Mathematics 2243: Calculus and Analytic Geometry III Name:______________________________
Examination II
February 22, 2013

I. For the series $\sum_{n=1}^{\infty} a_n$: [4 points]

1. Define the nth partial sum s_n .

2. Define what it means to say that the series *converges*.

II. Using Riemann sums, prove the divergence of the harmonic series: [4 points]

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

Include a sketch in your proof.

III. The Alternating Series Test does not apply to any of the following series. For each of the series, tell why it does not apply. [6 points]

1. $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^2}{n^2}$

2. $\sum_{n=1}^{\infty} (-1)^n b_n$ where $b_n = \frac{1}{n}$ if n is even, and $b_n = \frac{1}{n^2}$ if n is odd.

IV. Verify that the series [6 points]

$$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$$

converges, using the following two methods:

1. By using the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$:

2. By using the comparison test with a direct comparison to the terms of $\sum_{n=1}^{\infty} \frac{1}{n^2}$:

V. For each of the following power series, determine the convergence behavior. That is, find the interval of convergence and determine where the convergence is conditional and where it is absolute. Follow any special instructions given. [15 points]

1. $\sum_{n=2}^{\infty} \frac{x^n}{(\ln(n))^n}$ (Use the Root Test.)

2. $\sum_{n=2}^{\infty} (\ln n)^n x^n$

3. $\sum_{n=1}^{\infty} \frac{(x-1)^n}{\sqrt{n}}$ (Use the Ratio Test.)

VI. Suppose that a function $f(x)$ can be written as $c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$ for all x such that $0 < |x| < R$.

[5 points]

1. Show that $c_0 = f(0)$.

2. Use $f'(x)$ to find c_1.

3. Use $f''(x)$ to find c_2.

4. Use $f'''(x)$ to find c_3.

5. Use $f^{(4)}(x)$ to find c_4.

6. Use the previous 5 steps to ascertain a formula for c_n.

VII. For each of the following functions, write the Maclaurin series both as a summation and as an infinite list of terms. For example, $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ [4 points]

1. e^x

2. $\cos x$

VIII. Write the general formula for the Taylor series of $f(x)$ at $x = a$. Use the formula to calculate the Taylor series of $\frac{1}{x}$ at a=1. [7 points]

IX. Show that if $0 < b_n < \frac{1}{n}$ for all n, then $\sum_{n=1}^{\infty} \frac{b_n}{n}$ converges. [4 points]