Trigonometry Lesson 9: The Rest of the Addition and Subtraction Formulas
textbook section 8.2

Introduction: can we use what we learned in lesson 8 – that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ – to develop a similar formula for sine? What machinery (identities) do we have that allow us to switch between sine and cosine? Which one is preferable in this situation?

1: Is there a way that we can use these subtraction formulas to figure out the sine and cosine of a *sum* as well (this was touched on briefly in the previous lesson as well)? For example, how can we modify and use what we know already to evaluate $\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$ and $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$?

2: Use the same method as in part 1 to find general formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$:

3: Use the addition and subtraction formulas for sine and cosine to evaluate the following (this demonstrates that we don't actually need to derive and remember a similar formula for tangent, though many textbooks do):

$\tan\left(\frac{\pi}{12}\right)$

$\tan\left(\frac{7\pi}{12}\right)$

4: Given that $\cos\alpha = \frac{5}{13}$ and $\tan\beta = -2$, with $0 \le \alpha < \frac{\pi}{2}$ and $\frac{\pi}{2} \le \beta < \pi$, find $\tan(\alpha + \beta)$.

5: Use the addition and subtraction formulas to simplify the following expressions (the first two are known as the *double angle formulas*, which will feature prominently in the next section; the latter three exercises prove certain repetitive lengths for sine, cosine, and tangent in a different way than we had previously):

$\sin(2\theta)$

$\cos(2\theta)$

$\tan(\theta + \pi)$

$\sin(\theta + 2\pi)$

$\cos(\theta + 2\pi)$

Section 8.2 Recommended Homework: 7-21 odd, 35, 37