Calculus III: Homework Solutions

Answers to odd-numbered textbook exercises can be found in the back of the textbook (and consequently are not included here).

Section 10.5

Solutions to Even-Numbered Textbook Exercises:

10.5 # 28 (textbook):

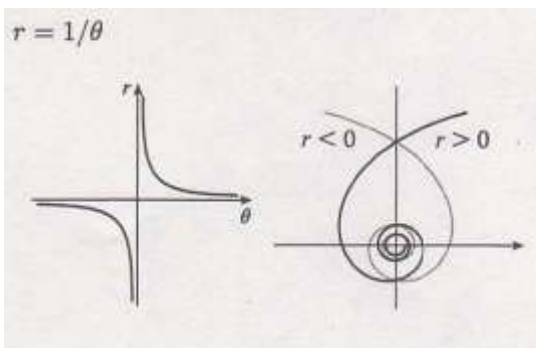
$$r = -3\sec\theta \Rightarrow r = \frac{-3}{\cos\theta} \Rightarrow r\cos\theta = -3 \Rightarrow x = -3\text{, a vertical line through } (-3, 0)$$

10.5 # 34 (textbook):

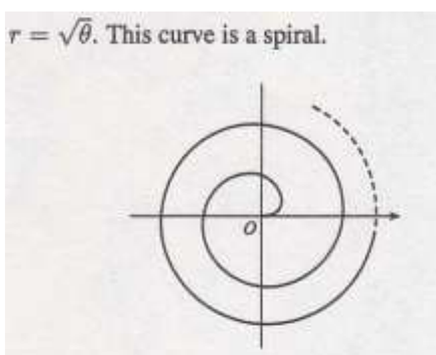
$$r^2 \sin 2\theta = 2 \Rightarrow 2r^2 \sin\theta\cos\theta = 2 \Rightarrow (r\sin\theta)(r\cos\theta) = 1 \Rightarrow xy = 1\text{, hyperbola with focal axis } y = x$$

Solutions to Supplemental Exercises:

10.5 # 12 (supplemental):



10.5 # 13 (supplemental):



10.5 # 14 (supplemental):

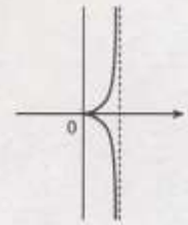
To show that $x = 1$ is an asymptote we must prove $\lim_{r \to \pm\infty} x = 1$.

$x = (r)\cos\theta = (\sin\theta\tan\theta)\cos\theta = \sin^2\theta$. Now, $r \to \infty \Rightarrow$

$\sin\theta\tan\theta \to \infty \Rightarrow \theta \to \left(\frac{\pi}{2}\right)^-$, so $\lim_{r \to \infty} x = \lim_{\theta \to \pi/2^-} \sin^2\theta = 1$. Also,

$r \to -\infty \Rightarrow \sin\theta\tan\theta \to -\infty \Rightarrow \theta \to \left(\frac{\pi}{2}\right)^+$, so

$\lim_{r \to -\infty} x = \lim_{\theta \to \pi/2^+} \sin^2\theta = 1$.



Therefore, $\lim_{r \to \pm\infty} x = 1 \Rightarrow x = 1$ is a vertical asymptote. Also notice that $x = \sin^2\theta \geq 0$ for all θ, and $x = \sin^2\theta \leq 1$ for all θ. And $x \neq 1$, since the curve is not defined at odd multiples of $\frac{\pi}{2}$. Therefore, the curve lies entirely within the vertical strip $0 \leq x < 1$.