<u>Modern Algebra I: Class Activities</u>

Thursday, February 28th

Task 1

Definition: A *cyclic group* is a group that is comprised completely of powers of one element, i.e. there exists a $g \in G$ such that $G = \{g^n : n \in \mathbb{Z}\}$. If G is an additive group, this becomes $G = \{ng : n \in \mathbb{Z}\}$. The element g is called the *generator* and the shorthand for "G is generated by g" is $G = \langle g \rangle$.

Determine if the following groups are cyclic (and, if so, state all possible generators):

$$(\mathbb{Z}_4, +_4), (\mathbb{Z}_4^\times, \cdot_4), (\mathbb{Z}_6, +_6), (\mathbb{Z}_6^\times, \cdot_6), (\mathbb{Z}, +)$$

Task 2

Suppose $(G, \bullet)$ and $(H, *)$ are groups and $\varphi: G \to H$ is an isomorphism. If G is cyclic, then H is cyclic.

Task 3

If $\varphi: G \rightarrow H$ is an isomorphism then $\varphi^{-1}: H \rightarrow G$ is an isomorphism.

Task 4

Recall the Even – Odd group:

+	EVEN	ODD
EVEN	EVEN	ODD
ODD	ODD	EVEN

How does it make sense for this to be a group?

- How many elements does it have?
- What are the elements?
- What exactly does it mean to add these elements together?

Task 5

Agree, as a class, on a standardized operation table for D_8.

$\circ$	e	R	R^2	R^3				
e								
R								
R^2								
R^3								