Properties of Real Numbers: The Commutative, Associative, and Distributive Laws

Commutative Law of Addition	When *adding* numbers, order does not matter.
a + b = b + a	For example, 2 + 3 is the same as 3 + 2.
Commutative Law of Multiplication	When *multiplying* numbers, order does not matter.
ab = ba	For example, 2 • 3 is the same as 3 • 2.
Associative Law of Addition	When *adding* numbers, they can be "grouped" in any way.
a + (b + c) = (a + b) + c	For example, (2 + 3) + 4 gives the same result as 2 + (3 + 4).
Associative Law of Multiplication	When *multiplying* numbers, they can be "grouped" in any way.
a(bc) = (ab)c	For example, 2 • (3 • 4) gives the same result as (2 • 3) • 4.

Note: Subtraction and division are NOT commutative or associative operations!

Distributive Law **a (b + c) = ab + ac**	The Distributive Law provides a useful way of removing parentheses 2(x+3) can be written as 2•x + 2•3 = 2x + 6. It also allows us to "factor" an expression: 3x + 21 can be written as 3 • x + 3 • 7 which can then be written as 3(x + 7)

Examples

Use the Commutative Law of Addition to write an equivalent expression for 5 + 2x.

The Commutative Law says we can change the ORDER of the addition so 2x + 5 would be an equivalent expression.

Which law is being used here? x (2 + z) = (2 + z) x

Be careful! What changed from one side of the equals to the other? All that changed was the ORDER of the multiplication of the x and the (2 + z). This is an example of the Commutative Law of Multiplication.

Use the Associative Law of Addition to write an equivalent expression for (2x + 3y) + 5.

The Associative Law says we may change the GROUPING of the addition. Let's group the 3y with the 5 instead of with the 2x. We will keep the ORDER the same (if we changed the order, we would be using the Commutative Law). 2x + (3y + 5) is an equivalent expression.

Use the Distributive Law to remove parentheses from 4(2x + 3).

4 (2x + 3)	
4 • 2x + 4 • 3	BOTH terms of the sum must be multiplied by 4.
8x + 12	Simplify when possible.

Use the Distributive Law to factor the expression 5x + 10y

5x + 10y	
5 • x + 5 • 2y	We must rewrite each term with a common multiplier (5).
5 (x + 2y)	"Factor out" the 5 and rewrite according to the Distributive Law.

Problems

1. Use the Commutative Law of Addition to rewrite the expression $6x + 4$.
2. Use the Commutative Law of Multiplication to rewrite the expression $6x(4)$.
3. Use the Associative Law of Addition to rewrite the expression $(6x + 4) + 8$.
4. Use the Associative Law of Multiplication to rewrite the expression $6x (4 \cdot 8)$.
5. Use the Distributive Law to rewrite the expression $6(x + 4)$.
6. Use the Distributive Law to rewrite the expression $4x + 8$.

<u>Identify which law is being used.</u>

7. $x (yz) = x (zy)$
8. $2 (x + y) = 2x + 2y$
9. $(x + y) + z = (y + x) + z$
10. $(x + y) + z = x + (y + z)$
11. $2 [x (a + d)] = [2x] (a + d)$

Answers

1. $4 + 6x$
2. $4(6x)$
3. $6x + (4 + 8)$
4. $(6x \cdot 4) \cdot 8$
5. $6x + 24$
6. $4(x + 2)$
7. Commutative Law of Multiplication
8. Distributive Law
9. Commutative Law of Addition
10. Associative Law of Addition
11. Associative Law of Multiplication