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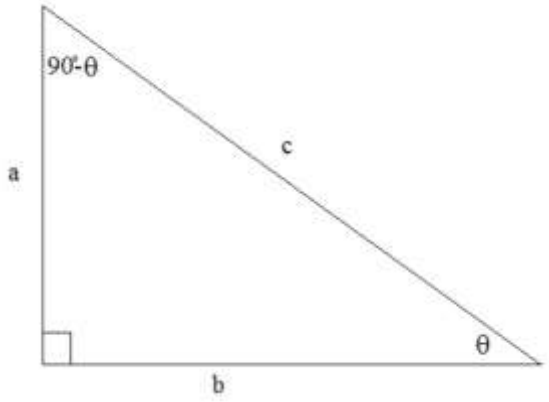
Mathematics 1613: Trigonometry Exam #1

1. This question involves deriving the sine and cosine of some basic angles.

   (1) *Derive* the values for the sine and cosine of 30° and 60°. Justify and explain your steps.

   (2) *Derive* the values for the sine and cosine of 45°. Justify and explain your steps.

2. Given the following right triangle,



   use the right triangle definitions of sine and cosine to find the indicated values:

   a. $\sin\theta$

   b. $\sin(90° - \theta)$

   c. $\cos\theta$

   d. $\cos(90° - \theta)$

   e. Based on this information, formulate a conjecture about the relationships between these values.

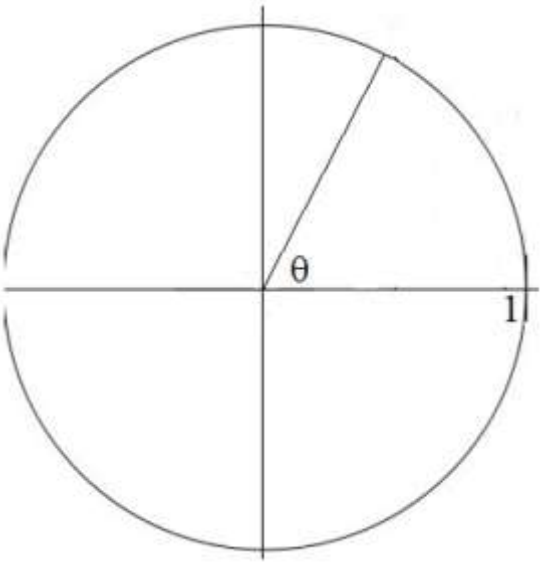
Name:_____

3. This question involves radian measure.

   (1) Explain what it means for an angle to be measured in radians.

   (2) Explain why converting from degrees to radians involves multiplying by $\frac{\pi}{180°}$:

4. Given the following angle on the unit circle, construct and label the lengths corresponding to the six basic trigonometric functions. (You do *not* need to justify, simply label.)



Using the diagram(s) above, derive the identities $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$ (that is, explain why they are true).

Name:_____

5. Find the following trigonometric values. Justify your answers by showing your work.

(1) $\cos\left(-\frac{7\pi}{6}\right)$

(2) $\sin\left(\frac{27\pi}{3}\right)$

(3) $\csc\left(\frac{5\pi}{2}\right)$

(4) $\sec\left(\frac{11\pi}{3}\right)$

(5) $\cot\left(-\frac{23\pi}{6}\right)$

(6) $\cos\left(-\frac{7\pi}{4}\right)$

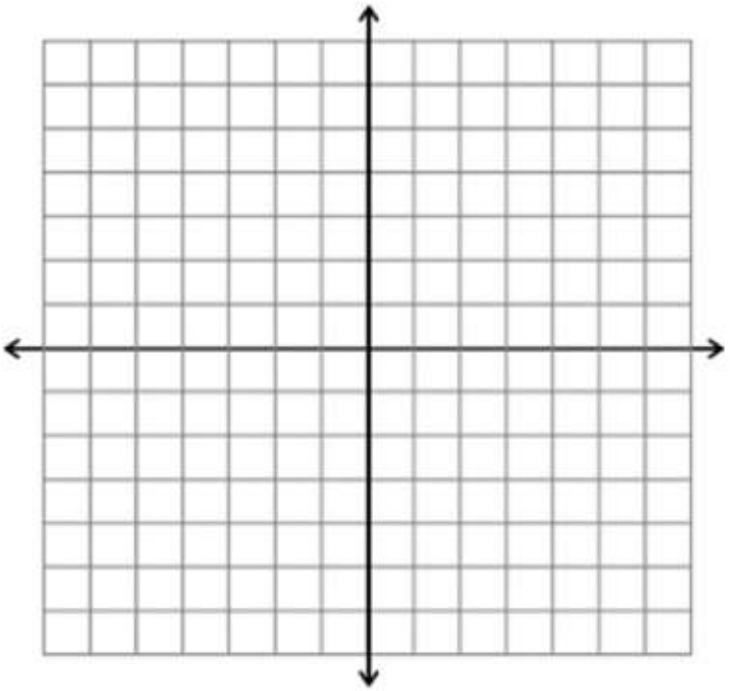
(7) $\tan\left(-\frac{25\pi}{2}\right)$

6. Using the even/odd properties of cosine and sine, prove that tangent is odd.

Name:_____

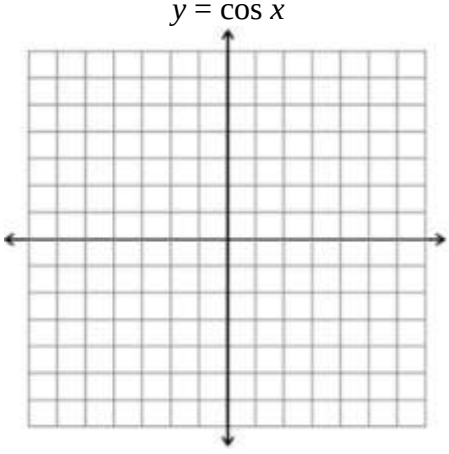
7. Use the given table to graph the functions $y = \csc x$ and $y = \sin x$ *on the same coordinate plane*. Use increments of .4 on the *y*-axis and $\frac{\pi}{6}$ on the *x*-axis. You may need to use the decimal approximations $\frac{\sqrt{3}}{2} \approx .87$ and $\frac{2}{\sqrt{3}} \approx 1.15$. If desired, you may use any relevant properties of these functions (though please mention and explain your work).

x	$y = \sin x$	$y = \csc x$
$-\pi$		
$-5\pi/6$		
$-2\pi/3$		
$-\pi/2$		
$-\pi/3$		
$-\pi/6$		
0		
$\pi/6$		
$\pi/3$		
$\pi/2$		
$2\pi/3$		
$5\pi/6$		
π		

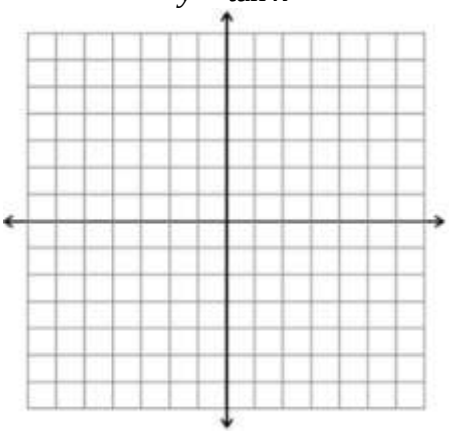


8. Provide *quick, accurate sketches* of the graphs for the given trigonometric functions. You may use any scale you wish, so long as it is consistent and clearly indicated.

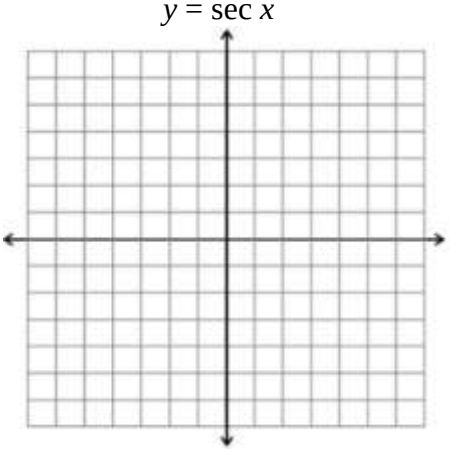
$y = \cos x$



$y = \tan x$



$y = \sec x$



Name:_____

9. Using the above graphs as a guide, fill in the following tables:

Function	Domain	Range
$y = \sin x$		
$y = \cos x$		
$y = \tan x$		
$y = \csc x$		
$y = \sec x$		
$y = \cot x$		

10. Define the term *period* of a function, and state the period of each of the basic trigonometric functions:

Sine

Cosine

Tangent

Cosecant

Secant

Cotangent

11. Solve the following trigonometric equations:

(1) $2\cos\theta + 1 = 0$

(2) $\tan^2\theta = 3$

12. Is $\frac{7\pi}{4}$ a solution to sin 4*x*=0? Justify your assertion without actually solving the equation.

Name:_____

13. Find the following values:

(1) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

(2) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

(3) $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

(4) $\arccos(0)$

(5) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$

14. Explain why $\arcsin\left(-\frac{1}{2}\right) \neq \frac{11\pi}{6}$ even though $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$.

15. Fill in the following table:

Function	Domain	Range
arcsin *x*		
arccos *x*		
arctan *x*		