Calculus III: Supplemental Homework Exercises

Section 3.5:

Directions: For #s 1-4, Eliminate the parameter to find a Cartesian equation of the curve. (b) Sketch the curve and indicate with an arrow its direction

(1) $x = \sin\theta$, $y = \cos\theta$ $\quad 0 \le \theta \le \pi$

(2) $x = \sin^2\theta$, $y = \cos^2\theta$

(3) $x = e^t$, $y = e^{-t}$

(4) $x = \cosh t$, $y = \sinh t$

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(5) Show that the curve $x = \cos t$, $y = \sin t \cos t$ have two tangent lines at $(0,0)$ and find their equations. Sketch the curve.

(6) At what points on the curve $x = t^3 + 4t$, $y = 6t^2$ is the tangent parallel to the line with equations $x = -7t$, $y = 12t - 5$.

(7) Find the area bounded by the curve $x = \cos t$, $y = e^t$, $0 \le t \le \frac{\pi}{2}$, and the lines $y = 1$ and $x = 0$.

(8) Use the parametric equations of an ellipse, $x = a\cos\theta$, $y = b\sin\theta$, $0 \le \theta \le 2\pi$, to find the area it encloses.

Sections 6.3, 6.5

(9) Setup, but do not evaluate, an integral that represents the length of the curve $x = t\sin t$, $y = t\cos t$, $0 \le t \le \frac{\pi}{2}$.

(10) Find the distance travelled by a particle with position (x,y) as t varies in the given time interval:

$$x = \cos^2 t, \quad y = \cos t, \quad 0 \le t \le 4\pi$$

(11) Find the surface area generated by rotating the given curve about the y-axis: $x = e^t - t$, $y = 4e^{t/2}$, $0 \le t \le 1$

Sections 10.5, 10.6

Sketch the curve with the given equation in the r-θ plane and the xy plane.

(12) $r = 1/\theta$ (13) $r = \sqrt{\theta}$

(14) Show that the curve $r = \sin\theta\tan\theta$ (called a cissoid of Diocles) has the line $x = 1$ as a vertical asymptote. Show also that the curve lies entirely within the vertical strip $0 \le x < 1$. Use this to help sketch the curve.

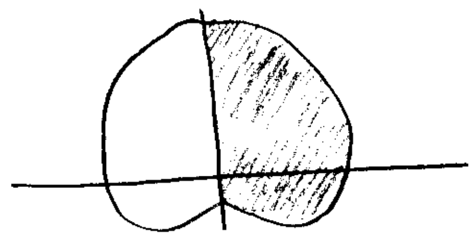
(15) Find the slope of the tangent line to the polar curve $r = 1/\theta$ at $\theta = \pi$.

Find the points on the given curve where the tangent is horizontal or vertical:

(16) $r = e^{\theta}$ (17) $r^2 = \sin 2\theta$

Section 10.7

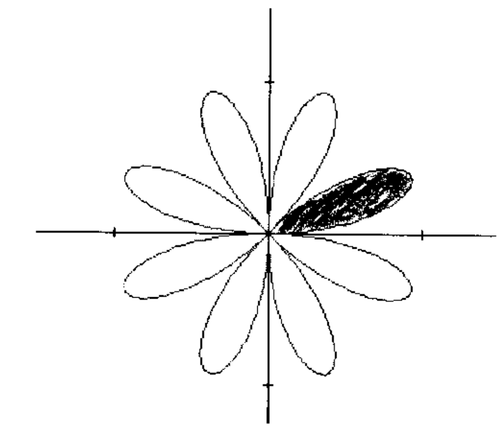
(18) Find the area of the shaded region:



$r = 4 + 3\sin\theta$

(19) Find the area of the shaded region:

$r = \sin(4\theta)$



Section 11.1

Determine whether the sequence converges or diverges. If it converges, find the limit.

(20) $a_n = \dfrac{n!}{(n+2)!}$

(21) $a_n = \dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2} + \dots + \dfrac{n}{n^2}$

(it may help to write out $a_1, a_2, a_3, a_4, \dots$)

(22) Determine if $a_n = \cos\left(\frac{n\pi}{2}\right)$ is increasing, decreasing, or not monotonic. Is the sequence bounded?

Section 11.2

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(23) $1 + .4 + .16 + .064 + \ldots$

(24) $\displaystyle\sum_{n=0}^{\infty} \frac{4^{n+1}}{5^n}$

(25) If the nth partial sum of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ is $S_n = \frac{n-1}{n+1}$, find a_n and $\displaystyle\sum_{n=1}^{\infty} a_n$.