Modern Algebra I: Homework 3

due Thursday, January 24th

9. Recall from a previous homework that the set $\{a, b, c\}$ has six permutations (that is, six bijective functions from the set to itself). Explicitly describe each function (use a function diagram instead of a formula!). The set of these permutations is called the *symmetric group on three elements*, or S_3.
    a. Prove that *in general* the composition of two bijective functions is also a bijective function.
    b. Construct an operation table that shows the result of composing any pair of these functions.
    c. How does this table compare to the table of symmetries of an equilateral triangle? Justify your response.
    d. Construct an operation table for the set of functions generated by $f : R \setminus \{0,1\} \to R$ given by $f(x) = \dfrac{1}{x}$, and $g : R \setminus \{0,1\} \to R$ given by $g(x) = \dfrac{1}{1-x}$ (which you worked on a previous homework). How does this table compare to the symmetries of an equilateral triangle? To the symmetric group on three elements?

10. Consider the symmetries of a square (ALL of them this time). How many are there? The set all symmetries of a square is called the *dihedral group of a square*, denoted by D_4.
    a. Just as for the symmetries of an equilateral triangle, write each symmetry in terms of a flip across a vertical axis of symmetry F, and a 90° clockwise rotation R.
    b. Make a table that shows the result of combining any two of these symmetries. (Hint: Try to develop a set of rules like those for the triangle and fill in the table by calculating. Make sure that closure is apparent in your table as well.)
    c. How does this table compare to the composition table for the 1-1 onto functions from the set $\{a, b, c, d\}$ to itself? *Hint: You should not have to make a table or even find all of the functions to answer this question*. Justify your response.
    d. How does this set with composition compare to the set of matrices generated by $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (which you worked on a previous homework)? To the symmetries of a square? Justify your response.