

The following study guide is formatted to match the competencies and skills suggested in the “*Florida Teacher Certification Examination Test Preparation Guide: Elementary Education K-6*”. This study guide is published by the FDOE, and is usually sent to students who register for the FTCE exam. However, it can also be downloaded at www.fl DOE.org if a student is attempting to prepare for the exam before he/she has actually registered. Each competency and skill in this study guide exactly matches the ones suggested in the FDOE’s preparation booklet. The information used to answer each skill was either found at a free public website through a search engine (with credit given to each website for any information found on that website, and so that you can visit that website for further explanation if necessary) or from Shannon Black’s “Elementary Education Study Guide”. After a review of the different study guide resources available to education students, I believe that Shannon Black’s “Elementary Education Study Guide” is the best resource for students who need to prepare for this exam. The CMC in the Education Complex has 20 copies available for check out. Each student is strongly recommended to check this book out, as this study guide will reference Shannon Black’s book in various areas. Thank you, and GOOD LUCK!

FTCE Math Competency #1

1. Associate multiple representations of numbers using word names, standard numerals, and pictorial models for real numbers (whole numbers, decimals, fractions, and integers).

Word names- Ex: 43 = Forty-Three

Standard numerals- Ex: 43 = 43

Pictorial models for real numbers:

Whole numbers- The whole numbers are the counting numbers and 0. The whole numbers are 0, 1, 2, 3, 4, 5, ...)Can not be negative.

The above information was found at:

<http://www.mathleague.com/help/wholenumbers/wholenumbers.htm>

Decimals-Decimals are a method of writing fractional numbers without writing a fraction having a numerator and denominator. The fraction 7/10 could be written as the decimal 0.7. The period or decimal point indicates that this is a decimal.

The above information was found at:

<http://www.aaamath.com/B/dec37ax3.htm>

Fractions-A fraction is part of an entire object. Ex: 1/5, 3/7, 12/50

1 = Numerator

5 = Denominator

The above information was found at:

<http://www.ccsreach.net/mathcomp/Practice/Fractions.htm>

Integers-are also called whole numbers, negative whole numbers, and zero. For example: 560, -35, 2, 0, -197744

Shannon black describes more ways to express numbers on pg 130.

2. *Compare the relative size of integers, fractions, and decimals, numbers expressed as percents, numbers with exponents, and/or numbers in scientific notation.*

Integers are negative WHOLE numbers, 0, and positive WHOLE numbers. So these are placed in order from negative to positive. You can use a number line.

Fractions are parts of integers and therefore fit between them, when comparing size. In order to compare fractions, there DENOMINATORS must be equal.

For example...

2 2/3 is in between 2 and 3. However, be careful that it is not a mixed fraction such as 2 10/8 because this is actually equal to 3 2/8 (bigger than 3). So this would be in between 3 and 4, not 2 and 3.

If you see a mixed fraction such as 41/8, simplify it into 5 1/8 before comparing it to other numbers. Also, when comparing many fractions with different denominators, the easiest way to compare them is to give them a common denominator.

For example...

If given 3/5, 6/8, 9/10, 2/4, the best way to compare them is to find the smallest number that they all fit into for their common denominator.

1. Look at the largest denominator in your fractions (in this case it is 10). Do all of the other denominators fit into 10? No. So we need to go higher. The next multiple of 10 is 20. Do they all fit into 20 without a remainder? No. So we keep going to 30, and they still do not all fit evenly. But they do all fit into 40.

2. So, in this case, our COMMON DENOMATOR would be 40. So now, we need to multiply both the numerator and denominator of each fraction by the same number so that all the denominators will equal 40. So, in 3/5 you would multiply 5 by 8 to get 40. Then you would also multiply 3 by 8...and you would end up with 3/5= 24/40. You would do the same to the others and get

$$6/8 = 30/40$$

$$9/10 = 36/40$$

$$2/4=20/40$$

3. So, you order them as $20/40$, $24/40$, $30/40$, $36/40$ and so we know the correct order here would be $2/4$, then $3/5$, then $6/8$, and the biggest is $9/10$... Notice the fractions get larger as they approach the number 1.

Percents- The best way to compare a percent to other number expressions is either to convert it to a decimal and leave it as that, or convert it to a fraction (depending on how the other numbers are expressed).

For example...

For the percent 76%, you could either turn it into a decimal by moving the decimal point two spaces the left and leaving it as .76 **OR** you could take it as .76 and convert that to a fraction, which is $76/100$. This can be simplified to $16/25$. It really depends on what the other numbers are and how they are being expressed.

Here is an example problem using percents:

A sweater is \$235. There is a markdown of 25%. What will be the new price?

Answer: $235 - 235(.25)$

You want to find what 25% of 235 is. This can be done by multiplying $235(.25)$. Then to find out the price after discount, subtract that number from the original price of \$235.

Decimal- This is another way to write a percent or a fraction. $2/10 = .2$, since 2 is in the tenths place. $50\% = .50$ since 50% equals $50/100$ and 50 is in the hundredths places. Here is an example question to see if you can tell which decimal is the biggest and of you can convert decimals to fractions and back again:

What number is the largest?

- a) $.2 \div .06$
- b) $.2 \div 6$
- c) $.02 \div .06$
- d) $.02 \div 6$

Answer: a) $.2 \div .06$

The first way to do this problem is to move the decimal place to the right in each number so that all numbers are whole numbers. Then set them up the numbers using the division bar and divide to find which number is smallest.

- a) **$.2 \div .06$** (and move the decimal place over two places in each number)

So, you then have 20 divided by 6. When you put 6 into 20 and divide, you will find the answer is **3.3** or **20/6**.

- b) $.2 \div 6$ (you only need to move the decimal one place in each number to make them whole numbers)

So you then have 2 divided by 60. When you put 60 into 2, you get $2/60$, which equals **1/30** (and if you divided using long division you would get **.03**)

- c) $.02 \div .06$ (again move the decimal place two spaces to the right for each number)

So, you then have 2 divided by 6. When you divide 6 into 2, you get $2/6$, or **1/3**, or **.33**

- d) $.02 \div 6$ (you have to move the decimal 2 places to the right to make both numbers whole)
So, then you have 2 divided by 600. This can be shown by $2/600$, or **1/300**.

Answer: $.2 \div .06$ because the answer is 3.3

The other way to do this problem is to make each number a fraction and divide. To divide fraction you must use the inverse of the second fraction (or flip it upside down). Let's look at how to do this.

- a) $.2 \div .06$
- b) $.2 \div 6$
- c) $.02 \div .06$
- d) $.02 \div 6$

- a) $.2 \div .06$

In fraction form, we would have $2/10$ divided by $6/100$. We would then insert or flip $6/100$ to $100/6$ and change the division to multiplication

The new expression would be $2/10 \times 100/6 = (2)(100) / (10)(6) = 200/60$
This can be reduced to **100/3 or 30 1/3**.

- b) $.2 \div 6$

In fraction form we would have $2/10$ divided by $6/1$. We would flip $6/1$ to $1/6$ and multiply.

This can be viewed as $2/10 \times 1/6 = (2)(1) / (10)(6) = 2/60 = 1/30$

***(c) and (d) will yield the same results as we saw above and the answer will still be (a).**

Exponents- To compare numbers with exponents, it is best to multiply them out, or estimate what they might be. If a number is being raised to a negative exponent, then treat it as though it were positive and place the answer under the number 1.

For example...

2^2 is equal to 4....if you had 2^{-2} (negative two) then the answer would be $\frac{1}{4}$ (or 1 over 2^2)

When you have a number that is being raised to zero it always equals 1. ($10^0 = 1$)

Scientific Notation- You will want to work with numbers in scientific notation in much the same way that you work with numbers with exponents. You will want to multiply out the scientific notation.

*If a number is being multiplied by 10 (to a positive number, such as 2), the decimal place is moved to the right however many places comes after the 10! (Ex. $6.89 \times 10^2 = 689$)

*If a number is being multiplied by 10 (to a negative number, such as -2), you move the decimal place to the left however many places comes after the 10! (Ex. $5367 \times 10^{-2} = 53.67$)

For example...

If you have 3.4567×10^3 you will want to move the decimal point three spaces to the right and will end with 3456.7 before attempting to compare it to other numbers.

If you have 26543×10^{-3} (negative three) you will want to move the decimal three spaces to the left and end with 26.543 before attempting to compare it to other numbers.

Try an example question:

How do you write 3,200,000 in scientific notation?

Answer: 3.2×10^6

This is because if you start at the end...to the right of the last zero of 3,200,000 and move the decimal 6 places until you get to 3.2 (only one number to the left of the decimal place).

3. Apply ratios, proportions, and percents in real-world situations.

The above conversion suggestions should help you with this area.

A ratio is just another way to write a fraction. If the ratio is 2:3 it means two out of three, or 2/3.

4. Represent numbers in a variety of equivalent forms, including whole numbers, integers, fractions, decimals, percents, scientific notation, and exponents.

Again, this relates to the above information. They simply want you to be able to change the numbers from one form to another and then be able to compare them. Here is an example:

In a multiple-choice test, there is a 20% chance of guessing. What are the odds that you may get a correct guess?

- a- 1:4
- b- 4:5
- c- 1:5
- d- 5:4

Answer: C (1:5)

This can be demonstrated by viewing 1:5 as a fraction, or $1/5$. If you do not automatically know that $1/5$ is equal to 20% you can either...

- a). Divide 5 into 1 by putting a decimal point above the 1, adding a zero next to the 1. 5 goes into, 2 times so you put a 2 next to the decimal on the answer line. 5 times 2 is 10, so you put that under the 10 under the division sign and subtract. 10-10 is zero, and 2 goes into zero, zero times, so the answer is .20, which is 20%.
- b) Make $1/5$ equal to a 100^{th} . ($1/5 \times 2$) equals $2/10$, because 1 times 2 is 2, and 5 times 2 is 10. Therefore, $(2/10 \times 2)$ equals $4/20$. And $(4/20 \times 5)$ equals $20/100$, which equals 20%.

5. Recognize the effects of operations on rational numbers and the relationships among these operations (i.e., addition, subtraction, multiplication, and division).

Just to review common vocabulary in mathematics:

$$\frac{80}{8} = 10 \quad 80 = \text{dividend} \quad 8 = \text{divisor} \quad 10 = \text{quotient}$$

$$6 \times 9 = 54 \quad 54 = \text{product}$$

$$2 + 4 = 6 \quad 6 = \text{sum}$$

$$10 - 3 = 7 \quad 7 = \text{difference of}$$

There is an inverse relationship between multiplication and division and also between addition and subtraction.

A number fact is made up of three numbers. These three numbers can be used to make up other number facts. Knowing one fact can help children with other facts. Look at the number facts we can make with the numbers 3, 4, and 7.

<u>Addition</u>	<u>Subtraction</u>
<u>Facts</u>	<u>Facts</u>
$3 + 4 = 7$	$7 - 3 = 4$
$4 + 3 = 7$	$7 - 4 = 3$

Generally, subtraction facts are harder for children to learn than addition facts. If a child knows that $6 + 9 = 15$, and he or she sees the subtraction sentence $15 - 9 = \underline{\hspace{2cm}}$, the child can think, 9 and

what are 15? This use of thinking of the related addition fact when children encounter a subtraction fact they don't know should be encouraged

The above information came from the following website:

<http://www.eduplace.com/math/mathsteps/1/b/index.html>

There are also fact families for multiplication and division. It is also helpful to know the terms, dividend, divisor, and quotient, so here is a picture to help you learn them.

Using operations can be tricky for fractions. Here is a quick review:

Addition and subtraction: The DENOMINATORS MUST BE EQUAL! Then you just subtract or add across numerators and LEAVE the denominator the same.

For example: $5/7 - 3/7 = 2/7$ or $4/5 + 1/5 = 5/5$

For multiplication the denominators do not need to be the same. In multiplication, you multiply across both the numerator and denominator.

For example: $3/7 \times 3/5 = 9/35$ or $4/6 \times 5/8 = 20/48$

For division of fractions the denominators do not need to be the same. BUT when you are doing division, you need to FLIP the second fraction and multiply. This is called an INVERSE. The inverse of $1/3$ is $3/1$. The inverse of $4/7$ is $7/4$. So you leave the first fraction as it is, invert the second fraction and then multiply. Let's look at the examples above, but with division this time.

$$3/7 \div 3/5 = 3/7 \times 5/3 = 15/21 \quad \text{or} \quad 4/6 \div 5/8 = 4/6 \times 8/5 = 32/30$$

6. Select the appropriate operation(s) to solve problems involving ratios, proportions, and percents and the addition, subtraction, multiplication, and division of rational numbers.

See above and here is another example:

There are 60 boxes in a library. Each box contains 20 books. It took the librarian 2.5 hours to unpack the boxes. How many books did the librarian unpack per minute?

Answer: 14 books per minute

Ok, first you want to look at what they are asking you to find. That is NUMBER OF BOOKS and MINUTES. So we want to know how many boxes of books there were all together, and how many total minutes were spent unpacking them.

-First, we will find how many total books there were in the library. This is found by multiplying 60 boxes times 20 books.

$$\begin{array}{r} 60 \\ \times 20 \\ \hline \end{array}$$

$$\begin{array}{r}
 00 \\
 +1200 \\
 \hline
 1200
 \end{array}$$

So, we have 1,200 books!

Now, we need to find how many total minutes were spent unpacking these books. They told us it took $2\frac{1}{2}$ hours. We can convert this into minutes by first taking the whole number (2). 2 hours equals 120 minutes. Next, we look at the $\frac{1}{2}$ hour. One $\frac{1}{2}$ hour equals 30 minutes. And $120 + 30 = 150$ minutes.

So, we have a total of 150 minutes!

The last step is to divide. The question asks us “*How many books did the librarian unpack per minute*. This can be calculated by finding books per minute OR another way to write it is: books/minutes. For this question, we will set up a proportion, and cross multiply:

a).

$$\frac{1,200 \text{ books}}{150 \text{ minutes}} = \frac{(x) \text{ books}}{1 \text{ minute}}$$

To cross multiply we will set it up as follows:

b.)

$$1,200(1) = 150(x)$$

c.)

$$\text{So... } 1,200 = 150x$$

Then, divide both sides by 150...

$1200/150$ can be done by dropping the zero on both numbers and simplifying to $120/15$...then divide 15 into 120... the answer is 8. So (x) equals 8, and that is our answer.

The librarian unpacked 8 books per minute!

For the above few skills you may also reference Shannon Black's Elementary Education Study Guide on pg. 131-133.

7. Use estimation in problem-solving situations.

An example is that you can round to the nearest tenth, so 10.67894 could be estimated to be 10.7, or even 11. Another example is estimating fractions, such as, $16/30$ can be estimated to be $\frac{1}{2}$ because $15/30$ is equal to $\frac{1}{2}$.

8. Apply number theory concepts (e.g., primes, composites, multiples, factors, number sequences, number properties, and rules of divisibility).

Prime numbers: A prime number is a number with only two factors, itself and 1.

Examples are 2, 3, 5, 7, 11, 13, 17

***2 is the only even prime number and 1 is not considered to be a prime number!**

The prime numbers between 2 and 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

Composite Numbers: A composite number has factors in addition to one and itself. All even numbers are divisible by two and so all even numbers greater than two are composite numbers.

*****The numbers 0 and 1 are neither prime nor composite.**

The above information was found at this website:

<http://www.aaamath.com/B/fra63ax2.htm>

Multiples vs. Factors:

Multiples: A multiple of a number is the product of that number and any whole number.

Multiples of 3 include 6, 9, 12, 15...

Factors: A factor of a number is a number that can be divided into the original number evenly (meaning there is no remainder). For example factors of 24 are 2, 3, 4, 6, 8, 12...

Here is a question to test your knowledge so far:

What is the smallest multiple of 12, 15 and 20?

Answer: 60

Factors are the smallest number that can be multiplied into the larger number. For example, 2 is a factor of 4.

Multiples are numbers that are created from those factors when they are multiplied. For example, 4 is a multiple of 2.

For this problem, start with the largest number, 20...

a) Do 12 and 15 go into 20 evenly?

NO! They each go in once with a remainder!

b) So, we go to the next multiple of 20, which is 40.

Do 12 and 15 go into 40 evenly?

NO! 12 goes in three times with a remainder, and 15 goes in twice with a remainder.

c) So, we go to the next multiple of 20, which is 60.

Do 12 and 15 go into 60 evenly? YES! 12 goes into 60, five times and 15 goes into 60, four times.

SO THE ANSWER IS 60.

Number Sequences:

Sequences on this test will most likely be harder than simply multiplying each number by a certain number or adding one number to each number in the sequence. If you see a sequence on your test, you want to look for things such as...

Each number being raised to an exponent: 1, 3, 9, 27... ($3^0, 3^1, 3^2, 3^3$)

Each number may be a prime number in order (2, 3, 5, 7...)

or skipping every other prime (2, 5, 11, 13...)

Each number might be the sum of that number plus the number before it (2, 4, 6, 10, 16...)

Number Properties:

Commutative Property of Addition and Multiplication

Addition and Multiplication are commutative: switching the order of two numbers being added or multiplied does not change the result. Examples...

$$100 + 8 = 8 + 100$$

$$100 \times 8 = 8 \times 100$$

Associative Property

Addition and multiplication are associative: the order that numbers are grouped in addition and multiplication does not affect the result. Examples...

$$(2 + 10) + 6 = 2 + (10 + 6) = 18$$

$$2 \times (10 \times 6) = (2 \times 10) \times 6 = 120$$

Distributive Property

The distributive property of multiplication over addition: multiplication may be distributed over addition. Examples...

$$10 \times (50 + 3) = (10 \times 50) + (10 \times 3)$$

$$3 \times (12+99) = (3 \times 12) + (3 \times 99)$$

The Zero Property of Addition

Adding 0 to a number leaves it unchanged. We call 0 the additive identity. Example...

$$88 + 0 = 88$$

The Zero Property of Multiplication

Multiplying any number by 0 gives 0. Example...

$$88 \times 0 = 0$$

$$0 \times 1003 = 0$$

The Multiplicative Identity

We call 1 the multiplicative identity. Multiplying any number by 1 leaves the number unchanged. Example...

$$88 \times 1 = 88$$

The above number properties can be found at the following website:

<http://www.mathleague.com/help/wholenumbers/wholenumbers.htm#commutativeproperty>

For rules of divisibility, refer to Shannon Black's Elementary Education Study Guide pg. 132.

9. Apply the order of operations.

P.E.M.D.A.S. (Please Excuse My Dear Aunt Sally)

Rule 1: Do the operations inside the Parentheses.

Rule 2: Simplify all Exponents, working from left to right.

Rule 3: Perform all Multiplications and Divisions, working from left to right.

Rule 4: Perform all Additions and Subtractions, working from left to right.

FTCE Math Competency # 2

1. Apply given measurement formulas for perimeter, circumference, area, volume, and surface area in problem situations.

They will give you a sheet with the formulas on the test, but you will need to know what the letters stand for and how to complete the formula:

l = length

w = width

p = perimeter

A = area

b = base

h = height

V = volume

d = diameter (across entire circle/oval, from one side to another)

r = radius (across half of oval/circle, from middle point to point on outer edge)

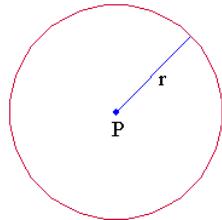
C = circumference (perimeter around circle)

$$\Pi = 3.14 \text{ or } 22/7$$

***Perimeter:** You always add all the sides of any given object!

Circle

A circle is the collection of points in a plane that are all the same distance from a fixed point. The fixed point is called the center (P). A line segment joining the center to any point on the circle is called a radius (r).



***Circumference:** is the distance around the outside of a circle. The circumference of a circle is:
 $C = 2\Pi r$ (2 multiplied by Π (or 3.14), which is multiplied by the length of the radius)

***Area of a circle:** $A = \Pi r^2$ (square the radius and multiply by Π)

The above information can be found at the following website:
<http://www.mathleague.com/help/geometry/polygons.htm#circle>

***Area:** (only refers to 2-dimensional objects).

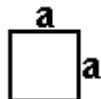
***Volume and Surface Area** (only refer to 3-dimensional objects). These formulas are different for each object, and will be covered below: However, the test will give you a list of the formulas. You will just need to know how to apply them.

AREA:

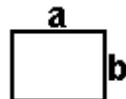
Area is measured in "square" units. The area of a figure is the number of squares required to cover it completely, like tiles on a floor.

Square/Rectangle (2 dimensional):

Area: $A = L \times W$ (the length is multiplied by the width)



The area for a square is always $= a^2$
 $(a) \times (a) = a^2$



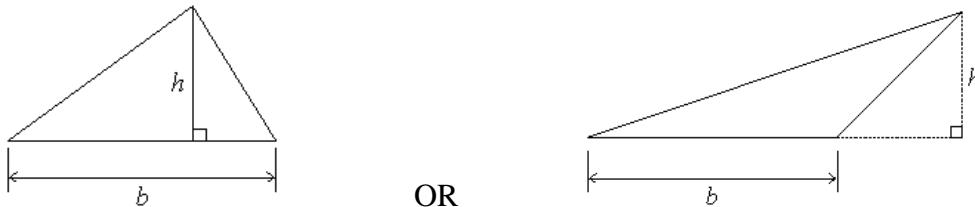
The area for this rectangle is $= ab$
 $(a) \times (b) = ab$

The above information can be found at the following website:

<http://www.math.com/tables/geometry/areas.htm>

Triangle (2 dimensional):

Area: $A = \frac{1}{2}bh$ ($\frac{1}{2}$ is multiplied by the base, which is multiplied by the height)



Consider a triangle with base length b and height h .

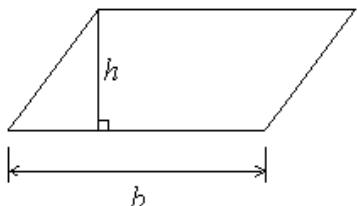
The area of the triangle is $\frac{1}{2} \times b \times h$.

Note: For the triangles that look like the obtuse triangle on the right, YOU will need to draw the height (h). Most likely, it will not be given to you, like it is here with the dotted line.

Parallelogram (2 dimensional):

Area: $A = bh$ (the base multiplied by the height)

The area of a parallelogram is $b \times h$, where b is the length of the base of the parallelogram, and h is the corresponding height. To picture this, consider the parallelogram below:

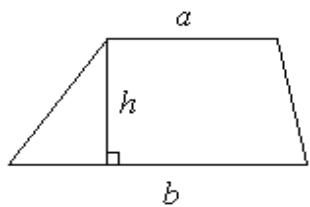


Trapezoid (2 dimensional):

Area: $A = \frac{1}{2}h(b_1 + b_2)$ or $A = \frac{1}{2}h(a + b)$

*If given this formula on the test, it will probably be shown using b_1 and b_2 , instead of (a) and (b) . It really doesn't matter, but just make sure you don't get confused just because they label them differently. It is still saying the same thing. The examples below use " a " and " b ".

Note: Make sure to add " a " and " b " before multiplying anything! (Remember the order of operations (Please Excuse My Dear Aunt Sally): parentheses first, multiply later)



If a and b are the lengths of the two parallel bases of a trapezoid, and h is its height, the area of the trapezoid is:

$$1/2 \times h \times (a + b)$$

The above information for triangles, parallelograms, and trapezoids can be found at the following website:

<http://www.mathleague.com/help/geometry/area.htm#area>

VOLUME:

Volume is measured in "cubic" units. The volume of a figure is the number of cubes required to fill it completely, like blocks in a box. Volume is found using a different formula for different objects. The most common examples are below, but remember that the test will give you these formulas. You do not need to memorize them. Just need to know how to apply them.

cube = a^3

rectangular prism (or triangular prism) = $a b c$

pyramid = $(1/3) b h$

cone = $(1/3) b h = 1/3 \pi r^2 h$

*because the base is a circle, so the area is equal to πr^2

sphere = $(4/3) \pi r^3$

cylinder = $b h = \pi r^2 h$

The above information was found at the following website:
<http://www.math.com/tables/geometry/volumes.htm>

Surface Area:

Surface area (SA) is the TOTAL area of ALL sides or parts of a 3D object. So, to find the SA, you ADD the all areas of all the sides. So first you would need to find the area of each side. Then you would add them all together.

Example:

$$\text{Surface Area of a Cube} = 6 a^2$$

(Because a cube has six faces and the area of each face is (a) times (a) or a^2 . So, $(6) \times (a^2) = 6a^2$

*Make sure you add ALL faces! For example, a cylinder has two circles, so make sure you add both areas plus the rest!!!

2. Evaluate how a change in length, width, height, or radius affects perimeter, circumference, area, surface area, or volume.

This can be figured out using the above formulas!

3. Within a given system, solve real-world problems involving measurement, with both direct and indirect measures, and make conversions to a larger or smaller unit (metric to customary).

A direct measurement is one where the reading observed represents the quantity measured, without a need to add, take averages or use geometric formulas to compute the value desired. (Examples: l, w, h)

An Indirect Measurement is a measurement that can't be made using a measuring device, but instead is calculated or derived based on direct or other indirect measurements. (Example: Area is an indirect measurement because it is derived using the formula $length \times width$, both of which are direct measurements.)

The above information was found at the following website:

<http://www.profsurv.com/archive.php?issue=15&article=163>

Conversions:

This mnemonic device can help you remember how to convert metric measurements:

K H D B D C M

King **H**enry **D**ied **B**y **D**rinking **C**hocolate **M**ilk

Kilo **H**ecto **D**eka **B**ASE **D**eci **C**enti **M**illi

Note: The BASE is whatever measurement you are working with.
(For example: liter, gram, or meter)

How to use this chart:

If the question says: How many MILLIMETERS are in 1.4 DEKAMETERS?

- a. You would write on your paper 1.4
- b. You BEGIN with the LAST unit in the question. Here that is DEKAMETER.
- c. Place your pencil on DEKAMETER and count how many places you have to move from DEKA to MILLI. You will count 4 to the right.
- d. So, you move your decimal, 4 places to the right. And 1.4 will turn into 14,000
- e. So your answer would be that 14,000 Millimeters is equal to 1.4 Dekameters.

Example #2: How many KILOGRAMS are in 560 DECIGRAMS?

- a. You would write 560 on your paper.
- b. You would begin with the last unit. Here that is Decigrams.
- c. Place your pencil on decigram and count how many places you need to move to get to kilogram. You will count four places to the left.
- d. So, you will move your decimal, four places to the left. And 560 will turn into .056
- e. So your answer is that .056 kilograms is equal to 560 decigrams.

Different measurements in the Metric and Customary System:

Metric: any of the prefixes above (meter, gram, liter) also Celsius

Customary: feet, inches, yards, Fahrenheit, mile, acre, pint, quart, gallon, ounces, pounds

4. Solve real world problems involving estimates and exact measures.

Always round to nearest tenth (one decimal place) or to closest whole number!

5. Select appropriate units to solve problems.

You never want a number that is too big or too small. Here a few examples:

What would you use to measure a pencil? Centimeters

What would you use to measure a pool? Gallons

How do we measure body warmth? Temperature

What is the best way to measure the amount of medicine you are taking? Milliliters (mL)

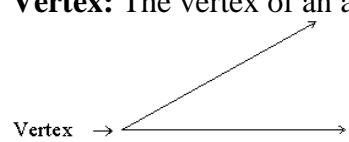
Note: Shannon Black also addresses different types of measurement on pages 135-138 of her book. Please refer to it if you desire more information.

FTCE Math Competency #3

1. Identify angles or pairs of angles as adjacent, complementary, supplementary, vertical, corresponding, alternate interior, alternate exterior, obtuse, acute, or right.

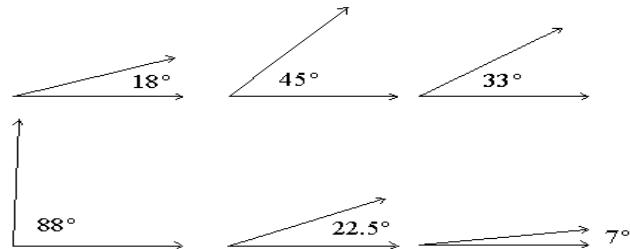
You may also want to refer to Shannon Black's Book which refers to angles, lines, and other geometric concepts on pages 140 and 141.

Vertex: The vertex of an angle is the point where the two rays that form the angle intersect.

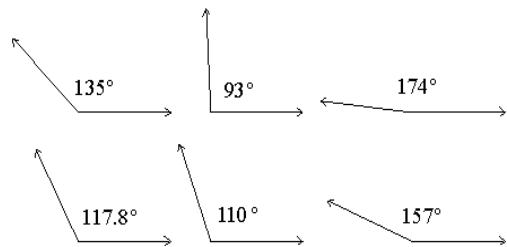


The above information can be found at the following website:
<http://www.mathleague.com/help/geometry/polygons.htm#vertex>

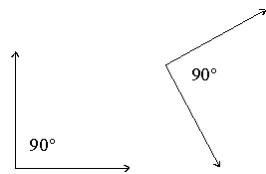
Acute Angles: An acute angle is an angle measuring between 0 and 90 degrees. The following angles are all acute angles.



Obtuse Angles: An obtuse angle is an angle measuring between 90 and 180 degrees. Example: The following angles are all obtuse.



Right Angles: A right angle is an angle measuring 90 degrees. Two lines or line segments that meet at a right angle are said to be perpendicular. The following angles are both right angles.



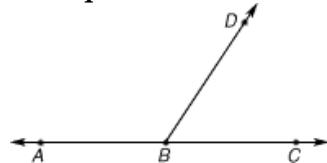
Note: Never assume that an angle is a right angle, just because it looks like it is. Angles are only right angles if test specifically states it, or if a small square is drawn in the angle, to designate it as a right angle.

The above information about acute, obtuse, and right angles, can be found at the following website:

<http://mathleague.com/help/geometry/angles.htm#complementaryangles>

Adjacent Angles: Angles that are side by side and have a common vertex and ray
(Note: A ray is a line that has one side that ends and one side that continues on)

Example:



$\angle ABD$ is adjacent to $\angle DBC$.

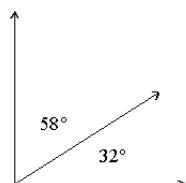
The above information can be found at the following website:

http://www.asmachronicle.com/issue_7_vol7.htm

Complementary Angles: Two angles are called complementary angles if the sum of their degree measurements equals 90 degrees. One of the complementary angles is said to be the complement of the other. Example: These two angles are complementary:



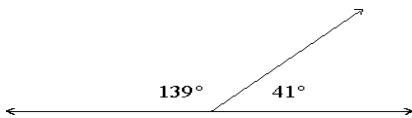
Note that these two angles can be "pasted" together to form a right angle!



Supplementary Angles: Two angles are called supplementary angles if the sum of their degree measurements equals 180 degrees. One of the supplementary angles is said to be the supplement of the other. Example: These two angles are supplementary.



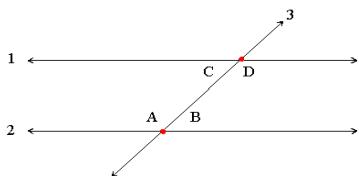
Note that these two angles can be "pasted" together to form a straight line!



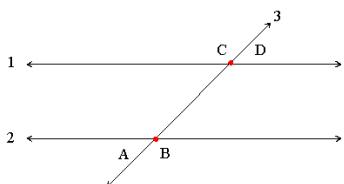
The above information for both complementary and supplementary angles can be found at the following website:

<http://mathleague.com/help/geometry/angles.htm#complementaryangles>

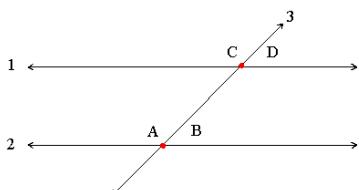
Alternate Interior Angles: For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle D are called alternate interior angles. Alternate interior angles have the same degree measurement. Angle B and angle C are also alternate interior angles.



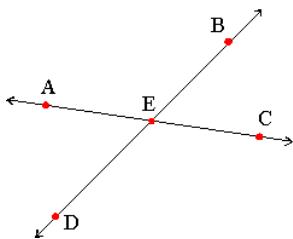
Alternate Exterior Angles: For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle D are called alternate exterior angles. Alternate exterior angles have the same degree measurement. Angle B and angle C are also alternate exterior angles.



Corresponding Angles: For any pair of parallel lines 1 and 2, that are both intersected by a third line, such as line 3 in the diagram below, angle A and angle C are called corresponding angles. Corresponding angles have the same degree measurement. Angle B and angle D are also corresponding angles.



Vertical Angles: For any two lines that meet, such as in the diagram below, angle AEB and angle DEC are called vertical angles. Vertical angles have the same degree measurement. Angle BEC and angle AED are also vertical angles.



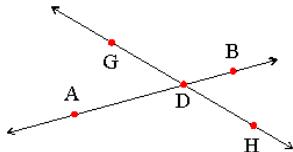
The above information about alternate interior, exterior, corresponding, and vertical angles can be found at the following website:

<http://mathleague.com/help/geometry/angles.htm#alternateinteriorangles>

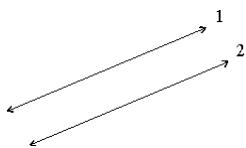
2. Identify lines and planes as perpendicular, intersecting, or parallel.

Perpendicular: Any two lines that form a right angle!

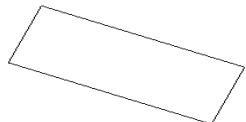
Intersecting: The term intersect is used when lines, rays, line segments or figures meet, that is, they share a common point. The point they share is called the point of intersection. We say that these figures intersect. Example: In the diagram below, line AB and line GH intersect at point D:



Parallel: Two lines in the same plane which never intersect are called parallel lines. We say that two line segments are parallel if the lines that they lie on are parallel. If line 1 is parallel to line 2, we write this as line 1 \parallel line 2. When two line segments DC and AB lie on parallel lines, we write this as segment DC \parallel segment AB. Example: Lines 1 and 2 below are parallel.



Example: The *opposite* sides of the rectangle below are parallel. The lines passing through them never meet.



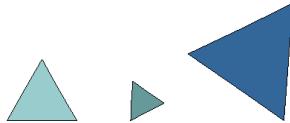
The above information about parallel, intersecting, and perpendicular lines can be found at the following website:

<http://www.mathleague.com/help/geometry/basicterms.htm>

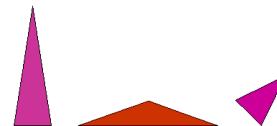
3. Apply geometric properties and relationships, such as Pythagorean Theorem, in solving problems.

Triangle: A three-sided polygon. The sum of the angles of a triangle is ALWAYS 180 degrees.

Equilateral Triangle: A triangle having all three sides of equal length. The angles of an equilateral triangle all measure 60 degrees. Examples:



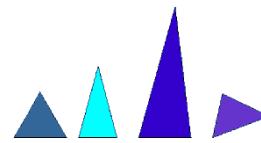
Isosceles Triangle: A triangle having two sides of equal length.



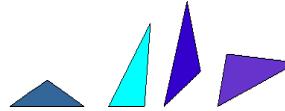
Scalene Triangle: A triangle having three sides of different lengths.



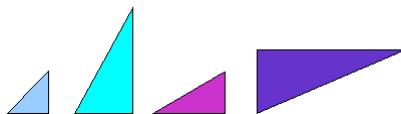
Acute Triangle: A triangle having three acute angles.



Obtuse Triangle: A triangle having an obtuse angle.



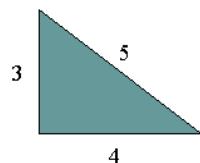
Right Triangle: One of the angles of the triangle measures 90 degrees. The side opposite the right angle is called the hypotenuse. The two sides that form the right angle are called the legs.



A right triangle has a special property: The sum of the squares of the lengths of the 2 legs, equals the square of the length of the hypotenuse. This is known as the Pythagorean Theorem.

Example: For the right triangles above, the lengths of the legs are A and B, and the hypotenuse has length C. Using the Pythagorean Theorem, we know that $A^2 + B^2 = C^2$.

Example:



In the right triangle above, we can use the Pythagorean Theorem and see that $3^2 + 4^2 = 5^2$. ($9 + 16 = 25$). This will be true for all right angles, and may help you solve problems on the test, where you are asked to find the length of one of the sides of a right triangle.

To review: You are shown a triangle and one of the angles measures 60° , what's going to be the total amount of all the angles? **180 degrees (All triangles angle add up to 180 degrees)**

The above information can be found at the following website:
<http://www.mathleague.com/help/geometry/polygons.htm#triangle>

It is important to know the total angle amounts for different geometric shapes:
(The * Below stands for a degrees sign)

- Triangle (3 sides) 180° - so in an equilateral triangle, each angle is 60° ($180/3$)
- Square (four sides) 360° - so each angle is 90° ($360/4$)
- Pentagon (5 sides) 540° - so each angle is 108° ($540/5$)
- Hexagon (6-sided figure) 720° - so each angle is 120° ($720/6$)
- Heptagon (7-sides) 900° - so each angle is 128.57° ($900/7$)
- Octagon (8-sides) 1080° , etc. – so each angle is 135° ($1080/8$)

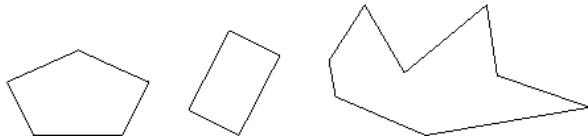
They give you the shape, and all of the angles except for one, and if you know the total, you can figure it out. If you don't want to memorize a million totals, you can use this formula to figure it out:

$$180^\circ \times (n - 2) \text{ degrees. } (n = \text{number of sides})$$

4. Identify the basic characteristics of, and relationships pertaining to, regular and irregular geometric shapes in two and three dimensions.

Polygon: A polygon is a closed figure made by joining line segments, where each line segment intersects with two others.

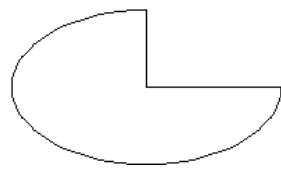
Examples:



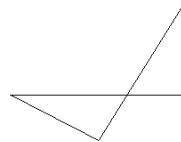
The following are examples of polygons:



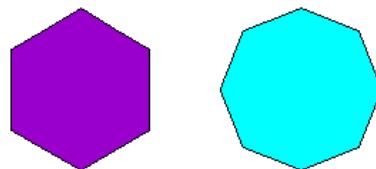
This figure is not a polygon, since it is not a closed figure:



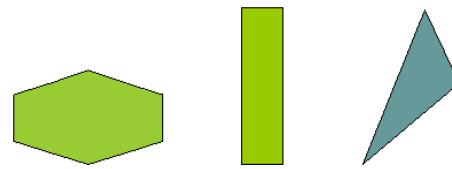
This figure is not a polygon, since it is not made of line segments:



Regular Polygon: A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same. (Ex. square, equilateral triangle)



The following are examples of regular polygons:



The following are not examples of regular polygons:

Note: The sum of the angles of a polygon with n sides, is $180^\circ \times (n - 2)$ degrees.

For example, in a triangle you would have $180 \times (3-2) = ?$ You would then get $180 \times 1 = 180$. So, the total of all angles for a triangle is 180. We already know that from above, but this was just to demonstrate how to use this formula. For a square you would have $180 \times (4-2)$. Then you would get $180 \times 2 = 360$. So the sum of all the angles of a square is 360. This only works for **polygons**!

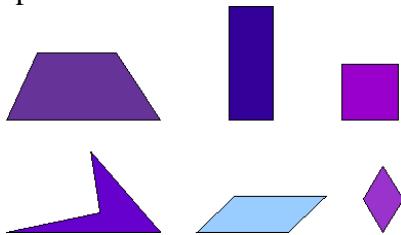
Try one a little harder: **You see an octagon with a measure on one of the sides as 5 in. The question asks... If each side is increased by 2, then what is the total perimeter?**

Answer: 56 inches

Since an octagon had 8 sides, and each of the sides is equal, and the question tells you that one side is 5in. Then you would multiply 5×8 and see that the perimeter is 40 without the increase. If you increase each side by two then each side will be 7in. instead of 5in. and then you will multiply 7 times 8. The answer will be 56 inches.

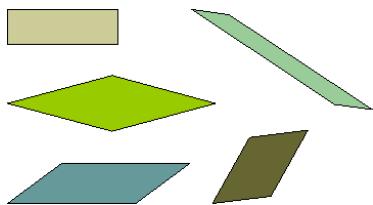
Quadrilateral: A four-sided polygon. The sum of the angles of a quadrilateral is 360 degrees.

Examples:



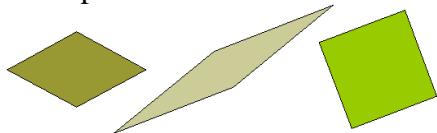
Parallelogram: A four-sided polygon with two pairs of parallel sides. The sum of the angles of a parallelogram is also 360 degrees.

Examples:



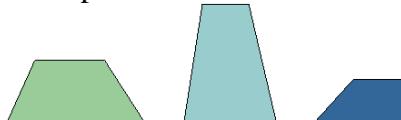
Rhombus: A four-sided polygon having all four sides of equal length. The sum of the angles of a rhombus is also 360 degrees.

Examples:

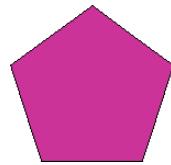


Trapezoid: A four-sided polygon having exactly one pair of parallel sides. The two sides that are parallel are called the bases of the trapezoid. The sum of the angles of a trapezoid is also 360 degrees.

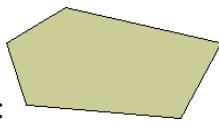
Examples:



Pentagon: A five-sided polygon. The sum of the angles of a pentagon is 540 degrees. You do not need to memorize this if you know the above formula. $180 \times (5-2) = ?$... So, $180(3) = 540$



Regular pentagon:



Irregular pentagon:

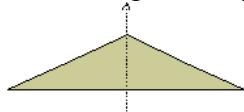
Other shapes that have regular and irregular versions are the Hexagon (6 sided), Heptagon (7 sided), Octagon (8 sided), Nonagon (9 sided), Decagon (10 sided). And all of the sums of angles can be calculated using the above formula.

All of the above information about polygons can be found at the following website:
<http://www.mathleague.com/help/geometry/polygons.htm#polygon>

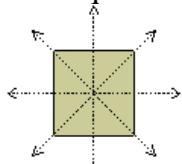
5. Apply the geometric concepts of symmetry, congruency, similarity, tessellations, transformations, and scaling.

Symmetry: A figure is symmetric if that figure can be folded flat along a line so that the two halves match perfectly. Such a line is called a line of symmetry.

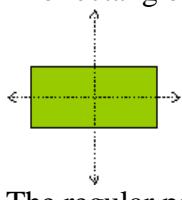
This triangle is a symmetric figure. The dotted line is the line of symmetry.



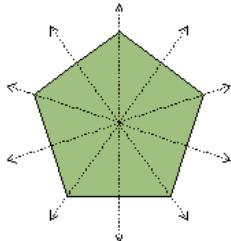
This square is a symmetric figure. It has four different lines of symmetry shown below.



The rectangle below is a symmetric figure. It has two different lines of symmetry shown below.

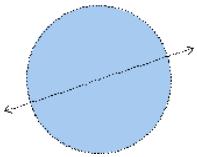


The regular pentagon below is a symmetric figure. It has five different lines of symmetry shown below.

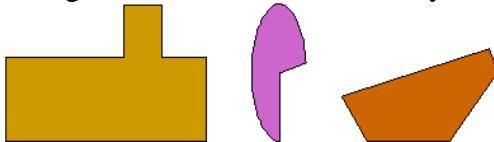


The circle below is a symmetric figure.

Any line that passes through its center is a line of symmetry!

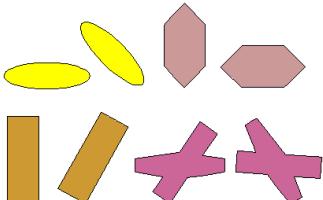


The figures shown below are not symmetric.

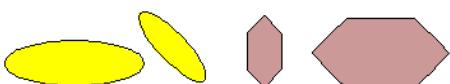


Congruency and Similarity: Two figures are congruent if they have the same shape and size. They are similar if they look like, and have the same scaled size. So if square 1 has 4inch sides and square 2 has 16inch sides. They would be similar, but not congruent.

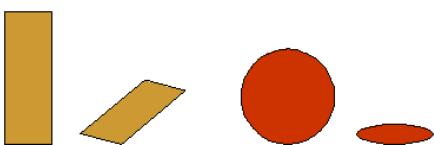
The following pairs of figures below are congruent. **Note: If two figures are congruent, they must be similar.**



The pairs below are similar but not congruent.



The pairs below are not similar or congruent.



Here is a question that can help illustrate how to use similarity to solve a problem:

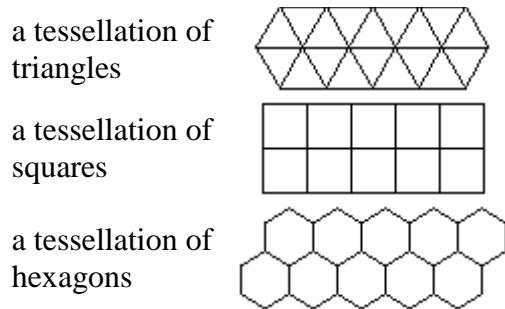
Triangle A has sides that measure 6, 2 and 4. Triangle B has only one number on one of the sides, and it is 10. (The 10 is marked on the same side of triangle B, as 2 is marked on triangle A. You need to find out the measures for the other 2 sides of triangle B.)

The triangles here are similar to one another. Triangle B is 5 times bigger than Triangle A since $2 \times 5 = 10$. Therefore, all three sides are also similar to one another. So, since (2) is the same as side (10) then to find all sides of the larger triangle, you need to multiply all sides by 5. So, the side (6) on Triangle A would equal (30) on Triangle B, and the side (4) on Triangle A, would equal (20) on Triangle B.

Tessellations: A **regular tessellation** means a tessellation made up of congruent regular polygons. [Remember: *Regular* means that the sides of the polygon are all the same length. *Congruent* means that the polygons that you put together are all the same size and shape.]

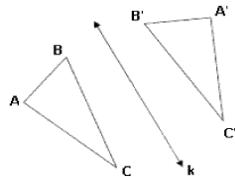
Only three regular polygons tessellate in the Euclidean plane: triangles, squares or hexagons. We can't show the entire plane, but imagine that these are pieces taken from planes that have been tiled. You can also make tessellations of irregular polygons as long as there is no empty space and the fit together perfectly.

Here are the examples of the THREE regular tessellations:

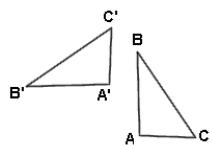


Transformations: A transformation can either be a reflection, rotation, or translation:

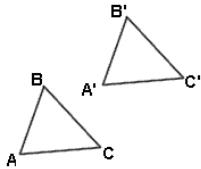
Reflection- A reflection is a transformation in which each point of the original figure (pre-image) has an image that is the same distance from the line of reflection as the original point but is on the opposite side. The resulting figure is congruent to the original.



Rotation- A rotation is a transformation that moves every point around a fixed point (usually the origin)... Kind of in a circular motion. Rotations > 0 are counterclockwise. Rotations < 0 are clockwise. The resulting figure is congruent to the original figure.



Translation- A translation is a transformation that slides every point of a figure the same distance in the same direction. The resulting image is also congruent to the original image.



The above information about transformations can be found at the following website:
<http://regentsprep.org/Regents/mathb/7B7/transformlesson.htm>

Scaling: A scale drawing is a drawing that represents a real object. The scale of the drawing is the ratio of the size of the drawing to the actual size of the object.

Example: The length of a stadium is 100 yards and its width is 75 yards. If 1 inch represents 25 yards, what would be the dimensions of the stadium drawn on a sheet of paper?

Solution:

- a. You write down what the problem states: 100 yards by 75 yards
- b. Then convert the yards into inches: $100 \text{ yards} = 4 \text{ inches}$ (HINT: $100 / 25$)
 $75 \text{ yards} = 3 \text{ inches}$ (HINT: $75 / 25$)

Therefore, the dimensions would be 4 inches by 3 inches.

Or you can establish a proportion:

(Notice that the inches are all on the top and the yards are all on the bottom for this solution. Other combinations are possible.)

Length:	Width:
$\frac{1}{25} = \frac{x}{100}$	$\frac{1}{25} = \frac{y}{75}$
$25x = 100$	$25y = 75$
$x = 4 \text{ inches}$	$y = 3 \text{ inches}$

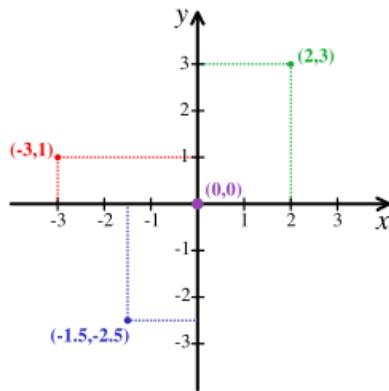
The information about scaling was found at the following website:
<http://regentsprep.org/Regents/math/scale/Lscale.htm>

6. Determine and locate ordered pairs in all four quadrants of a rectangular coordinate system.

An ordered pair is a set of two numbers in which the order has an agreed-upon meaning, such as the Cartesian coordinates (x, y) . The first coordinate represents the horizontal position, and the second coordinate represents the vertical position.

The above definition was found at the following website:
http://www.math.com/school/glossary/defs/ordered_pair.html

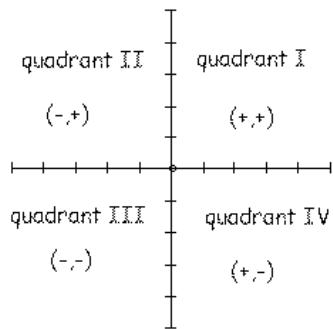
Below is an example of a Cartesian coordinate system. Note the x-axis is horizontal and the y-axis is vertical. The points are graphed using the x-coordinate first, then the y-coordinate. The center (0,0), is called the origin.



The above picture was found at the following website:

<http://en.wikipedia.org/wiki/Image:Cartesian-coordinate-system.svg>

The **rectangular coordinate system**, also called the **Cartesian coordinate system** or the **x-y coordinate system** is shown below.



Notice that the rectangular coordinate system consists of 4 **quadrants**, a **horizontal axis**, a **vertical axis**, and the **origin**. A point of the graph is called an **ordered pair**. The first coordinate (or **abscissa**) is known as the x-coordinate, while the second coordinate (or **ordinate**) is the y-coordinate. These tell us how far and in what direction we move from the origin.

The above information was found at the following website:

<http://a-s.clayton.edu/garrison/Math%200099/sect31.htm>

FTCE Math Competency #4

1. Extend and generalize patterns or functional relationships.

-Functions are rules that describe the relationship between two variables.

- Common examples of functional relationships include...
 - *The relationship between the distance traveled at a certain rate over a period of time
 - *The relationship between the unit cost of an item and the cost for multiple numbers of that item
 - *The relationship between temperatures in degrees Celsius and degrees Fahrenheit.
- Functions are commonly expressed as equations. When you know the value of one variable, you can use a rule or an equation to find the value of the other variable.

-For example: In the function $y = 2x + 6$, if you know that $x = 3$, you can calculate that the value of y is 12. (Since $3 \times 2 = 6$. And $6 + 6 = 12$.)

It is important for students to understand that for **each value of a variable, there is **exactly one related value for the other variable**. In the previous example when $x = 3$, y can **only** equal 12.

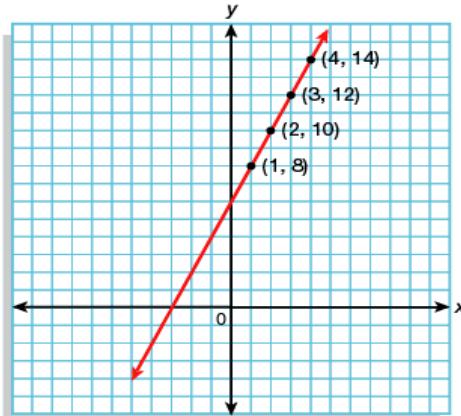
-The two variables in a function have names that explain how they relate to one another. One variable, usually **x**, is known as the input or **independent variable**. This is the variable upon which the operation or operations are performed. The other variable, usually **y**, is called the output or **dependent variable**. The value of this variable is dependent upon the value of the independent variable.

-Functions are also displayed or described in the form of function tables or graphs. A function table is a table of ordered pairs following the rule for that function. Function tables can be made up of an infinite number of ordered pairs. A function table for the equation $y = 2x + 6$ is shown below.

$$y = 2x + 6$$

<u>x</u>	<u>y</u>
1	8
2	10
3	12
4	14

-Once you have created a function table, you can graph the ordered pairs from the table on the coordinate plane. When graphing the values in a function table, the value of the input variable is the x -coordinate and the corresponding value of the output variable is the y -coordinate. The graph for the function $y = 2x + 6$ is shown here.

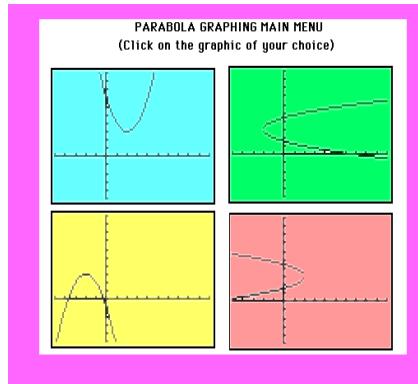


The above information can be found at the following website:

http://www.eduplace.com/math/mw/background/5/04b/te_5_04b_overview.html

*****When a functional relationship is graphed, a vertical line will only pass through the line graph once. *****

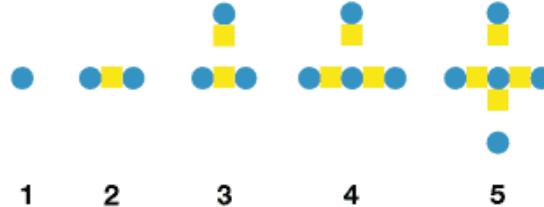
-So both the straight line above and the parabolas (U-shaped figures) in the blue and yellow boxes below are functional relationships. However, the parabolas in the green and red boxes are not functional relationships because a vertical line would go through these graphs twice!



The above picture can be found at the following website:

<http://ellerbruch.nmu.edu/classes/CS560W96/students/SHEWITT/para.tutorial/para.mainmenu.html>

-Functions also relate to patterns. A pattern is a design or an arrangement of numbers that models a general rule. Each design or number in the pattern is called a *term*. In the pattern below, the first term shows 1 circle and 0 squares; the second term shows 2 circles and 1 square; the third term shows 3 circles and 2 squares.



-By identifying the pattern, students can determine how many circles or squares are needed for any term. In this pattern the number of squares is always one less than the number of the term. This relationship can be written as an equation. So if x stands for the term, and y stands for the number of squares, the function for the squares in this pattern is $y = x - 1$.

-Then, there is a different function and therefore a different equation to describe the relationship between the term and the number of circles. Students should understand that in multi-shape or multi-color patterns, each shape or color may have its own function. **MAKE SURE YOU READ THE QUESTIONS ON THE TEST CAREFULLY!!**

The above information can be found at the following website:

http://www.eduplace.com/math/mw/background/5/04b/te_5_04b_overview.html

2. Interpret tables, graphs, equations, and verbal descriptions to explain real-world situations involving functional relationships.

-Scientists, mathematicians, economists, and many other people use functions to describe a relationship between two or more variables. The scientists and economists use functions because they wish to show how things are dependent on other things. They like to have this knowledge so that they can predict the future using their functions.

The above paragraph can be found at the following website:

<http://www.msu.edu/user/steffeyl/functions.html>

-As mentioned above, common examples of functional relationships in everyday life also include the relationship between the distance traveled at a certain rate over a period of time, the relationship between the unit cost of an item and the cost for multiple numbers of that item, and the relationship between temperatures in degrees Celsius and degrees Fahrenheit.

-Functional relationships exist all around us in our everyday experiences. Below are some common examples:

1. As adults, we experience these relationships when we are spending our money. For example, a large retail store is having a sale on bananas. The sale price is 4 pounds for a dollar. The regular price is \$0.35 per pound. The sale price of 4 pounds per dollar is actually \$0.25 per pound. Therefore, you are getting more bananas for your dollar while the bananas are on sale.

The difficulty is realizing that the word "sale" does not always mean that the consumer saves money. The use of functional relationships allows an educated consumer to make the smart decision by getting "the most for their buck." As adults, we need to buy staple products such as laundry detergent. Some stores sell the "economy" box of detergent (276 ounces). Since this is

the largest box, many consumers think it is the better deal. A closer look sometimes reveals the opposite. For example, the prices of three different sized boxes are as follows:

276 ounce box	\$12.95
184 ounce box	\$6.25
92 ounce box	\$3.00

If we look at the relationship between the number of ounces and the price, we can determine how much we are actually paying for each ounce. **This is done by dividing the cost of each box by the number of ounces we find:**

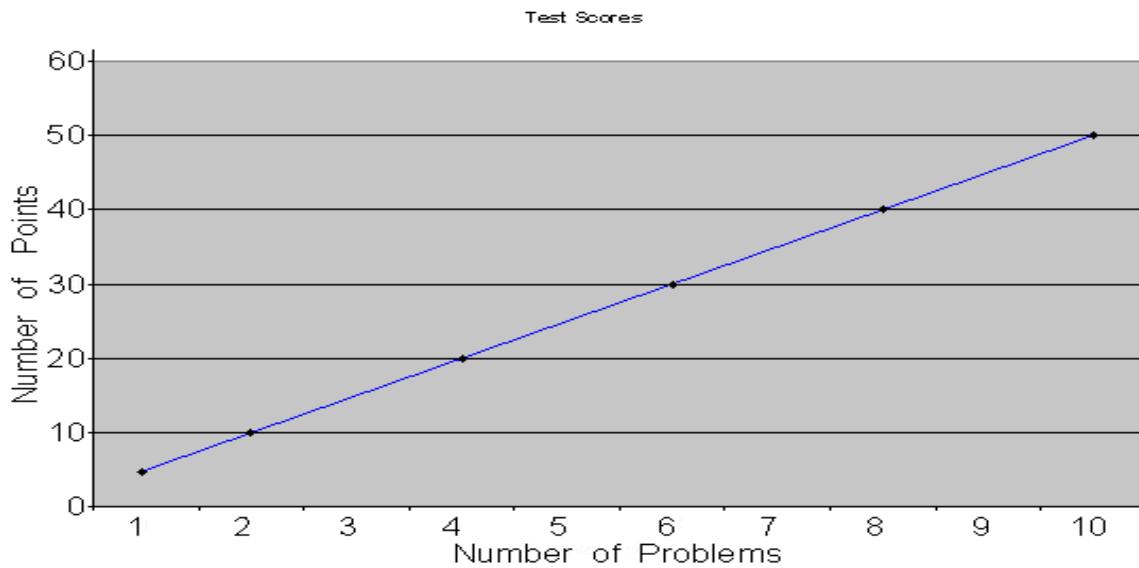
Box	price	approx. cost per ounce
276 ounce box	\$13.95	\$0.05
184 ounce box	\$7.50	\$0.04
92 ounce box	\$3.00	\$0.03

An educated consumer finds that the smaller box is actually a better buy. Three smaller boxes would cost you \$9.00 for a total of 276 ounces. Bigger is not always better.

A linear relationship is demonstrated by a straight line, and is like a functional relationship in that each x corresponds with only one y . An example of a linear relationship that we encounter; may be taking a test in school. The table below shows the results of a ten-question test.

Number correct	Points
1	5
2	10
3	15
4	20
5	25
6	30
7	35
8	40
9	45
10	50

Based on the information in the table above, each problem was worth 5 points, with a total score of 50. A graph of the data is shown below.



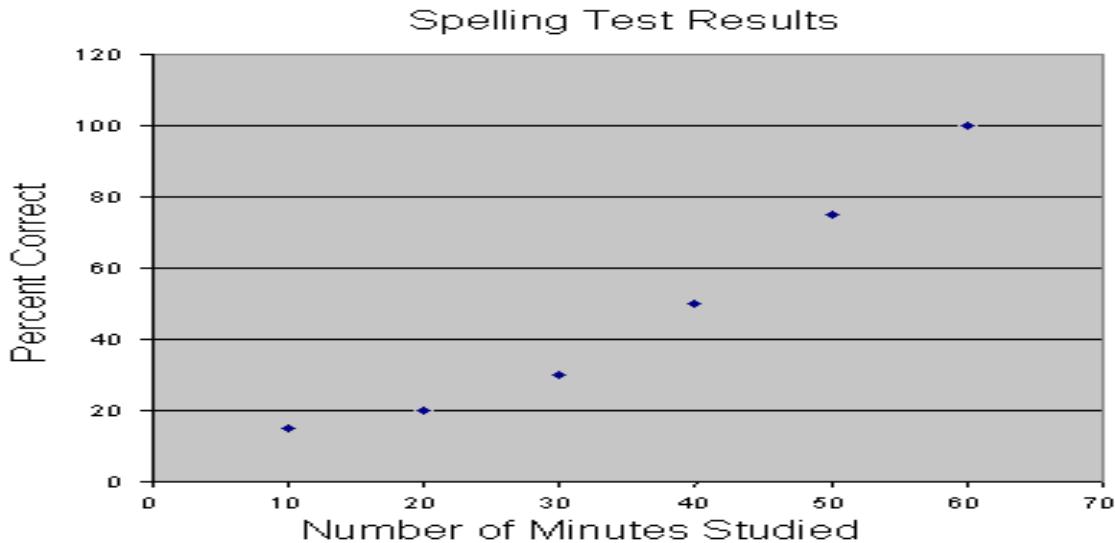
Notice that when all the data points are connected, the result is a **straight line**. This indicates a **linear relationship**. The word linear means line. A linear relationship indicates that as the number of questions answered correctly increases, the number of points will also increase by a value of 5 points per problem. **The rate at which the number of points increases is constant.**

Not all relationships are linear. For example, look at the table below. The table represents the number of minutes students studied for a spelling test and their actual grade on the test.

Time (minutes) Grade (percent correct)

10	15
20	20
30	30
40	50
50	75
60	100

A graph of the data is shown below.



Notice that if you connect the dots, you would not form a straight line, but a curve. Therefore, this is not a linear relationship. In other words, as the number of minutes increases, the percent correct does not increase at the **same rate** throughout the graph. The number of minutes increases by 10 each time, but the percent correct increases by values such as 5, 10, 20 and even 25. The goal is for students to recognize that the rate of increase is **not constant** (linear). An important characteristic of the relationship of study time versus percent correct is that the more the person studied, the greater the percent correct on the test. In other words, there was a positive result for the increase in study time, but not a linear one.

The above information was found at the following website:
<http://www.eduplace.com/math/mathsteps/3/e/index.html>

3. Select a representation of an algebraic expression, equation, or inequality that applies to real world situations.

-When you take a real-world situation and translate it into math, you are actually “expressing” it; hence the mathematical term “expression”.

-Everything that is **left** of the equal sign is considered to be **something you are expressing**. And everything to the **right** of the equal sign (or inequality) is **yet another expression**.

-Simply stated, an **EXPRESSION** is a combination of numbers, variables (letters) and operations. Expressions have numerical value. **EQUATIONS** are sometimes confused with expressions. To keep these two terms separate, simply ask yourself if you can answer it with a true/false. If so, you have an equation, not an expression which would have numerical value. When simplifying equations, one often drops expressions such as $7-7$ that equals 0.

A few samples:

Word Expression	Algebraic Expression
x plus 5	$x + 5$

10 times x y minus 12	$10x$ $y - 12$
--------------------------	-------------------

Testing Tip: *Word problems consist of sentences. You will need to read the problem through carefully to ensure you have some understanding of what you are being asked to solve. Pay close attention to the problem to determine the key clues. Focus on the final question of the word problem. Read the problem again to make sure you understand what you're being asked for. Then, jot down the expression:*

Here are some examples:

1. On my last birthday I weighed 125 pounds. One year later, I have put on x pounds. Which expression gives my weight one year later?
 a) $x + 125$ b) $125 - x$ c) $x + 125$ d) $125x$

2. If you multiply the square of a number n by 6 and then added 3 to the product, the sum is equal to 57. One of the expressions equals 57, which one is it?
 a) $(6n)^2 + 3$ b) $(n + 3)^2$ c) $6(n^2 + 3)$ d) $6n^2 + 3$

Answer for 1 is a) $x + 125$

Answer for 2 is d) $6n^2 + 3$

On your own:

Sample 1:

The price of a new radio is p dollars. The radio is on sale for 30% off. What expression will you write that will tell the savings that is being offered on the radio?

Answer: $0.3p$

Sample 2

Jane and her three college friends are going to be sharing the cost of a 4 bedroom apartment. The cost of rent is n dollars. What expression can you write that will tell you what Jane's share is?

Answer: $n/4$

The above information can be found at the following website:

<http://math.about.com/library/weekly/aa071002a.htm>

*In this section, you will learn how to solve inequalities. "Solving" an inequality means finding all of its solutions. A "solution" of an inequality is a number which when substituted for the variable makes the inequality a true statement.

Here is an example: Consider the inequality:

$$x - 2 > 5.$$

-When we substitute 8 for x , the inequality becomes $8-2 > 5$. Thus, $x=8$ is ONE solution of the inequality. On the other hand, substituting -2 for x yields a false statement $(-2)-2 > 5$. Thus $x = -2$ is NOT a solution of the inequality. **Inequalities usually have many solutions.**

-As in the case of solving equations, there are certain manipulations of the inequality which do not change the solutions. Here is a list of "permissible" manipulations:

Rule 1. Adding/subtracting the same number on both sides

Example: The inequality $x-2>5$ has the same solutions as the inequality $x > 7$.
(The second inequality was obtained from the first one by adding 2 on both sides.)

Rule 2. Switching sides and changing the orientation of the inequality sign

Example: The inequality $5-x > 4$ has the same solutions as the inequality $4 < 5 - x$.
(We have switched sides and turned the ``>'' into a ``<'', but notice that the expression $5-x$ stays in the same direction, it just switches sides.)

Last, but not least, the operations which makes inequalities difficult:

Rule 3a: **Multiplying/dividing** by the same POSITIVE number on both sides

Rule 3b: **Multiplying/dividing** by the same NEGATIVE number on both sides AND
changing the orientation of the inequality sign.

$2x \leq 6$ $x \leq 3$
Example 1: The inequality $\frac{2x}{2} \leq \frac{6}{2}$ has the same solutions as the inequality $x \leq 3$.
(We divided by 2 on both sides).

Example 2: The inequality $-2x > 4$ has the same solutions as the inequality $x < -2$.
(We divided by (-2) on both sides and switched ">" to "<".)

ANYTIME YOU MULTIPLY OR DIVIDE BY A NEGATIVE NUMBER YOU MUST
REMEMBER TO SWITCH THE SIGN!!!

$x^2 > x$
***The inequality $x^2 > x$ DOES NOT have the same solutions as the inequality $x > 1$.
(We were planning on dividing both sides by x , but we can't, because we do not know at this point whether x will be positive or negative!) In fact, it is easy to check that $x = -2$ solves the first inequality, but does not solve the second inequality.

***Only ``easy'' inequalities without exponents are solved using these three rules! ***

Example 1:

$$2x + 5 < 7.$$

The basic strategy for inequalities and equations is the same: isolate x on one side, and put the "other stuff" on the other side. Following this strategy, let's move +5 to the right side. We accomplish this by subtracting 5 on both sides (Rule 1) to obtain...

$$(2x + 5) - 5 < 7 - 5,$$

After simplifying the equation we obtain

$$2x < 2.$$

Once we divide by +2 on both sides (Rule 3a), we have succeeded in isolating x on the left:

$$\frac{2x}{2} < \frac{2}{2},$$

Or simplified,

$$x < 1.$$

All real numbers less than 1 solve the inequality. We say that the "set of solutions" of the inequality consists of all real numbers less than 1. In *interval notation*, the set of solutions is the interval:

$$(-\infty, 1)$$

Example 2:

$$5 - x \leq 6.$$

Let's start by moving the "5" to the right side by subtracting 5 on both sides (Rule 1):

$$(5 - x) - 5 \leq 6 - 5,$$

Or simplified,

$$-x \leq 1.$$

***How do we get rid of the “-” sign in front of x ? Just multiply by (-1) on both sides (Rule 3b),

AND REMEMBERING TO CHANGE \leq to \geq along the way:

$$(-x) \cdot (-1) \geq 1 \cdot (-1),$$

Or simplified

$$x \geq -1.$$

All real numbers greater than or equal to -1 satisfy the inequality. The set of solutions of the inequality is the interval

$$[-1, \infty)$$

Example 3:

Solve the inequality

$$2(x - 1) > 3(2x + 3).$$

Let us simplify first:

$$2x - 2 > 6x + 9.$$

There is more than one route to proceed; let's take this one: subtract $2x$ on both sides (Rule 1).

$$(2x - 2) - 2x > (6x + 9) - 2x,$$

And simplify:

$$-2 > 4x + 9.$$

Next, subtract 9 on both sides (Rule 1):

$$-2 - 9 > (4x + 9) - 9;$$

Simplify to obtain

$$-11 > 4x.$$

Then, divide by 4 (Rule 3a):

$$\frac{-11}{4} > \frac{4x}{4},$$

And simplify again:

$$\frac{-11}{4} > x.$$

It looks nicer, if we switch sides (Rule 2).

$$x < \frac{-11}{4}.$$

$$\left(-\infty, -\frac{11}{4}\right)$$

In interval notation, the set of solutions looks like this:

Example of a word problem:

Suzy's quiz score is greater than or equal to Jimmy's quiz score of 90:

If Suzy is represented by an S , then: $S \geq 90$

The above information can be found at the following website:

<http://www.sosmath.com/algebra/inequalities/ineq01/ineq01.html>

Math Competency #5

1. Apply the concepts of range and central tendency (mean, median, and mode).

Mean: The Mean is the arithmetic average. This is the average you are probably used to finding for a set of numbers:

* add up the numbers and divide by how many there are:
 $(80 + 90 + 90 + 100 + 85 + 90) / 6 = 89 \frac{1}{6}$.

Median: The Median is the number in the middle. In order to find the median, you have to put the values in order from lowest to highest, and then find the number that is exactly in the middle:

80 85 90 90 90 100

Since there is an even number of values, the median is between these two, or it is 90. Notice that there is exactly the same number of values above the median as below it!

-If there is an even number of values, and the two middle numbers are different, you take the average:

80 85 90 92 95 100

Here, the two middle numbers are 90 and 92, so you would take the average, which is 91.

Mode: The mode is the value that occurs most often. In this case, since there are 3 90's, the mode is 90. A set of data can have more than one mode.

Range: The range is the difference between the lowest and highest values. In this case $100 - 80 = 20$, so the range is 20. The range tells you something about how spread out the data are. Data with large ranges tend to be more spread out.

The above information can be found at the following website:

<http://mathforum.org/library/drmath/view/58326.html>

You may also reference Shannon Black's Book on page 142.

2. Determine probabilities of dependent or independent events.

First, it is important to understand 'sample space'. The sample space is the set of all possible outcomes for an event or experiment. For example, the sample space of a die (one of a pair of dice) is six: $S = \{1, 2, 3, 4, 5, 6\}$. The sample space is 6 because there are 6 possible outcomes. The sample space for a coin is two: $S = \{H, T\}$, H for heads and T for tails. The sample space is 2 because there are 2 possible outcomes.

Now, an independent event occurs when two events A and B occur, and when one of them happens, it doesn't affect the other one happening or not. Let's use a real life example...

Let's say that you have a coin and a die (one of a pair of dice). You want to find the probability of tossing the coin, getting heads one time, and then tossing the die and getting a five one time. We'll call the coin toss event A. The plain old probability of tossing a coin and getting heads is $1/2$. That is:

$$A = \frac{\text{The number of favorable outcomes}}{\text{Total possible outcomes (Sample space)}} = \frac{1}{2}$$

The probability of getting a five when you toss the die will be event B and that is:

$$B = \frac{\text{The number of favorable outcomes}}{\text{Total possible outcomes (Sample space)}} = \frac{1}{6}$$

Now for the independent part. Does your chance of getting a five when you toss the die have anything to do with whether you get heads or tails when you toss the coin? It does not. That's why they are independent events.

The probability of independent events occurring is found by multiplying the probability of the first event occurring by the probability of the second event occurring. Generally, it looks like this: (P) stands for probability of, (A) stands for event A, and (B) stands for event B

$$P(A, B) = P(A) * P(B)$$

In our example it looks like this:

$$P(H, 5) = 1/2 * 1/6 = 1/12$$

Now for **dependent events**. A dependent event is one where the outcome of the second event is influenced by the outcome of the first event.

For example... Let's say we have a box with 6 marbles: 3 red, 1 blue, 1 green and 1 yellow. What's the probability of picking a yellow marble? We know that probability is 1/6. What's the probability of picking a blue marble? Can it be 1/6 also? Well, it could be if we put back the first marble we picked. But if we don't put back the first marble, our sample space will have changed. We started with six marbles, picked one, and now we only have five marbles in the sample space, so the probability of picking a blue marble is now 1/5. And in such a case we have dependent events, because something about the first one changed the probability of the second.

The probability of two dependent events occurring, one right after the other, is still found by using the same formula: $P(A, B) = P(A) * P(B)$

*The big difference is that the individual probabilities won't have the same sample spaces. So from our example, what is the probability of picking a yellow marble and then a blue marble, without putting the first marble back?

$$P(\text{Yellow}) = 1/6$$

$$P(\text{Blue}) = 1/5$$

$$P(Y, B) = 1/6 * 1/5 = 1/30$$

This is a very different number from what we would get if the events were independent: that is if the sample space remained the same because we put the first marble we picked back into the box.

$$P(\text{Yellow}) = 1/6$$

$$P(\text{Blue}) = 1/6$$

$$P(Y,B) = 1/6 * 1/6 = 1/36$$

So the trick is to figure out ahead of time if the events are independent or dependent, and then use the formula: $P(A, B) = P(A) \times P(B)$

The above information can be found at the following website:

<http://mathforum.org/library/drmath/view/56494.html>

You may also refer to Shannon Black's book on page 143.

3. Determine odds for and odds against a given situation.

Odds are another way to write probability. The odds of pulling out a yellow marble could still be expressed as 1/6. Now, the odds that it would not occur, can be determined by subtracting 1 - 1/6 ... so the odds of NOT pulling a yellow marble would be 5/6. When you add the odds for and odds against a situation, they should ALWAYS equal 1.0

4. Apply fundamental counting principles such as combinations to solve probability problems.

Fundamental Theorem of Arithmetic - Every integer greater than one is either prime or can be expressed as a unique product of prime numbers. (Prime factor tree)

Fundamental Counting Principle - If there are M ways to do one thing, and N ways to do another, then there are $M \times N$ ways of doing both. The Fundamental Counting Principle is the guiding rule for finding the number of ways to accomplish two tasks. Examples using the counting principle...

Let's say that you want to flip a coin and roll a die. There are 2 ways that you can flip a coin and 6 ways that you can roll a die. There are then $2 \times 6 = 12$ ways that you can flip a coin **and** roll a die.

If you want to draw 2 cards from a standard deck of 52 cards without replacing them, then there are 52 ways to draw the first and 51 ways to draw the second, so there are a total of $52 \times 51 = 2652$ ways to draw the two cards.

Example 1: If you are ordering a sandwich and you can have either turkey or ham, lettuce, tomato, or onion, and either cheddar or American cheese...how many different combinations can you make if you can only pick one thing from each category?

2 (turkey or ham) **X** **3** (lettuce, tomato, or onion) **X** **2** (American or Cheddar) = **12**

Example 2: There are 2 yellow marbles and 5 red marbles. If you pick one marble without replacement, what's the probability that you will get a red marble and then a red again?

20/42 or 10/21

This is what we call a dependent event, because picking the red marble the second time is dependent upon picking the red marble the first time. The probability of picking the first red marble is 5/7 because 5 out of the 7 marbles are red. Since we are not replacing the marble, than the probability of picking a red marble again will be 4/6. This is because we already took out one red marble, so now instead of 5 red marbles we have four red marbles, and we also no longer have 7 marbles we are choosing from, but now we have 6.

The last step is to multiply the two probabilities together since we want to find the probability of them occurring together. So, we set up $5/7 \times 4/6$, and get 20/42. ($4 \times 5 = 20$, and $7 \times 6 = 42$)

20/42 can be simplified into 10/21!

Permutations - A permutation is an arrangement of objects, without repetition, and order being important. Another definition of permutation is the number of such arrangements that are possible. $n!$ is equal to the number, times each number below it. For example...

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

This is expressed as:

$${}_nP_r = P(n, r) = \frac{n!}{(n-r)!}$$

Since a permutation is the number of ways you can arrange objects, it will always be a whole number. The denominator in the formula will always divide evenly into the numerator.

*The **n** value is the total number of objects to choose from.*

*The **r** is the number of objects you're actually using.*

The two key things to notice about permutations are that there is no repetition of objects allowed and that order is important. Examples of permutations...

Example 1: List all permutations of the letters ABCD

ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

Now, if you didn't actually need a listing of all the permutations, you could use the formula for the number of permutations.

There are 4 objects and you're taking 4 at a time, so: ${}_4P_4 = 4! / (4-4)! = 4! / 0! = 24 / 1 = 24$

Example 2: List all three letter permutations of the letters in the word HAND

HAN	AHN	NHD	DHA
HNA	ANH	NDH	DAH
HAD	AHD	NAH	DAN
HDA	ADH	NHA	DNA
HND	AND	NAD	DHN
HDN	ADN	NDA	DNH

Now, if you didn't actually need a listing of all the permutations, you could use the formula for the number of permutations. There are 4 objects and you're taking 3 at a time, so:

$${}_4P_3 = 4! / (4-3)! = 4! / 1! = 24 / 1 = 24.$$

Combinations - A combination is an arrangement of objects, without repetition, and order not being important. Another definition of combination is the number of such arrangements that are possible.

$${}_nC_r = C(n, r) = \binom{n}{r} = \frac{n!}{(n-r)! \cdot r!}$$

The **n** in the formula stand for the total number of objects to choose from
And **r** stands for the number of objects in the arrangement.

The key points to a combination are that there is no repetition of objects allowed and the order isn't important.

For example...

List all combinations of the letters ABCD in groups of 3. There are only four combinations (ABC, ABD, ACD, and BCD). Listed below each of those combinations are the six **permutations** that are equivalent as combinations.

ABC	ABD	ACD	BCD
ABC	ABD	ACD	BCD
ACB	ADB	ADC	BDC
BAC	BAD	CAD	CBD
BCA	BDA	CDA	CDB
CAB	DAB	DAC	DBC
CBA	DBA	DCA	DCB

Examples of Combinations

1. The old Illinois Lottery had 54 balls, of these 54 balls, six are chosen. None of the six can be repeated and the order of the six is not important. That makes it a combination:

$54! / (54!-6!)6!$ This can be done by hand or a calculator to reach... $C(54, 6) = 25,827,165$.

2. I was told that on January 17, 1998, the Illinois Lottery will be changing to 48 balls, six of which are chosen. Try to reach the answer on your own before looking below.

Now, the number of possibilities will be $C(48, 6) = 12,271,512$

3. How many 5 card poker hands are there with 3 clubs and 2 diamonds?

Well, there is no repetition of cards in a hand, and the order doesn't matter, so we have a combination again. Since there are 13 clubs and we want 3 of them, there are $C(13,3) = 286$ ways to get the 3 clubs. Since there are 13 diamonds and we want 2 of them, there are $C(13,2) = 78$ ways to get the 2 diamonds. **Since we want them both to occur at the same time, we use the fundamental counting principle and multiply 286 and 78 together to get 22,308 possible hands.**

Difference between Permutations and Combinations

The distinguishing feature between Permutations and Combinations is not whether or not there is repetition. **Neither one allows repetition. The difference between the two is whether or not order is important.** If you have a problem where you can repeat objects, then you must use the Fundamental Counting Principle, you can't use Permutations or Combinations.

The above information was taken from the following website:

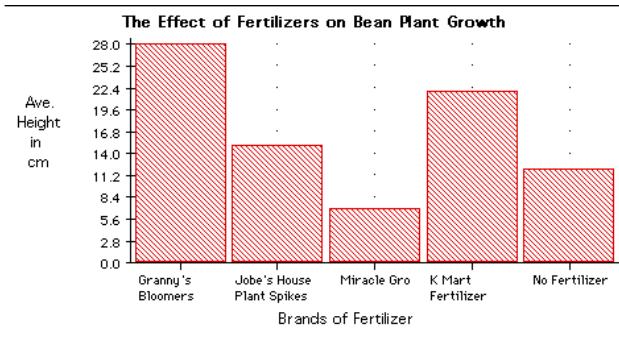
<http://www.richland.edu/james/lecture/m116/sequences/counting.html>

5. Interpret information from tables, charts, line graphs, bar graphs, circle graphs, box and whisker graphs, and stem and leaf plots.

Be sure to give your charts and graphs an appropriate title that explains what the data measures. On line and bar graphs, the x and y axes must be appropriately labeled with correct unit of measure (in metrics where applicable). There are three basic graph forms. The bar graph, the line graph, and the circle (or pie) graph.

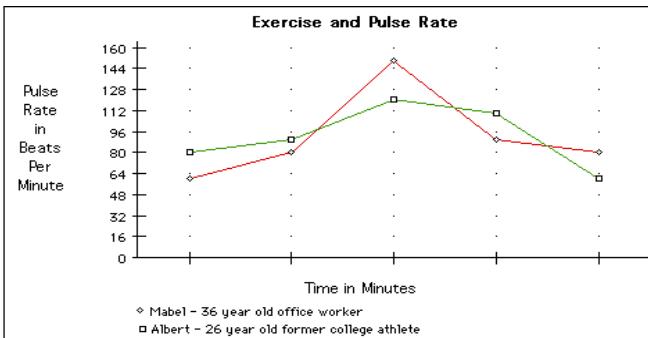
Bar graph:

A bar graph is used to show relationships between groups. The two items being compared do not need to affect each other. It's a fast way to show big differences. Notice how easy it is to see what was done in the experiment below with bean plant growth and different brands of fertilizer. A typical chart or table for this graph might look like this:



Line graph:

A line graph is used to show continuing data; how one thing is affected by another. It's clear to see how things are going by the rises and falls a line graph shows. This kind of graph is needed to show the effect of an independent variable on a dependent variable. In the sample below, the pulse rate of a person is shown to change over time. As time continues, the pulse rate changes. A typical chart or table for this graph might look like this:



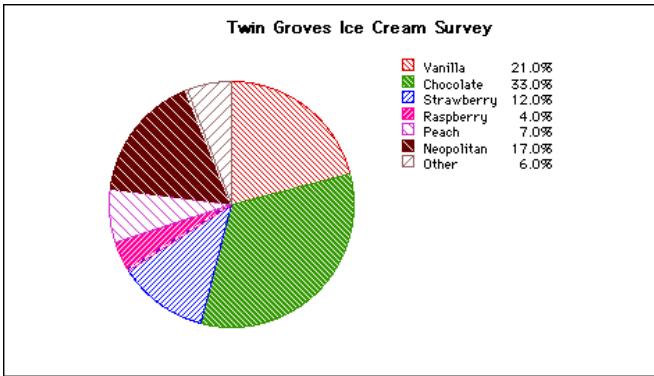
An example of a question using a line graph might be as follows:

What is the range of average temperature in Florida between April and November? To illustrate the question, you might see a line graph. The y-axis might have the temperatures and the x-axis might have the months.

For this question, you want to find the temperature in April and the temperature in November. You want to subtract the lowest month from the highest month. The answer will be the range. For example, if the temperature in April is 85 degrees, and the temperature in November is 25 degrees, then $85-25$ is 60. So the answer would be 60 degrees.

Circle (pie) graph:

A circle graph is used to show how a part of something relates to the whole. This kind of graph is needed to show percentages effectively. A typical chart or table for this graph might look like this:



An example of a question using a pie chart might be as follows:

A family has a budget of \$45000. How much did they spend in clothing and food? To illustrate this example, you might see a pie chart with percentages. For example, one section says: food 30%, another one Clothing 10%, another one mortgage 20% and so on.

For this question, you will need to find the percentages of 45,000 that they ask for. 10% can be found by removing the zero at the end of 45,000. When you take away the last zero, 45,000 turns into 4,500. So, 10% of 45,000 is 4,500.

Answer: Clothing (10%): \$4,500

Then, to find 20 percent, you can either double the 10%, since 10% plus 10% is 20%, and 10% was equal to 4,500, then 4,500 plus 4,500 (9,000) equals 20%. OR you can multiply 45,000 x .20.

$$\begin{array}{r}
 45,000 \\
 \times \quad .20 \\
 \hline
 00000 \\
 + 900000 \\
 \hline
 900000.0
 \end{array}$$

900000.0, and then move the decimal place TWO spaces to the left because it is TWO spaces to the left of (.20). So, the answer is 9,000, as we said before.

Answer: Mortgage (20%): \$9,000

The above information can be found at the following website:

<http://www.twingroves.district96.k12.il.us/ScienceInternet/ChartsGraphs.html>

Box and Whisker Plot:

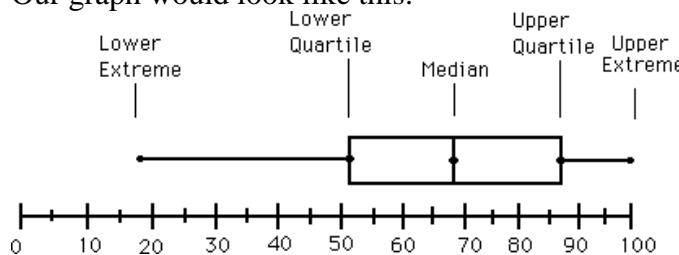
A box-and-whisker plot can be useful for handling many data values. They allow people to explore data and to draw informal conclusions when two or more variables are present. It shows only certain statistics rather than all the data. *Five-number summary* is another name for the visual representations of the box-and-whisker plot. The five-number summary consists of the median, the quartiles, and the smallest and greatest values in the distribution.

The first step in constructing a box-and-whisker plot is to first find the *median*, the *lower quartile* and the *upper quartile* of a given set of data. Example: The following set of numbers is the amount of marbles fifteen different boys own (they are arranged from least to greatest).

18 27 34 52 54 59 61 68 78 82 85 87 91 93 100

1. First find the median. The median is the value exactly in the middle of an ordered set of numbers... **68 is the median**
2. Next, we consider only the values to the left of the median: 18 27 34 52 54 59 61. We now find the median of this set of numbers. Remember, the median is the value exactly in the middle of an ordered set of numbers. Thus 52 is the median of the scores less than the median of all scores, and therefore is the lower quartile... **52 is the lower quartile**
3. Now consider only the values to the right of the median: 78 82 85 87 91 93 100. We now find the median of this set of numbers. The median 87 is therefore called the upper quartile... **87 is the upper quartile**
4. You are now ready to find the inter-quartile range (IQR). The inter-quartile range is the difference between the upper quartile and the lower quartile. In our case the $IQR = 87 - 52 = 35$. The IQR is a very useful measurement. It is useful because it is less influenced by extreme values, it limits the range to the middle 50% of the values... **35 is the inter-quartile range**

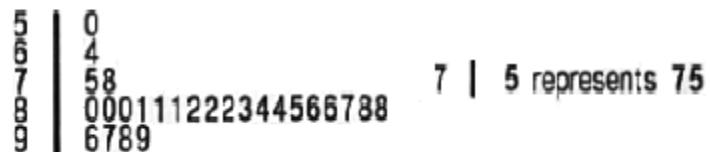
Our graph would look like this:



The above information can be found at the following website:
<http://ellerbruch.nmu.edu/cs255/jnord/boxplot.html>

Stem and Leaf Plot:

4th. Grade Test Scores



Another type of graph is the stem-and-leaf plot. The numbers on the left side of the vertical line are the stems. The numbers on the right side are the leaves. In our graph, the stems are the tens digits of the scores on the test, and the leaves are the unit digits. In this case, 9/6 represents a score of 96. So the 8/000111 means there were 3 scores of 80 and 3 scores of 81. To make an ordered stem-and-leaf plot, we arrange the leaves on their given rows from least to greatest, starting from the left.

The above information can be found at the following website:

<http://ellerbruch.nmu.edu/cs255/jnord/stemplot.html>

6. Make accurate predictions and draw conclusions from data.

This has been addressed above, in many different examples!

Congratulations! You are now done with the Math section!