Mathematics 2243: Calculus and Analytic Geometry III Name:______________________________
Examination 1
February 1, 2013

I. Consider the polar equation $r = 1 + \cos\theta$. [5 points]

1. In a (θ, r) coordinate system, sketch the graph of the equation $r = 1 + \cos\theta$.

2. In an (x, y) coordinate system, sketch the graph of the polar equation $r = 1 + \cos\theta$.

II. Using $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$, calculate a general formula for $\frac{dy}{dx}$ for a polar curve in which r is a function of θ. [4 points]

III. For the parametric equations $x = e^t, y = -e^{2t}$, solve for t in terms of *x* and *y* to eliminate the parameter t (that is, to obtain an *xy*-equation). Graph the equation in an *(x,y)* coordinate system, indicating the orientation as t ranges from $-\infty$ to ∞. (Hint: you may use the fact that $\ln\left(a^b\right) = b\ln a$.) [6 points]

IV. On the ellipse $x = \cos t,\ y = \sqrt{3}\sin t$, there are two points where the tangent line has slope 1. [6 points]

1. Calculate an expression for $\frac{dy}{dx}$ in terms of t.

2. Find a value of t between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ where the tangent line has slope 1.

3. Find the exact x and y coordinates where that tangent line occurs on the ellipse.

V. Calculate, if possible, the exact value of the given series. [4 points]

$$\sum_{n=0}^{\infty} \frac{1}{e^{2n}}$$

VI. Consider the line segment $x = t, y = 1 - t, 0 \leq t \leq 1$. It runs from $(0, 1)$ to $(1, 0)$. [6 points]

1. Use *ds* and an integral to verify that the length of this segment is $\sqrt{2}$.

2. Sketch the cone obtained by rotating this line segment about the *x* axis. Use *ds* and an integral to calculate the surface area of this cone.

VII. Consider the line segment $x + y = 1, 0 \leq x \leq 1$. [5 points]

1. Change the equation $x + y = 1$ to polar coordinates, and solve for *r* in terms of θ.

2. Write an integral in terms of θ whose value is the area of the triangle bounded by the line segment and the *x* and *y* axes. Supply the limits of integration, but do *not* evaluate the integral.

VIII. Give a precise mathematical definition of what it means to say: [up to 2 bonus points]

1. The sequence a_n converges to *A*.

2. The series $\sum_{n=0}^{\infty} a_n$ converges to *L*.

IX. Determine if the given sequence converges (and, if so, find its limit). If the sequence diverges, determine if it diverges to $+\infty, -\infty$, or neither. Give some explanation of indication of your reasoning. In particular, when the series is one of the basic types considered in class, such as n^p or r^n, note this and use your knowledge of the behavior of these sequences to determine the limit. [10 points]

1. $a_n = \dfrac{\sqrt{n}}{n+1}$

2. $b_n = \dfrac{1}{(.999999999999739)^n}$

3. $c_n = \dfrac{\sqrt{n}}{1+\sqrt{n}}$

4. $x_n = n^{-0.000000000001}$

5. $y_n = \tan^{-1} n$

X. Explain how one knows immediately that the following series diverges: [4 points + 1 bonus]

$$\sum_{n=1}^{\infty} \frac{n^2}{2n+3n^2}$$

True/false: If $\lim_{n\to\infty} a_n = 0$ for some infinite series $\sum_{n=0}^{\infty} a_n$, does this imply convergence? Explain or give a counterexample.