

Modern Algebra I: Homework 1

due Tuesday, January 15th

1. Consider $f : \mathbb{R} \setminus \{0,1\} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x}$, and $g : \mathbb{R} \setminus \{0,1\} \rightarrow \mathbb{R}$ given by $g(x) = \frac{1}{1-x}$. How many possible functions can be created by composing these two functions (and what are they)? (There is no limit to how many times a function can be used, and functions may be composed with themselves.)
2. Consider the matrices $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. How many possible matrices can be created by multiplying these two matrices (and what are they)? (There is no limit to how many times a matrix can be used, and matrices may be multiplied with themselves.)
3. Is function composition commutative? Is matrix multiplication commutative? Justify your assertions.
4. How many permutations are there of the elements of $\{1, 2, 3\}$? How is this related to the symmetries of an equilateral triangle?