Calculus III: Homework Solutions

Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 6.3

Solutions to Even-Numbered Textbook Exercises:

6.3 # 2 (textbook): 4

6.3 # 6 (textbook): π^2

Section 6.5

Solutions to Even-Numbered Textbook Exercises:

6.5 # 34 (textbook): $\dfrac{28\pi}{9}$

6.5 # 36 (textbook):

$$\frac{dx}{dt} = a(1-\cos t) \text{ and } \frac{dy}{dt} = a\sin t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{[\,a(1-\cos t)]^2 + (a\sin t)^2}$$
$$= \sqrt{a^2 - 2a^2\cos t + a^2\cos^2 t + a^2\sin^2 t} = \sqrt{2a^2 - 2a^2\cos t} = a\sqrt{2}\sqrt{1-\cos t} \Rightarrow S = \int 2\pi y\, ds$$
$$= \int_0^{2\pi} 2\pi\, a(1-\cos t)\cdot a\sqrt{2}\sqrt{1-\cos t}\, dt = 2\sqrt{2}\,\pi\, a^2 \int_0^{2\pi} (1-\cos t)^{3/2}\, dt$$

Solutions to Supplemental Exercises:

6.5 # 9 (supplemental):

$x = t\sin t$, $y = t\cos t$, $0 \le t \le \frac{\pi}{2}$. $dx/dt = t\cos t + \sin t$ and $dy/dt = t(-\sin t) + \cos t$, so

$$\begin{aligned}(dx/dt)^2 + (dy/dt)^2 &= (t\cos t + \sin t)^2 + (\cos t - t\sin t)^2 \\ &= t^2\cos^2 t + 2t\sin t\cos t + \sin^2 t + \cos^2 t - 2t\sin t\cos t + t^2\sin^2 t \\ &= t^2\left(\cos^2 t + \sin^2 t\right) + \sin^2 t + \cos^2 t = t^2 + 1\end{aligned}$$

and $L = \int_0^{\pi/2} \sqrt{t^2+1}\, dt$.

6.5 # 10 (supplemental):

$x = \cos^2 t,\ y = \cos t,\ 0 \le t \le 4\pi.\ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = (-2\cos t \sin t)^2 + (-\sin t)^2 = \sin^2 t\left(4\cos^2 t + 1\right)$

$$
\begin{aligned}
\text{Distance } &= \int_0^{4\pi} |\sin t|\,\sqrt{4\cos^2 t + 1}\,dt = 4\int_0^{\pi} \sin t\sqrt{4\cos^2 t + 1}\,dt \\
&= -4\int_1^{-1} \sqrt{4u^2+1}\,du\ \ [u = \cos t,\ du = -\sin t\,dt] = 4\int_{-1}^{1} \sqrt{4u^2+1}\,du = 8\int_0^1 \sqrt{4u^2+1}\,du \\
&= 8\int_0^{\tan^{-1} 2} \sec\theta\,\tfrac{1}{2}\sec^2\theta\,d\theta = 4\int_0^{\tan^{-1} 2} \sec^3\theta\,d\theta \overset{71}{=} \left[2\sec\theta\tan\theta + 2\ln|\sec\theta + \tan\theta|\right]_0^{\tan^{-1} 2} \\
&= 4\sqrt{5} + 2\ln\left(\sqrt{5} + 2\right)
\end{aligned}
$$

$L = \int_0^{\pi} |\sin t|\,\sqrt{4\cos^2 t + 1}\,dt = \sqrt{5} + \frac{1}{2}\ln\left(\sqrt{5} + 2\right)$

6.5 # 11 (supplemental):

$x = e^t - t,\ y = 4e^{t/2},\ 0 \le t \le 1.\ \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(e^t - 1\right)^2 + \left(2e^{t/2}\right)^2 = e^{2t} + 2e^t + 1 = \left(e^t + 1\right)^2.$

$$
\begin{aligned}
S &= \int_0^1 2\pi\left(e^t - t\right)\sqrt{\left(e^t - 1\right)^2 + \left(2e^{t/2}\right)^2}\,dt = \int_0^1 2\pi\left(e^t - t\right)\left(e^t + 1\right)dt \\
&= 2\pi\left[\tfrac{1}{2}e^{2t} + e^t - (t-1)\,e^t - \tfrac{1}{2}t^2\right]_0^1 = \pi\left(e^2 + 2e - 6\right)
\end{aligned}
$$