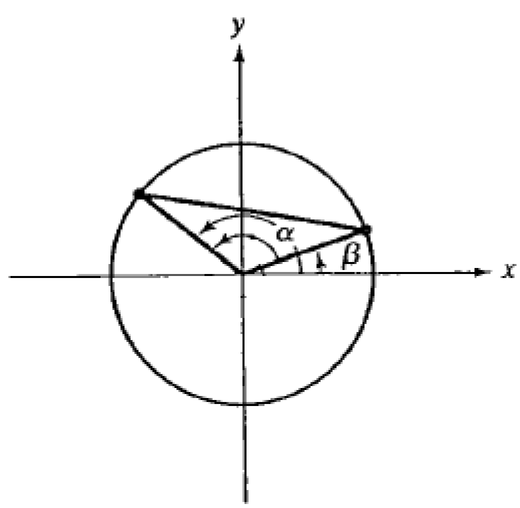
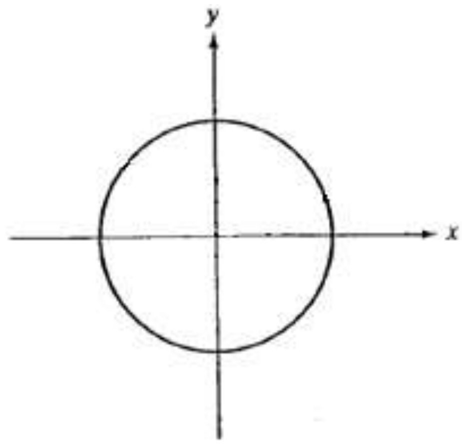
Trigonometry Lesson 8: The Cosine Subtraction Formula
textbook section 8.2

Introduction: how can we use what we already know about the angles on the unit circle to "fill in the gaps"? For example, is there a way that we can find the cosine of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ (in degrees: 15˚=60˚-45˚)? Does $\cos(\alpha - \beta) = \cos(\alpha) - \cos(\beta)$?

1: Label the coordinates in the following diagram. What is the angle between α and β?



2: Rotate the angle $\alpha - \beta$ so that it is in standard position, and label its new coordinates. Despite its new coordinates, did this rotation change the length of the line segment? Why or why not?



3: Using the distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (along with both unit circle diagrams above), find two different expressions for the length of the line segment above.

4: Based on the fact that a rotation is an *isometry*, we may assume that the distance between the two points on the unit circle did not change after rotating $\alpha - \beta$ into standard position. Set both expressions equal to each to solve for a trigonometric function of the angle $\alpha - \beta$.

5: Use the formula you derived above to find the following trigonometric values:

$$\cos\frac{\pi}{12}$$

$$\cos\frac{5\pi}{12}$$

$$\cos\frac{7\pi}{12}$$

7: Given that $\sin(\alpha) = -\frac{4}{5}$ and $\cos(\beta) = \frac{12}{13}$, find $\cos(\alpha - \beta)$.

8: Use the cosine subtraction formula to evaluate and simplify the following expressions:

$\cos(\theta + 2\pi)$

$\cos\left(\frac{\pi}{2} - \alpha\right)$

Recommended Homework: 7, 11, 17