<u>Modern Algebra I: Class Activities</u>

Tuesday, February 12th

Task 1

Consider this conjecture:

"Any closed subset of a group is a subgroup."

Prove or disprove this conjecture.

Task 2

Consider the following "proof" of the conjecture that closure is sufficient:

"Because the subset is closed, only elements from the subset appear in a row of the table. Because the entire group table has the Sudoku property, any closed subset does as well. So each element of the subset must appear in any row. Since an element a must appear in the row for symmetry a, the identity must be in the subset. Then since the identity must appear in the row for a, the inverse of a must be in the subset. Finally, since associativity holds for the whole group it must hold for the subset. Therefore the subset is a group and hence a subgroup."

Is this a valid argument? If not, are there cases where this proof works? Can this be used to make a new subgroup theorem?

Task 3

Could this be an operation table for the symmetries of an equilateral triangle?

The *Mystery* Group

*	**A**	**B**	**C**	**D**	**E**	**G**
A	B	A	D	C	G	E
B	A	B	C	D	E	G
C	G	C	B	E	D	A
D	E	D	A	G	C	B
E	D	E	G	A	B	C
G	C	G	E	B	A	D

Task 4

Why doesn't the correspondence

A ↔ F, B ↔ I, C ↔ FR^2, D ↔ R, E ↔ FR, G ↔ R^2

work for showing the mystery group is really D_3?

Task 5
Suppose that I wanted to match up the elements by saying that $C \leftrightarrow F$ and $G \leftrightarrow R$. What would the other four correspondences have to be?

Task 6
We use the term *isomorphic* to express the idea that two groups are essentially the same (equivalent but not necessarily equal). So the group given by the mystery table is isomorphic to $\boldsymbol{D_6}$. Write a definition for *isomorphic*.

Definition: Let $(\boldsymbol{G}, \bullet)$ and $(\boldsymbol{H}, *)$ be groups. $\boldsymbol{G}$ is *isomorphic* to $\boldsymbol{H}$ if…