Name: Solutions

Mathematics 1613: Trigonometry Quiz #11

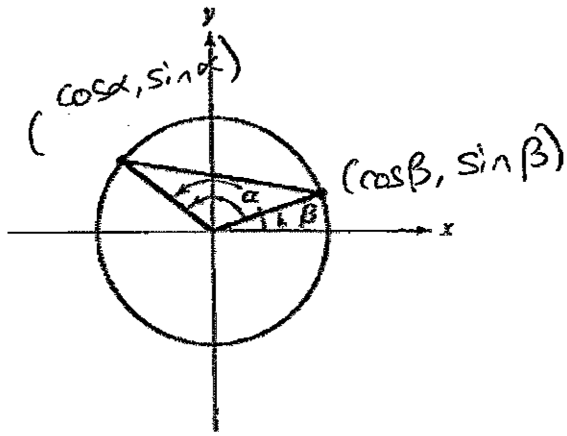
32: Does $\cos(\alpha - \beta) = \cos\alpha - \cos\beta$? Give an example to support your assertion.

No!

$$\cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) \overset{?}{\neq} \cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = 0$$

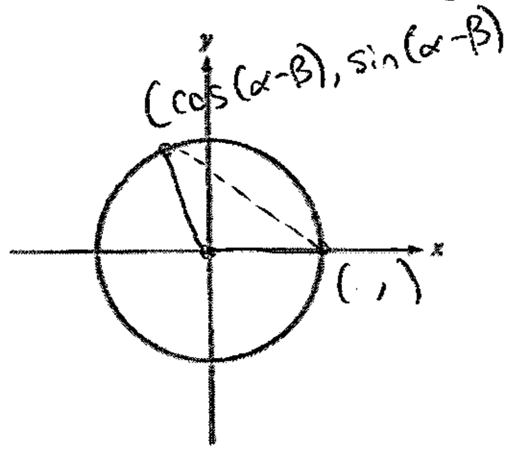
$$= \cos(0) = 1$$

33: Label the coordinates in the following diagram. What is the angle between α and β? $\alpha - \beta$



34: Rotate the angle $\alpha - \beta$ so that it is in standard position, and label its new coordinates. Despite its new coordinates, did this rotation change the length of the line segment? Why or why not?

→ no; rotation does not change distance



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35: Find two different expressions for the length of the line segment above. Simplify each one as much as possible, explaining your steps.

distance formula

$$\boxed{\#1}\ \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{(\cos\alpha-\cos\beta)^2+(\sin\alpha-\sin\beta)^2}$$

$$= \sqrt{\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta}$$

pythagorean identities $=1$ $=1$

$$= \sqrt{2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}$$

$$\boxed{\#2}\ \sqrt{(\cos(\alpha-\beta)-1)^2+(\sin(\alpha-\beta)-0)^2} = \sqrt{\underbrace{\cos^2(\alpha-\beta) - 2\cos(\alpha-\beta) + 1 + \sin^2(\alpha-\beta)}_{\cos^2(\alpha-\beta)+\sin^2(\alpha-\beta)=1}}$$

$$= \sqrt{2 - 2\cos(\alpha-\beta)}$$

36: Use the two expressions to find a trigonometric angle subtraction formula. Explain your reasoning.

distance #1 = distance #2

$$\sqrt{2 - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta} = \sqrt{2 - 2\cos(\alpha-\beta)} \quad \text{(square both sides)}$$

$$\cancel{2} - 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta = \cancel{2} - 2\cos(\alpha-\beta) \quad \text{(cancel 2s)}$$

$$\frac{-2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta}{-2} = \frac{-2\cos(\alpha-\beta)}{-2}$$

$$\therefore \boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$