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Modeling the Dynamics of Risky Choice

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Individuals make decisions under uncertainty every day. Decisions are based on incomplete information concerning the potential outcome or the predicted likelihood with which events occur. In addition, individuals' choices often deviate from the rational or mathematically objective solution. Accordingly, the dynamics of human decision making are difficult to capture using conventional, linear mathematical models. Here, we present data from a 2-choice task with variable risk between sure loss and risky loss to illustrate how a simple nonlinear dynamical system can be employed to capture the dynamics of human decision making under uncertainty (i.e., multistability, bifurcations). We test the feasibility of this model quantitatively and demonstrate how the model can account for up to 86% of the observed choice behavior. The implications of using dynamical models for explaining the nonlinear complexities of human decision making are discussed as well as the degree to which the theory of nonlinear dynamical systems might offer an alternative framework for understanding human decision making processes.

Guy Van Orden was a pioneer—a mentor, colleague, and collaborator who constantly challenged and inspired us to think of new ways to explore and understand human cognition and behavior. His general focus on fractal scaling, complexity science, and nonlinear dynamics is well documented (e.g., Van Orden, 2010; Van Orden, Holden, & Turvey, 2003, 2005). Perhaps less documented was his interest in how people respond during situations of ambiguity and uncertainty. It was this interest that inspired the research presented here, namely, to investigate

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the feasibility of extending the modeling techniques of nonlinear dynamical systems to current decision-making science.

Two major streams of research are generally distinguished within decision-making science: normative research and descriptive research. Whereas normative research seeks to define what would be the optimal decision in a given situation, descriptive research details how humans actually decide in everyday life (Johnson & Busemeyer, 2010). The significance of this later form of research is that it provides evidence that individuals' decision-making behavior is sensitive to loss-gain framing. For instance, when a decision is framed in terms of potential loss, the majority of individuals will avoid taking risk, but when the same decision is framed in terms of potential gain, the majority of individuals do take risk (Tversky & Kahneman, 1981). Objectively, the choices are exactly the same, but because of subtle differences in wording, they elicit opposite choices. Other descriptive research has demonstrated that risks with low probabilities are either grossly overweighed or completely neglected by decision makers (Tversky & Kahneman, 1983). Moreover, there is large heterogeneity among individuals with respect to perceived risk and gain. Specifically, there is significantly greater variability between individuals when decisions involve a potential loss compared with potential gain (Kahneman & Tversky, 1979; Tversky & Kahneman, 1981). Collectively, such findings indicate that human decision making under uncertainty is highly context specific, heterogeneous, and multistable.

The knowledge that decision making under uncertainty is multistable provides evidence that such behavior is characterized by nonlinear dynamics. A multistable system can, for the same input, settle in more than one possible stable state. A possible consequence of multistability is *hysteresis*, which occurs when a system's immediate history influences the current state of the system. Sir James Alfred Ewing first coined the term *hysteresis* while observing the phenomenon in magnetic materials (Ewing, 1881). Figure 1 displays hysteresis in the magnetization and demagnetization of a magnet as a result of varying strength of the magnetic force. Depending on the direction of change of the magnetic field, the change from magnetization in one direction to the opposite direction occurs at a different moment. Thus, the system has a primitive form of memory, remaining in an existing stable state longer than would otherwise be expected. The opposite of hysteresis, *reversed hysteresis,* can also occur in multistable systems. Rather than remaining in the existing stable state longer (as with hysteresis), the system changes to another stable state sooner.

In addition to physical systems, the behavior of many biological systems, including human behaviors, exhibits hysteresis or reversed hysteresis. Indeed, previous research has demonstrated how such behavioral dynamics are known to characterize body-scaled transitions between the actualization of different action modes (i.e., one- to two-hand object grasping; Lopresti-Goodman, Turvey, & Frank, 2011; Richardson, Marsh, & Baron, 2007), perceived action possi-

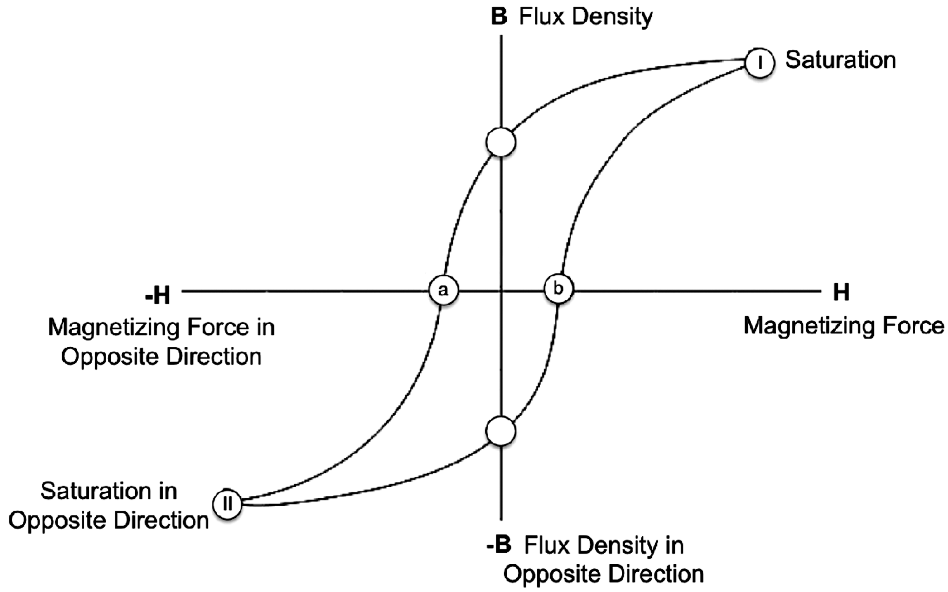

FIGURE 1 Hysteresis in magnets. The relation between magnetic force *H* and magnetic flux *B*. A magnet is magnetized by a magnetizing force *H* into direction *B* (state *I*). If the strength of *H* is then slowly decreased, the saturation of the magnet will change until it becomes fully magnetized into the opposite direction $-B$ (state *II*). If *H* is then increased again, the change toward saturation in the positive direction *B* happens at a different value for the strength of the magnetic force *H* (the difference between *a* and *b*).

bilities (Fitzpatrick, Carello, Schmidt, & Corey, 1994), speech categorization (Tuller, Case, Ding, & Kelso, 1994), and problem solving (Stephen, Boncoddo, Magnuson, & Dixon, 2009). Accordingly, the first step toward a nonlinear dynamical account of decision making under uncertainty entails empirically verifying whether hysteresis and/or reversed hysteresis are characteristic features of risky choice behavior. If so, the second step will be to attempt to develop a dynamical model with two attractors that is able to capture such decision-making dynamics. In order to test for hysteresis and reversed hysteresis in decision making, we adopted a standard model of risky decision behavior with the implicit assumption that real-world decisions under uncertainty have the same properties as a monetary gamble (Hertwig & Erev, 2009). A typical example of the type of monetary gamble researchers use to study risky decision behavior is the *risky choice* between a *sure loss* (*S*) of $750 and a *risky loss* (*R*) consisting of a 0.75 probability (*p*) to lose $1,000 (Tversky & Kahneman, 1981). The parameters in a risky choice are the probability to lose and the values of *R* and *S*. The outcome is either a *risk-seeking* choice for *R* or a *risk-avoiding* choice for *S*.

Two key components to finding hysteresis or reversed hysteresis in risky choice are (a) to change the context in two opposite directions and (b) to do this in a systematic way. It is necessary to find an input parameter that, at different values, results in opposite, or at least qualitatively different, values of the system's output. In risky choice, the key parameter that drives the choice between risk-seeking and risk-avoiding behavior is the amount of risk that is present in *R*. There are several ways to vary the amount of risk in *R*; we have opted to manipulate the value of the risky loss (in $, a high value of *R* corresponds with a high risk). Only when the value of *R* is first increased and then decreased or vice versa will there be an opportunity to observe hysteresis and/or reversed hysteresis. A *sequential* risky choice task is therefore a sequence of consecutive risky choices between *S* and *R*[1] in which the value of *R* is either increased or decreased in a stepwise fashion.

In a sequential choice task, hysteresis looks like this: A decision maker is presented with a risky choice where the risk in *R* is minimal (relative to *S*) and chooses *R*. Next, the decision maker is presented with a second risky choice in which the risk in *R* is slightly higher. Next, another risky choice occurs that is even riskier, and so on. All the while the decision maker continues choosing *R*. Then, at some *switch-point* (see definition later), when the risk in *R* has become too high, the decision maker will switch to choosing *S* and continue to do so until the risk in *R* is maximal (relative to *S*). Then, the whole process is reversed by decreasing the risk in *R* again, causing the decision maker to switch back from choosing *S* to choosing *R* at another switch-point. If the second switch occurs for a lower risk in *R* than the first, we have found an indication of hysteresis. If the second switch occurs for a higher risk in *R* than the first, we have found an indication of reversed hysteresis.

METHOD

Participants and Design

Thirty-six undergraduate students from the University of Cincinnati were presented with three sets of sequential risky choices between a risky and a sure loss. In the first and third set, the amount of risk in *R* was systematically varied, either in increasing and then decreasing order (*ID*) or vice versa (*DI*). The second set always contained the same choices but in randomized order to mediate carryover effects between the first and third sets. Half of the students were first presented with the *ID* set followed by the random set and the *DI* set. The other half were presented first with the *DI* set. The value of *R* ranged from $1,500 to $525 with

[1] Note that objectively, in each risky choice, *S* is the better choice as soon as the sure loss of *S* is lower than the expected value of *R*, whereas *R* is the better choice as soon as the expected value of *R* becomes lower than the sure loss of *S*.

increments of \$25. The probability to lose this amount $p = .75$, and $S = \$750$. The total amount of choices was 238.

Stimuli and Apparatus

All stimuli were variations of the risky choice example given in the introduction and contained the values for p, R, and S. In total, 40 different values of R (ranging from \$525 to \$1,500 with increments of \$25) were presented either on the left side of the screen, with the value of S on the right, or vice versa. The stimuli were presented on an iMac, and a cordless computer mouse was used to select the choices; both were run using PsychToolbox software (Brainard, 1997).

Procedure

Participants provided their written consent and received instructions about the sequential risky choice task. Participants were seated in front of the computer screen that displayed the various choices and were instructed to indicate their choice preferences using the mouse.

RESULTS

Choice outcomes of 22% of the participants showed no change at all. This is consistent with an earlier experiment with a smaller range of risk in R (from \$725 to \$1,175) in which 27% of the participants showed no change. This indicates either of the following: (a) for about one fourth of participants, the attractor for S is nonexistent or (b) the initial conditions strengthen the attractor for R relative to S such that the changing constraints provide too little perturbation to the system. A small follow-up study ($N = 16$) was conducted in which participants were instructed to decide as quickly as possible while still using the available information on the screen. It was hypothesized that this speed manipulation would destabilize the initial strength of the attractor for R. All participants switched at least once between S and R ($M = 10.8$ fluctuations, $SD = 12.5$), and the relation between the speed manipulation and the absence of "no change" participants is significant, $\chi(1, N = 52) = 4.20$, $p = .04$. The speed manipulation increased variability and caused participants to be more sensitive to changing risk constraints.

The remaining 28 participants exhibited a *phase transition* between risk-seeking and risk-averse choices at least once per sequence ($M = 3.8$ *fluctuations*,[2] $SD = 3.4$). Using an automated search algorithm, two *switch-points*[3] per

[2] A *fluctuation* is defined as each choice that is different from the previous choice.

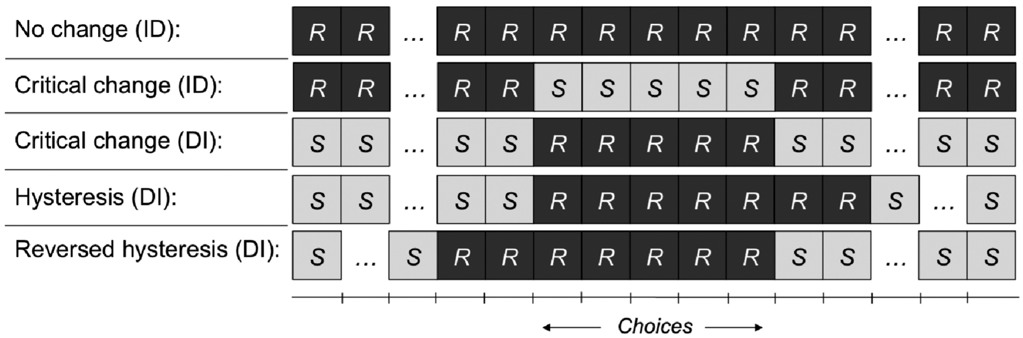

FIGURE 2 Model changes between choices for *R* and *S*. *No change* occurs when participants do not switch. *Critical change* occurs when participants switch from *S* (*R*) to *R* (*S*) for the same amount of risk in the first and second half of the *DI* (*ID*) sequence. *Hysteresis* occurs when participants switch from *S* (*R*) to *S* (*R*) *later* in the second half on the *DI* (*ID*) sequence. *Reversed hysteresis* occurs when participants switch from *S* (*R*) to *R* (*S*) *earlier* in the first half on the *DI* (*ID*) sequence.

ID and *DI* sequence were determined for each participant. Based on the locations of the switch-points, 48% of participants showed critical change followed by reversed hysteresis (39%), and hysteresis (13%); see Figure 2 for details. The average value of the risk in *R* for switches from *S* to *R* was $1,000 ($SD = \$215$) and from *R* to *S*, $941 ($SD = \$174$), indicating that overall, participants were slightly risk averse. The distance between the two switch-points for the *DI* and *ID* sequences was significantly larger compared with the random sequences $t(27) = 3.61,\ p = .001$.

DYNAMICAL MODELING

The types of phase transitions exhibited by participants in the aforementioned study, as well as the multistable nature of switching behavior, is problematic for most linear models but can be accounted for by a *nonlinear dynamical system* (e.g., Cho, Jones, Braver, Holmes, & Cohen, 2002; Roxin & Ledberg, 2008). A dynamical system is a mathematical concept where the time dependence of a state variable (a variable that describes a certain quantity of a system that we are interested in, e.g., position or concentration) is described using a fixed rule. In a *nonlinear* dynamical system this fixed rule is nonlinear. Therefore, the system does not satisfy the additivity and homogeneity properties that are necessary for

[3]A *switch-point* is defined as the closest fluctuation to the middle choice for which, in case of an *ID* sequence, the number of *R* choices in between this fluctuation and the first *S* choice in a continuous stretch of *S* choices spanning the middle is less than the number of *S* in between. In case of a *DI* sequence, it is the other way around.

linearity. Examples of applications of nonlinear dynamical modeling to human behavior include vision (Fürstenau, 2006), speech (Kelso, Saltzman, & Tuller, 1986; Tuller et al., 1994), language (Spivey, Grosjean, & Knoblich, 2005), motor and neural dynamics (Haken, Kelso, & Bunz, 1985; Kelso et al., 1992; Schöner & Kelso, 1988), and cognition (Bressler & Kelso, 2001). Applications of dynamical models to decision making under uncertainty have focused on either microlevel or macrolevel behavioral observations. For example, on the microlevel, Brown & Holmes (2001) modeled a simple choice task using a dynamical model of firing rates of neurons. On a macrolevel, dynamical models of multiagent decision-making processes have developed (for a brief overview, see Lu, Chen, & Yu, 2011). With respect to the decision-making behavior of individuals, however, we are not aware of any previous attempts to model such behavior using a nonlinear dynamical model. Therefore, what follows is our initial attempt to model the behavioral dynamics and phase transitions exhibited by participants in the aforementioned experimental study using a simple one-dimensional nonlinear dynamical model.

A ONE-DIMENSIONAL MODEL OF PHASE TRANSITIONS AND MULTISTABILITY IN RISKY CHOICE

To model the observed switching between *R* and *S*, we propose a nonlinear dynamical system that has previously been applied to other cases in which individuals switched between two different behaviors and where nonlinear phenomena like hysteresis and reversed hysteresis informed the use of a nonlinear dynamical model (e.g., Tuller et al., 1994). Equation 1 gives the potential function of the one-dimensional model,

$$V(x) = kx - \frac{x^2}{2} - \frac{x^4}{4} + \xi, \tag{1}$$

where x is the observed choice, k the control parameter, and a noise term ξ is added to each choice.

A potential function is the integral of the differential equation describing the evolution of the state variable x (in our case, the observed choices). This means that a minimum or maximum of the potential function corresponds with a stable state of the system. Our system's potential function therefore reveals the *attractor* and *repeller states* to which the system is attracted or repelled from (see Kelso, 1995, and Strogatz, 2000, for more background on dynamical systems). The behavior of our dynamical system is driven by a control parameter k. Figure 3 shows some examples of the shape of the potential function, or

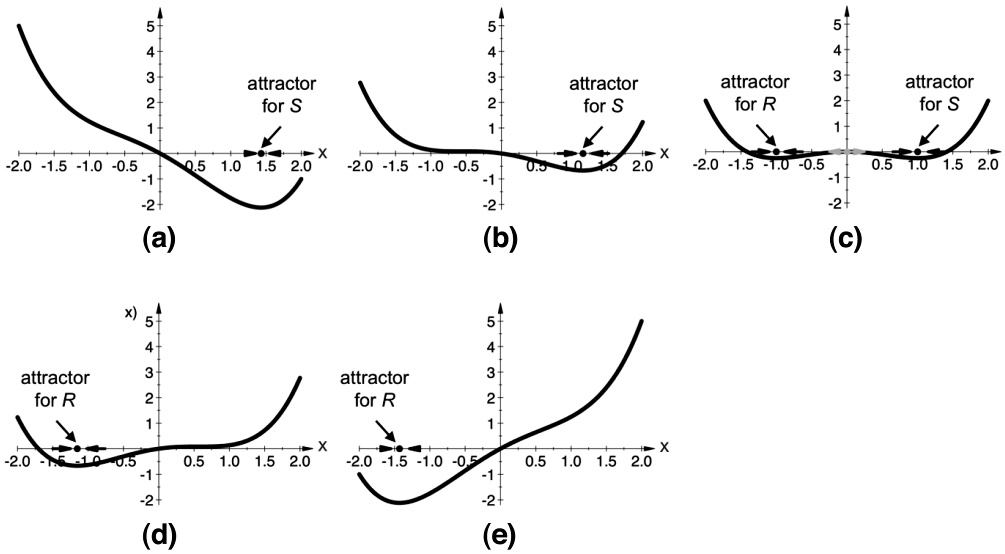

FIGURE 3 Potential landscape for five different values of k. Depending on the direction of change, a phase transition occurs between the two possible attractors for a critical value of k, $\pm k_c$.

attractor landscape, for different values of k. For a critical value of k, k_c, a *bifurcation* occurs (for both $k = k_c$ and $k = -k_c$), causing a phase transition between risk-seeking and risk-avoiding choices or vice versa. A phase transition occurs for a different value of k, depending on the direction of change, which explains hysteresis. By defining the two attractor states as the choice for R and S, respectively, this model thus explains phase transitions between risk-seeking and risk-avoiding choices as well as multistability through hysteresis (although not reversed hysteresis; see later for a more detailed discussion of reversed hysteresis).

Parameter Selection and Optimization

The potential function offers a way to simulate sequential choice data. The key to modeling the risky choice phenomena is the control parameter k, which must reflect the changing risk in R. We propose k as a simple linear function of a baseline value, k_0; the risk in R at choice j; and a parameter specifying the sensitivity to the changing risk, β, such that

$$k_j = k_0 - \beta R_j. \tag{2}$$

Because k_0 controls the vertical position of k relative to the critical values of the model, $\pm k_c$, and β controls the rate of change of k as a function of change in *R*, different combinations of values for k_0 and β result in different choice behaviors. A high value of β in relation to k_0 results in critical change, a high value of k_0 in relation to β results in no change, and balanced values of β and k_0 result in hysteresis.

By sampling k_0 and β from uniform distributions spanning all possible values of k between two extremes, and using Equations (1) and (2), we simulated an entire range of possible choice data for $j = 1{:}79$. The lower boundary for k_0 corresponds to the case where only the attractor for *S* exists, regardless of the value of the risk in *R*, and the upper boundary corresponds to only one attractor for *R*. The lower boundary for β corresponds to a large hysteresis effect and the upper boundary corresponds to critical change. Using a bootstrapped optimization with respect to the difference between the simulated and empirical choices on the *DI* and *ID* sequences of our main experiment ($N = 36$), we were able to simulate 86% of the observed choices. The differences in switch-points for reversed hysteresis are relatively small compared with the total range of values for *R* ($M = \$170.45$, $SD = \$183.08$). This accounts for the model's ability to generate a high proportion of correct choices despite not accounting for reversed hysteresis.

DISCUSSION

There are many models of risky choice (see Glöckner & Pachur, 2012, for a review). However, nonlinearity is a necessary assumption in order to account for multistability. The results presented here show multistability in risky choice for which we have provided a preliminary nonlinear dynamical model. The model provides a way to explore decision making under uncertainty within the framework of complexity theory and provides the blueprint for a nonlinear dynamical model that can potentially capture the entire range of observed choice behavior.

Remaining questions include (a) how to model reversed hysteresis and (b) what are the theoretical implications of modeling risky choice with a nonlinear dynamical model? Lopresti-Goodman, Turvey, & Frank (2012) provide a possible solution to the first question by using a so-called *auto-regulated* control parameter. Negative auto-regulation forces the dynamical system to remain close to a bifurcation and may reflect habituation to the amount of risk presented in the choices, rendering the choice for *S* or *R* unstable. We are currently investigating the possibility of adapting the control parameter k using this habituation in order to model reversed hysteresis in risky choice behavior. Because such a control parameter might determine characteristic switching behavior, it may also offer insight into the psychological processes behind when and why individuals make risky choices. Because a risky choice task measures whether individuals' choices are risk seeking or risk averse, the moment of switching in a *sequential* risky choice task could also be an indication of the individual's sensitivity to risk, given the choice and context. If that is indeed the case, then the control parameter, which controls the moment of switching, should capture some of that risk sensitivity.

In conclusion, this study provides initial evidence that the risky decision behavior of individuals can be understood and modeled using a nonlinear dynamical system. Although the degree to which adopting this approach to decision making will provide transformative insights remains unclear, future research aimed at uncovering the complex nonlinear dynamics of decision making are likely to result in novel theoretical and methodological questions that may challenge our existing understanding of human decision making. Being so greatly inspired by Guy Van Orden and his work, it is our hope that such challenges may move us closer to a theory about how people respond during situations of ambiguity and uncertainty grounded in the natural laws of self-organization and emergence.

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