

The SlateMath Approach to Early-Age Math Education

Raz Kupferman¹

raz@slateScience.com

Shimon Schocken²

shimon@slateScience.com

Slate Science White Paper, March 28, 2013

SlateMath is a series of software apps and teaching materials, designed to support math education at the elementary school level (kindergarten through grade 6) using tablets and personal computers. The apps are available in two versions: *SlateMath for Kids*, designed for home use, and *SlateMath for School*, equipped with student diagnostics and analytics and a teacher's dashboard. The first app in the series, *SlateMath for Kids K1*, prepares children for Kindergarten and first grade math, and can be downloaded freely from Apple's app store. Additional SlateMath apps, as well as Android and HTML5 versions, will be released in the first quarter of 2014.

This document is intended primarily for teachers who wish to go beyond the apps and explore the SlateMath pedagogy and general approach to early-age math education.

Why *SlateMath*? Thousands of years ago, humans began to record numbers by chalking up symbols on rectangular slabs of grey rock called *slate*. These erasable slate boards gave ancient students the ability to practice, record, and share work with others. Our approach to early-age math education is called *SlateMath* as a tribute to this precursor of modern learning technologies.

SlateMath in a Nutshell

SlateMath is a series of immersive math education apps for tablets and personal computers. The minimal learning object in SlateMath is called *episode*: a bite-sized interactive progression of several related and increasingly challenging hands-on activities, presented in a playful and animated user experience. Each SlateMath episode lasts about 5 to 15 minutes, and seeks to address a single, well-defined, curricular-driven mathematical concept, skill, or insight. In addition to these hands-on activities, we also provide numerous interactive worksheets that last as long as it takes to complete the given problem sets.

¹ Professor of mathematics and chair of the Einstein Institute at the Hebrew University, and co-founder of Slate Science.

² Professor of computer science and Founding dean, the Efi Arazi school of computer science at IDC Herzliya, and co-founder of Slate Science

The hands-on episodes focus on building an intuitive *understanding* of the subject matter, e.g. understanding, hands-on, the rationale behind the distributive rule $a \cdot (b + c) = a \cdot b + a \cdot c$; the worksheets are designed to build a *mastery* of the related skill set, e.g. solving algebraic problems that involve many manifestations of the distributive rule. Both the episodes and the worksheets are adaptive, as we describe shortly.

SlateMath is based on a collection of about 700 episodes and worksheets, designed to support math education from kindergarten through grade 6. The verb "support" is important. SlateMath is not intended to teach math, but rather to help teachers teach math. It provides an abundance of richly-indexed activities and exercises that can light up a lecture and turn a classroom into a lively math laboratory. Teachers normally use 2-3 SlateMath episodes in each class meeting, in a blended fashion that alternates between frontal teaching and self-paced student practice.

Perhaps the biggest challenge in early-age math education is that children develop math deficiencies that go unnoticed and uncorrected. These hidden deficiencies linger into the program, causing mounting frustration and accelerated incompetence. With SlateMath, the child's performance is continuously monitored. When the child's work indicates that he or she is remiss of some requisite skill, SlateMath proposes to the teacher remedial work, using relevant episodes. This student diagnostics is done in real-time, while the errors are still fresh and unsettled.

SlateMath comes with student authentication, tracking, and analytics capabilities. A teacher dashboard provides both on-line and periodical status reports about each student's performance.

Standard-based Learning

SlateMath was designed from the ground up to be consistent with standardized curricula. For example, consider the *Common Core State Standards for Mathematics*, adopted by 45 states in the USA. This standard specifies the math topics and skills that students should understand and master, respectively, in each K-12 age group. According to the Common Core, the math contents of each grade level is divided into about 30 topics like "tell and write time" (grade 1) and "use equivalent fractions as a strategy to add and subtract fractions" (grade 5).

SlateMath covers each one of the 183 Common Core K-6 algebra and geometry topics using several methodologies that we call "manipulatives", "literacy", "worksheets", "word problems" and "strategies". Each methodology comes to play via a subset of SlateMath episodes. The result is a portfolio of about 100 episodes for each age group. Figure 1 illustrates the relationships between the Common Core standard and SlateMath's methodologies and episodes.

The Common Core is one standard example; other math teaching standards are used in other countries, as well as in some states in the USA. Since the SlateMath episodes are highly modular and richly indexed, they can be readily mapped on practically any given math teaching standard, program, or table of contents of any given math textbook. These mappings play a key role in the SlateMath teacher's dashboard. In particular, the mappings drive the logic that helps teachers find and assemble subsets of relevant episodes and worksheets that can best support the teaching of one's next class meeting, as we describe later in the paper.

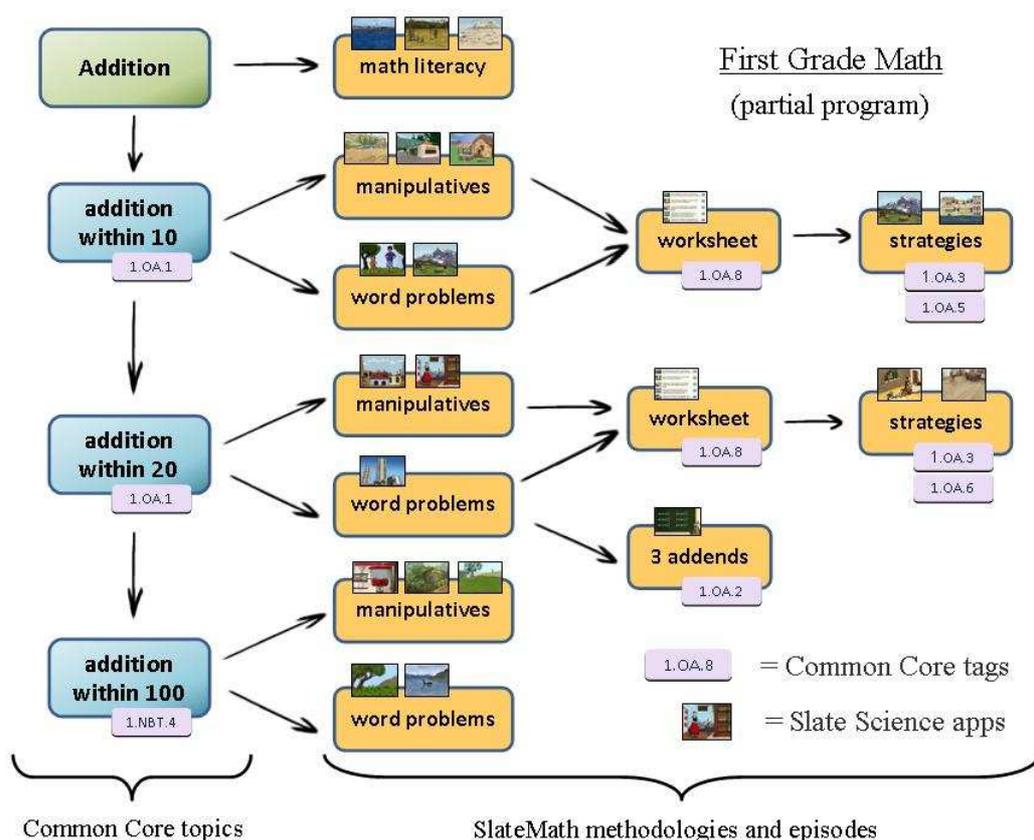


Figure 1: The relationship between SlateMath and the Common Core Math Standard. The yellow bars represent some of the methodologies used in SlateMath to cover different Common Core topics; each methodology comes to play through one or more learning episodes. The arrows represent recommended learning paths. The entire first grade SlateMath program consists of 105 learning episodes.

Episode Example

Much of early-age math education evolves around understanding and mastering the four basic mathematical operations: *addition*, *subtraction*, *multiplication*, and *division*. For example, according to the Common Core Standard, children should start being exposed to simple forms of division in grade 3 (see CCSS.Math.Content.3.OA.A.2). One way to set the stage for division is to introduce children to the notion of *parity*, which is based on a simple form of division – division by two. This section describes a SlateMath episode designed to help teach parity.

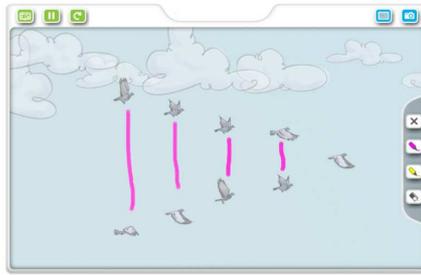
How can you tell if a given number, say 28, is *odd* or *even*? The simplest way to go about it is to look at the number's rightmost digit; if it's divisible by two, the number is even; otherwise, it's odd. This trick works well, but it provides no insight whatsoever for understanding the fundamental algebraic notion of "being odd" or "being even" – a property commonly known in mathematics as *parity*. As usual, we wish children to feel parity in the marrow of their bones. With that in mind, SlateMath features a dedicated episode and a teaching guide that focus on solving parity questions using an interactive, hands-on "parity lab". Figure 2 shows some snapshots from this activity.

The child is invited to play with flocks of birds, each having a different geometry and symmetry. The teacher explains that there are two ways to determine if the number of birds in a given flock is odd or even: we can either try to divide the birds into pairs, or we can try to divide them into two subsets containing an equal number of birds. If we succeed to do so, and no bird is left out of the division, we say that the number of birds is *even*. If one bird is left out of the division, we say that the number of birds is *odd*.



Is the number of birds **odd** or **even** ?

Is the number of birds odd or even?
Can you answer the question without counting?



Is the number of birds **odd** or **even** ?

Using a retractable overlay screen and crayons, we can arrange the birds in pairs. Note that one bird is left out of the division.



Is the number of birds **odd** or **even** ?

Creating two groups containing an equal number of birds. Once again, one bird is left out.



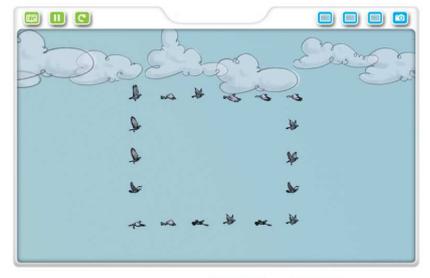
Is the number of birds **odd** or **even** ?

Using the camera tool to take and save a snapshot of one's work.



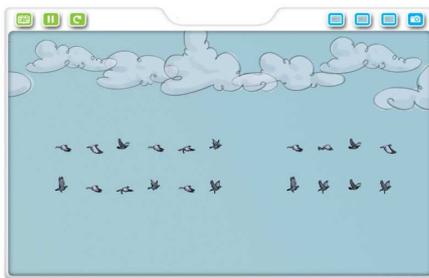
Is the number of birds **odd** or **even** ?

Another flock formation. The leading and trailing birds can be moved around, creating a pair.



Is the number of birds **odd** or **even** ?

Another flock formation, out of many more flocks of varying geometry and symmetry.



Is the number of birds **odd** or **even** ?

We use this formation to reason that the sum of two even numbers is an even number.



Is the number of birds **odd** or **even** ?

We use this formation to reason that the sum of two odd numbers is also an even number.



Is the number of birds **odd** or **even** ?

We saw that (i) the parity of a symmetric arrowhead is odd, and that (ii) odd plus odd gives even.

Figure 2: Screenshots from the parity episode.

Different flocks of birds fly into the screen; the task is to determine if the number of birds in each flock is *odd*, or *even*. To do so, the child is encouraged to move the birds around and rearrange them into groups. This is done using a retractable and transparent overlay screen, a set of crayons, an eraser, and a camera (all software artifacts).

This hands-on manipulation, when applied to different flocks of birds, leads to many interesting mathematical insights. For example, we quickly discover that irrespective of how many birds a flock contains, either one bird is left out of the division, or no birds are left out. And, because the software encourages moving the birds around and rearranging them in groups, the child discovers that "being odd" or "being even" is not a property of individual birds, but rather a property of the entire flock. Some of these learned insights run deep. For example, by playing with certain flocks, children discover that the sum of two even numbers is an even number, and that the sum of two odd numbers is, surprisingly, also an even number. They use these insights to reason about the parity of subsequent complex flocks without having to count the birds. In short, the children are exposed to the way mathematicians think, which is no less important than understanding parity proper. We note in passing that each SlateMath episode is accompanied by a teaching guide that suggests a set of activities, insights, and discussion points. For example, another important observation is that when we pair up any number of birds, *at most* 1 bird is left out of the division. This plants the seed for the notion of modular arithmetic, or the notion of a remainder.

The Parity Episode is just one of the hundreds of episodes contained in SlateMath. This elaborate body of instructional materials, and the pedagogy behind it, constitute an approach to early-age math education that can be described as *non-nonsense*, *versatile*, *object-based*, *bite-sized*, *customized*, and *spiral*. The rest of the paper describes each one of these properties of SlateMath.

Non-Nonsense learning

Many math teaching programs and textbooks sugarcoat the "boring math" with "gamification elements" like number-eating cows and similar contraptions, each following one author's fantasy on the best way to explain something. We don't believe in number-eating cows, for two reasons. First, cows don't eat numbers. Second, math is sufficiently attractive in its own right, and there is no need to decorate it with infantile metaphors. Our experience shows that SlateMath endears math on children as young as 4 year-olds by helping them master *common tasks* that unfold in *common settings*: counting animals, hanging balloons, decorating cakes, and so on. There is no need to have a dinosaur slap its tail on the smaller of two numbers when there are many interesting ordinary scenarios in which order comparisons come to play. Moreover, identifying which mathematical procedures are applicable to which everyday situations is in itself an important math literacy skill.

We do gamify the SlateMath working environment with peripheral motivators like avatars, badges, and a quest-like context. And yet, although we want children to feel engaged and rewarded, we don't believe in shortcuts and instant learning. Analytic reasoning, like any worthwhile intellectual art, takes time to develop in one's mind. Therefore, we want the magic of math to grow on children gradually, as they complete more SlateMath episodes and become increasingly more competent and confident in their work.

Versatile Learning

Consider *counting* – the fundamental ability to add 1 to a given number, and the related ability to associate quantities like ●●● with words like "three" and the agreed-upon symbols that represent them, e.g. "3". Adults take this ability for granted. However, for a child who is new to the world of numbers and decimal digits, counting is an elaborate skill that manifests itself in many different situations. Here are a few examples (see Figure 3):

1. *How many fish do you see?*

Explanation: a quantity of objects is given; the child is prompted to specify the number associated with the given quantity.

2. *Create a necklace using 6 beads.*

Explanation: a number is specified; the child is prompted to arrange the quantity of objects that the number represents.

3. *How many elephants are there in the whole area?*

Explanation: a panoramic scene is shown, divided into several horizontal panes; in order to count all the animals that possibly roam out there, the child has to swipe the panes, sideways. Counting in this context requires the ability to recall, update, and store in memory a running total representing the number of objects that were accounted so far. This *phased counting* skill builds a cognitive foundation for the subsequent ability to *add*, as we discuss below.

4. *How many birds flew over the roof?*

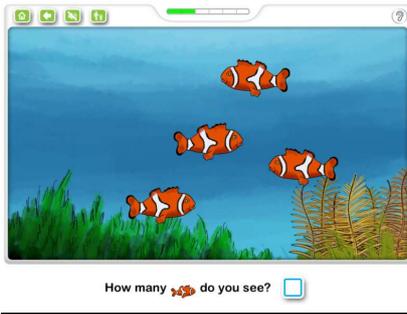
Explanation: the birds enter and leave the scene one bird at a time; counting them requires the ability to recall, update, and store in memory a running total over time, rather than over space, as we did before.

5. *How many meerkats do you spy? click "play" when ready to count.*

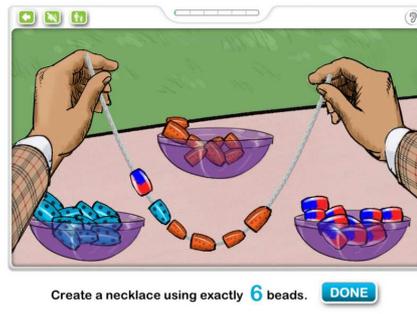
Explanation: the meerkats hide underground, invisible. When the child clicks "play", a few meerkats pop up momentarily, and then hide again. This episode trains the child in *speed-counting*, an important step toward making counting an "automatic" ability.

These examples illustrate some of the many situations in which counting manifests itself in real life. SlateMath offers 15 different such counting episodes. We take pains to vary not only the counting task, but also its setting. When children learn a mathematical operation using a single metaphor, or several trivial variations of the same theme, they tend to associate the operation with the single metaphor, especially if the metaphor is strikingly gamified. This fixation obliterates the child's ability to understand the abstract and universal nature of the underlying mathematical operation.

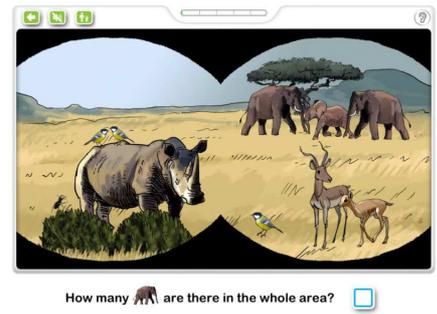
Another reason for episode variation is that one never knows which channel will hit home. Different children relate differently to different tasks and settings. Thus, the more angles one uses to shed light on the same math topic, the better.



Counting, from objects to number:
How many fish do you see?
 (when a fish is touched, it bloats)



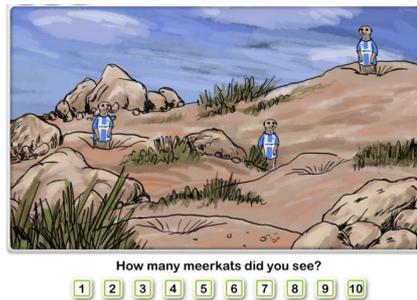
Counting, from number to objects:
Create a necklace using 6 beads.



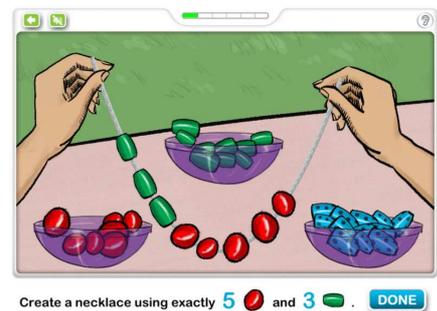
Phased counting, over space: *how many elephants in the entire scene?*
 (the scene continues, sideways)



Phased counting, over time:
How many birds flew over the roof?
 (snapshot in mid action)



Speed counting:
How many meerkats did you spy?
 (snapshot in mid action)



Addition episode: the next question is "*how many beads are in the necklace altogether?*"

Figure 3: Some Counting Episodes

We note in passing that the software features an optional voice-over channel that helps overcome reading difficulties. Answers to questions are entered either by choosing a number from a number wheel or, in tablets, using a stylus or one's finger to hand-write the number on the screen.

Object-based learning

We believe that early-age mathematical skills are best acquired through direct manipulation of familiar objects in familiar settings: counting animals, sorting cookie jars, cutting and pasting geometric figures, and so on. Indeed, child developmental psychologists, most notably Jean Piaget, observed that tactile manipulations of concrete objects help children form math concepts and operations using their own cognitive devices. This innate learning process can be significantly accelerated by teachers who can prepare, monitor, and guide such hands-on explorations.

Since it is unrealistic to expect teachers to prepare, and bring to the classroom, thousands of physical objects ranging from birds to beads to boats, SlateMath renders the interaction with such objects using computer technology. The SlateMath software employs a state-of-art graphical user interface and a physics engine to simulate the manipulation of numerous familiar objects in an accurate and engaging manner. These guided interactions are carefully designed to facilitate hands-on exploration of math concepts and insights within a playful and supportive learning environment.

Bite-sized learning

Consider an elementary school classroom in which children have access to tablets or desktop computers. Suppose that the teacher sends the children to practice some math skill on their own, using some app. What should be the ideal duration of such an uninterrupted self-practice session? We believe that the answer is 5 to 15 minutes. This duration enables teachers to easily switch back and forth between frontal teaching and self-practice. With that in mind, most SlateMath episodes entail 5 to 15 minutes of interaction (some episodes are longer, but can be terminated mid-way without losing the pedagogical thread). Field experience indicates that K-5 teachers use an average of 2-3 SlateMath episodes to support a single class meeting.

Customized learning

The short duration of individual SlateMath episodes has another virtue, which is *modularity*. Each year in the K-6 math program is supported by a portfolio of about 100 SlateMath episodes. Each one of these episodes entails a series of related tasks, resulting in hundreds of different interactive activities.

The SlateMath portfolios are highly modular, richly indexed, and recombinant. Every week, teachers select the SlateMath episodes that best support their weekly teaching program, and assemble them into a customized, weekly math learning bundle (we provide default weekly bundles, designed to cover the entire year). This way, teachers control the selection and order in which the episodes become available to their students. And, in a BYOD (Bring Your Own Device) setting, teachers can control the episodes availability and staging outside school as well. All these tasks are done through drag-and-drop operations on the teacher's dashboard.

The pre-determined contents of the weekly math bundle can be easily overridden. If the teacher feels that the class dynamics requires access to additional SlateMath episodes, he or she can easily make them available to the students. Episode selection is done by queries like "I need episodes that practice ordering objects according to their length" or "I need episodes that support the Common Core 1.MD.2 topic". Once again, all these tasks are done on the teacher's dashboard.

Spiral Learning

The edifice of mathematical knowledge is built in layers. Thus, when learning a new mathematical operation, it always helps to relate it to another operation that was already acquired and mastered. For example, one way to calculate (or motivate, or explain) $a + b$ is to add 1 to a , b times. Thus, the ability to add up numbers is related to, and builds upon, the more fundamental ability to count. With that in mind, SlateMath uses variants of the same episode to support families of related math abilities. For example, when learning to count, SlateMath presents the task "create a necklace using x beads". When learning to add up numbers several weeks later, SlateMath presents the task "create a necklace using x green beads and y yellow beads" (see Figure 3). Once the task has been completed, the child is prompted to answer the question "how many beads are in the necklace altogether?" This example is just one out of many available addition tasks and settings.

The ability to apply different operations within familiar settings (e.g. stringing beads into necklaces) helps to boost confidence and promote comprehension. Further, it serves to unravel the implicit relationships between different math artifacts, e.g. *counting* and *addition*. This gestalt view of mathematics, which is extremely important, is made possible since SlateMath was designed by mathematicians and early-age math education experts who sought to support a multi-year curriculum from the ground up. Rather than developing our contents in the order of "grade 1 first" and "grade 2 second", we developed the contents by creating families of themes that cut across, and serve, the entire K-6 curriculum. And since SlateMath features numerous such themes, children don't tire, and in fact welcome, revisiting them.

The ability to plan instructional materials that support a multi-year curriculum "vertically" requires expert mathematical knowledge and deep pedagogical insight. For example, a mathematician who is well versed in set theory can recognize that *order comparisons*, normally taught in kindergarten, are closely related to *sets* and *mappings*, normally taught several years later. Thus, in one of our order episodes, when we ask kindergarten children "are there more bees or flowers?" we encourage them to answer the question without counting. This can be done by dragging the flying bees to the flowers, and then observing if there is a surplus of unmatched objects. This reasoning strategy lays the foundation for understanding the notions of set cardinality and mapping later in the program.

Personalized Learning

When serving an episode that requires adding up integers, SlateMath can detect that the child's counting skills are not sufficiently developed; when serving an episode that involves fraction arithmetic, SlateMath can detect that the child has trouble computing common denominators; when serving an episode that involves calculating triangle areas using geometric manipulations, SlateMath can detect that the child does not know how to calculate rectangular areas. SlateMath continuously monitors the child's actions in each episode, and accumulates reams of data about the child's evolving math abilities. Importantly, this continuous monitoring is done *in vivo*, while the child interacts with the actual episodes, rather than in subsequent and out of context quizzes. The latter check if the child can pass math exams; the former check if the child understands math.

The diagnostic data that SlateMath collects is continuously mined, summarized, and presented on the teacher's dashboard. Teachers can get real-time information about each child's current performance, as well as tallied information about the child's cumulative progress. If the child's work indicates that he or she is remiss of some math topic, SlateMath recommends remedial action, using relevant episodes. The teacher can either accept these recommendations, or modify them to come up with another corrective procedure.

The fact that different children learn in different paces is one of the biggest challenges in mathematics education. In less tightly layered fields of study, missing a class or a concept can be easily rectified, or even side-stepped. Yet in math, if a child does not understand the notion of a *common denominator*, he will be at loss when learning *fraction arithmetic*. He may be able to do the exercises, following supplied algorithms, but he will not understand why the algorithms work. Such are the early fault lines from which math anxiety begins to emerge, setting the stage for a life-long math phobia that typically takes hold sometimes between elementary and middle school.

Conclusion

As the reader has hopefully noted, our pedagogical style is quite plain and "back to basics". At the same time, this style is quite difficult to implement using computers. It would have been much easier to develop interactive multiple-choice questions and similar modes of instruction that simply take existing worksheets and make them "digital". Yet, as we argued in this paper, we believe that the most effective way to learn math is to explore and experiment. Our pilots indicate that we are on the right track: teachers in schools that use SlateMath tell us that children fall in love with math, and can't wait to put their hands, literally speaking, on more SlateMath episodes.

We believe that in the era of tablets and smart phones, learning can occur anytime, anyplace. That said, no technology can replace human touch, especially in early-age math education. In particular, the teacher's role in staging and guiding the learning process is indispensable. That's why SlateMath does not pretend to teach math, but rather to help teachers teach math.

According to Bertrand Russel, "*Mathematics, rightly viewed, possesses not only truth, but supreme beauty — sublimely pure, and capable of a stern perfection such as only the greatest art can show.*" With that in mind, our goal is not limited to developing math skills and literacy. We are equally interested to endear math on children, and to kindle an affinity to quantitative reasoning and to science in general. Math proficiency provides a foundation for becoming a rational, productive, and self-reliant person, and opens numerous career possibilities; helping children build this foundation is our overriding passion and commitment.

About Slate Science

SlateMath was designed by a team of scientists, teachers and software experts who shared a common passion for endearing math on children and empowering early-age math teachers. This was the seed that later became *Slate Science*, a company dedicated to developing teaching platforms and contents for STEM (Science, Technology, Engineering, Mathematics) education.

The company was founded by software engineers who developed widely-used software products for Intel, Agfa, and Qualcomm, by mathematicians and technology education experts who worked at NYU, Harvard, Stanford, Rice and Berkeley, and by gaming specialists who developed gaming platforms for Microsoft, Yahoo, Comcast, and EA. We are excited to be able to focus this combined experience on transforming early-age math education on a global scale. Our software is now being used in 25 different languages, with more to come.