

Calculus III: Homework Solutions

Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 11.2

Solutions to Even-Numbered Textbook Exercises:

11.2 # 10 (textbook):

$$5 - \frac{5}{4} + \frac{5}{16} - \frac{5}{64} + \dots, \text{ the sum of this geometric series is } \frac{5}{1 - (-\frac{1}{4})} = 4$$

11.2 # 14 (textbook):

$$2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \dots = 2 \left(1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots \right); \text{ the sum of this geometric series is } 2 \left(\frac{1}{1 - (\frac{2}{5})} \right) = \frac{10}{3}$$

11.2 # 38 (textbook):

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \ln \left(\frac{n}{2n+1} \right) = \ln \left(\frac{1}{2} \right) \neq 0 \Rightarrow \text{diverges}$$

11.2 # 48 (textbook):

$$a = 1, r = \frac{3-x}{2}; \text{ converges to } \frac{1}{1 - (\frac{3-x}{2})} = \frac{2}{x-1} \text{ for } \left| \frac{3-x}{2} \right| < 1 \text{ or } 1 < x < 5$$

Solutions to Supplemental Exercises:

11.2 # 23 (supplemental):

$1 + 0.4 + 0.16 + 0.064 + \dots$ is a geometric series with ratio 0.4. The series converges to $\frac{a}{1-r} = \frac{1}{1-2/5} = \frac{5}{3}$ since $|r| = \frac{2}{5} < 1$.

11.2 # 24 (supplemental):

$$\sum_{n=0}^{\infty} 4 \left(\frac{4}{5} \right)^n \Rightarrow a = 4, |r| = \frac{4}{5} < 1, \text{ so the series converges to } \frac{4}{1 - 4/5} = 20.$$

11.2 # 25 (supplemental):

For $n = 1$, $a_1 = 0$ since $s_1 = 0$. For $n > 1$,

$$a_n = s_n - s_{n-1} = \frac{n-1}{n+1} - \frac{(n-1)-1}{(n-1)+1} = \frac{(n-1)n - (n+1)(n-2)}{(n+1)n} = \frac{2}{n(n+1)}$$

Also, $\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \frac{1 - 1/n}{1 + 1/n} = 1.$