Think about this for the case of D_4 and then see if your reasoning makes sense for the general case (constructing quotient groups starting from any group). Can you prove your assertion? *Hint: see what happens if the identity is not a closed subgroup.*



Task 3: Does the identity element of a quotient group need to contain the identity of the original group? Why? Think about this for the case of D_4 and then see if your reasoning makes sense for the general case (constructing quotient groups starting from any group). Can you prove your assertion? *Hint: suppose that the identity of the original group is an element in the second subset instead.*



Task 4: Does the identity element of a quotient group need to contain the inverse of each of its elements? Why? Think about this for the case of D_4 and then see if your reasoning makes sense for the general case (constructing quotient groups starting from any group). Can you prove your assertion? *Hint: Suppose that g is in the identity subset (the first one), but its inverse g^{-1} is in the second subset.*