METRIC EMBEDDINGS, ENERGY INTEGRALS AND SUMMING OPERATORS

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In this talk we show how some results from the theory of absolutely summing operators can be applied to a problem in metric geometry. We take a compact set $K \subset \mathbb{R}^n$ and endowed it with the metric

$$d_\alpha(x,y) = |x-y|^\alpha,$$

where $0 < \alpha < 1$. A classical result of Schoenberg and von Neumann asserts that there exist a minimum r for which the metric space (K, d_α) may be isometrically embedded on the surface of a Hilbert sphere of radius r. Our aim then is to estimate this minimun radius, and we do it for several centrally symmetric convex bodies K. To this end, we relate this radius with the energy integrals of the form

$$\int_K \int_K |x-y|^{2\alpha} d\mu(x) d\mu(y),$$

for finite Borel measures μ on K of total mass one. This allows us to estimate the radius r in terms of the 2α-summing norm of certain operator or the mean width of K. In the case where K is an ellipsoid or K is the unit ball of $(\mathbb{R}^n, \|\cdot\|_q)$ (for $1 < q \leq 2$), we obtain the correct asymptotical behavior of the least possible radius.

This is a joint work with Daniel Galicer and Damián Pinasco.