Calculus III: Homework Solutions

Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 10.7

Solutions to Even-Numbered Textbook Exercises:

10.7 # 8 (textbook):

$\mathrm{r} = 1$ and $\mathrm{r} = 2\sin\theta \Rightarrow 2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$; therefore

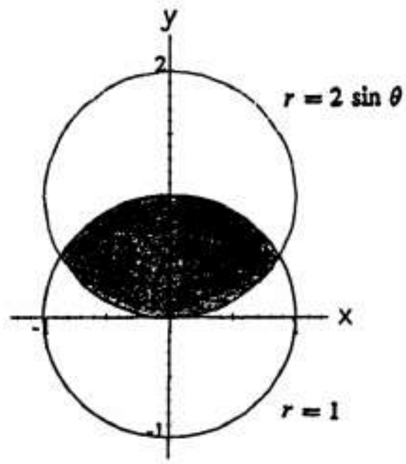
$$\mathrm{A} = \pi(1)^2 - \int_{\pi/6}^{5\pi/6} \tfrac{1}{2}\left[(2\sin\theta)^2 - 1^2\right] d\theta$$

$$= \pi - \int_{\pi/6}^{5\pi/6} \left(2\sin^2\theta - \tfrac{1}{2}\right) d\theta$$

$$= \pi - \int_{\pi/6}^{5\pi/6} \left(1 - \cos 2\theta - \tfrac{1}{2}\right) d\theta$$

$$= \pi - \int_{\pi/6}^{5\pi/6} \left(\tfrac{1}{2} - \cos 2\theta\right) d\theta = \pi - \left[\tfrac{1}{2}\theta - \tfrac{\sin 2\theta}{2}\right]_{\pi/6}^{5\pi/6}$$

$$= \pi - \left(\tfrac{5\pi}{12} - \tfrac{1}{2}\sin\tfrac{5\pi}{3}\right) + \left(\tfrac{\pi}{12} - \tfrac{1}{2}\sin\tfrac{\pi}{3}\right) = \frac{4\pi - 3\sqrt{3}}{6}$$



10.7 # 20 (textbook):

$$\mathrm{r} = \frac{e^{\theta}}{\sqrt{2}},\ 0 \le \theta \le \pi \Rightarrow \frac{dr}{d\theta} = \frac{e^{\theta}}{\sqrt{2}};\ \text{therefore Length} = \int_0^{\pi} \sqrt{\left(\frac{e^{\theta}}{\sqrt{2}}\right)^2 + \left(\frac{e^{\theta}}{\sqrt{2}}\right)^2}\, d\theta = \int_0^{\pi} \sqrt{2\left(\frac{e^{2\theta}}{2}\right)}\, d\theta$$

$$= \int_0^{\pi} e^{\theta}\, d\theta = \left[e^{\theta}\right]_0^{\pi} = e^{\pi} - 1$$

10.7 # 30 (textbook):

$$\mathrm{r} = \sqrt{2}e^{\theta/2},\ 0 \le \theta \le \frac{\pi}{2} \Rightarrow \frac{dr}{d\theta} = \sqrt{2}\left(\tfrac{1}{2}\right)e^{\theta/2} = \frac{\sqrt{2}}{2}e^{\theta/2};\ \text{therefore Surface Area}$$

$$= \int_0^{\pi/2} \left(2\pi\sqrt{2}e^{\theta/2}\right)(\sin\theta)\sqrt{\left(\sqrt{2}e^{\theta/2}\right)^2 + \left(\frac{\sqrt{2}}{2}e^{\theta/2}\right)^2}\, d\theta = \int_0^{\pi/2} \left(2\pi\sqrt{2}e^{\theta/2}\right)(\sin\theta)\sqrt{2e^{\theta} + \tfrac{1}{2}e^{\theta}}\, d\theta$$

$$= \int_0^{\pi/2} \left(2\pi\sqrt{2}e^{\theta/2}\right)(\sin\theta)\sqrt{\tfrac{5}{2}e^{\theta}}\, d\theta = \int_0^{\pi/2} \left(2\pi\sqrt{2}e^{\theta/2}\right)(\sin\theta)\left(\frac{\sqrt{5}}{\sqrt{2}}e^{\theta/2}\right) d\theta = 2\pi\sqrt{5}\int_0^{\pi/2} e^{\theta}\sin\theta\, d\theta$$

$$= 2\pi\sqrt{5}\left[\frac{e^{\theta}}{2}(\sin\theta - \cos\theta)\right]_0^{\pi/2} = \pi\sqrt{5}\left(e^{\pi/2} + 1\right) \text{ where we integrated by parts}$$

Solutions to Supplemental Exercises:

10.7 # 18 (supplemental):

$r = 4 + 3\sin\theta, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$$A = \int_{-\pi/2}^{\pi/2} \frac{1}{2}\left(4 + 3\sin\theta\right)^2 \, d\theta = \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(16 + 24\sin\theta + 9\sin^2\theta\right) d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} \left(16 + 9\sin^2\theta\right) d\theta$$

$$= \frac{1}{2} \cdot 2\int_{0}^{\pi/2} \left[16 + 9 \cdot \frac{1}{2}\left(1 - \cos 2\theta\right)\right] d\theta$$

$$= \int_{0}^{\pi/2} \left(\frac{41}{2} - \frac{9}{2}\cos 2\theta\right) d\theta = \left[\frac{41}{2}\theta - \frac{9}{4}\sin 2\theta\right]_0^{\pi/2} = \left(\frac{41\pi}{4} - 0\right) - (0 - 0) = \frac{41\pi}{4}$$

10.7 # 19 (supplemental):

$r = \sin 4\theta, 0 \le \theta \le \frac{\pi}{4}$. $A = \int_0^{\pi/4} \frac{1}{2}\sin^2 4\theta \, d\theta = \int_0^{\pi/4} \frac{1}{4}\left(1 - \cos 8\theta\right) d\theta = \left[\frac{1}{4}\theta - \frac{1}{32}\sin 8\theta\right]_0^{\pi/4} = \frac{\pi}{16}$