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# Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View*

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ABSTRACT

We develop a model in which, in order to provide managerial incentives, it is optimal to have costly bankruptcy. With commitment it is optimal for governments not to interfere with private contracts. Such noninterference is time inconsistent in the sense that, without commitment, governments have incentives to bail out firms by levying taxes to buy up the debt of distressed firms and renegotiating their contracts with managers. When governments lack commitment, the relevant notion of constrained efficiency incorporates this incentive to intervene and is called sustainable efficiency. Bailout outcomes are sustainably inefficient because of subsidy distortions and size externalities. Allowing governments the power of forcing debtors to incur losses through an orderly resolution procedure can eliminate subsidy distortions but not the size externalities. Allowing governments to regulate both debt levels and size eliminates both subsidy distortions and size externalities. Such regulation is particularly desirable in crisis times.

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Recent experience has shown that governments can and will intervene during financial crises. During such crises, many firms are faced with the prospect of costly bankruptcy and liquidation. To minimize these costs, governments bail out firms that are on the brink of bankruptcy by purchasing their debt. Governments then renegotiate the debt contracts to allow such firms to continue operations without having to go through bankruptcy. Governments pay for such bailouts with taxes.

To understand the role of bailouts we first need to understand why firms would enter into contracts that entail costly bankruptcy. We argue that such contracts can provide appropriate incentives for managers. We develop a model in which the presence of private information leads firms to design contracts with costly bankruptcy. We show that equilibrium outcomes without bailouts are efficient. Thus, if the government could commit to its policies, it would not engage in bailouts.

Without such commitment, a government that faces widespread bankruptcies is tempted to bail out the debt holders of distressed firms and prevent such firms from declaring bankruptcy. Such bailouts are financed by taxes. If the temptation is sufficiently strong, governments engage in bailouts in equilibrium. Expectations of such bailouts lead private agents to alter contracts, making the outcomes worse than they would be under commitment. In this sense, the government faces a time inconsistency problem.

We ask how policy interventions can improve on bailout outcomes. We think of such interventions as granting the government additional powers to intervene in private contracts. In analyzing the role of such additional powers, we take as given that governments in practice lack commitment and, regardless of the laws in place, will find ways to engage in tax-financed bailouts if they find it desirable to do so.

This perspective implies that outcomes that do not respect the incentives of the government to engage in bailouts are unattainable and leads us to label outcomes that do respect these incentives as *sustainable*. The relevant notion of constrained efficiency, then, is the best outcome that respects sustainability as well as the resource constraints and informational constraints in the economy. We label such outcomes *sustainably efficient*.

Our first main result is that, if the bailout temptation is sufficiently strong, outcomes with bailouts are sustainably inefficient. This inefficiency arises for two reasons. Bailouts create *subsidy distortions* by providing incentives for private agents to enter into debt contracts with inefficiently high debt levels which in turn induce the bailout authority to intervene to a greater extent. They create *size externalities* because no individual firm internalizes the effect of its size choices on the incentives of governments to undertake bailouts. We ask what policies can remedy these inefficiencies.

One proposed remedy reflects the popular view that bailouts lead to inefficiency because they subsidize firms that would otherwise go bankrupt. The Dodd-Frank Act contains provisions that reflect this popular view. The key feature of this lengthy act, which we refer to as an *orderly resolution* provision, grants to a governmental authority the power to impose losses on unsecured creditors. Our reading of the act is that, while the additional powers are intended to dampen the incentives of the government to engage in tax-financed bailouts, in practice the government cannot be prevented from engaging in such bailouts in terms of severe financial stress if it chooses to do so.

We show in our second main result that the orderly resolution provision remedies the subsidy distortion and hence improves on the outcomes of tax-financed bailouts. It does not, however, address the size externality and hence is not sustainably efficient.

Our third main result is that sustainably efficient outcomes can be implemented by granting a *regulatory authority* the ability to impose limits on both the debt to value ratio and firm size. The limit on the debt to value ratio addresses the subsidy distortion by preventing firms from taking on excessive debt and the limit on firm size addresses the firm size externality.

We begin with a simple one-period model that highlights the time inconsistency problem faced by governments. Our model has information frictions, and we allow private agents to renegotiate contracts. With these features, private agents optimally choose to enter into debt-equity contracts that specify the size of the firm, as well as payments to debt holders, equity holders, and managers. Debt levels are chosen high enough so that the firm enters into costly bankruptcy with positive probability. Such bankruptcy is needed to provide managers with incentives to exert effort. These contracts lead to efficient outcomes, and with commitment the government chooses not to intervene.

Here, bankruptcy plays no socially useful role after managers have made their effort decisions. Thus, without commitment, it is optimal for the government to eliminate all impending bankruptcies. It does so by buying up the debt of firms and financing these purchases by levying taxes on all firms. Such a policy effectively subsidizes payments to the firm in the event of impending bankruptcy. An individual firm has an incentive to increase the likelihood of bankruptcy so as to receive these subsidies because from its perspective the taxes it pays are not affected by its choices. Of course, in equilibrium each firm's expected subsidies equal its expected tax payments. These subsidies thus induce a distortion in effort and size choices even though no external funds are used to finance these subsidies. This distortion leads welfare to be strictly lower than it would be under commitment and, thus, our economy has a time inconsistency problem.

The one-period model is a useful prelude to our main analysis but it misses some important features of the data and lacks a critical insight into size externalities that we discuss below. In the data, we observe partial but not complete bailouts, and we typically observe sizable bailouts during crises rather than during normal times. Our dynamic model, an infinitely repeated stochastic version of the one-period model, generates both of these features. In our model, the economy can be in either *normal* times or *crisis* times. Output is higher and the resources lost to bankruptcy are smaller in normal times than in crisis times.

In our dynamic model, reputational considerations impose ex post costs on intervention by affecting private agents' beliefs about future policies. To see how such ex posts costs can arise, suppose that an unexpectedly large bailout today leads private agents to expect that all distressed firms will be bailed out in the future. Such expectations imply that a bailout authority that is contemplating an unexpectedly large bailout may be deterred from doing so, because the current gain from reducing bankruptcy may be outweighed by the future losses arising from distortions induced by anticipated bailouts.

We show that such logic implies that equilibrium outcomes respect the incentives of the government to intervene if and only if they satisfy a *sustainability constraint*. This constraint requires that the current gains from policy deviations must be outweighed by the future losses induced by changes in private expectations from such deviations. We show that if the discount factor is not too high, the sustainability constraint binds during crisis times but not during normal times. The bailout authority bails out some but not all distressed firms during crisis times and refrains from bailouts during normal times. In this sense, our model is consistent with the observation that bailouts are partial and occur during crisis times.

We show that bailout equilibrium outcomes are sustainably inefficient if the sustainability constraint is binding. This inefficiency arises from subsidy distortions and size externalities. By providing subsidies to firms with impending bankruptcies, bailouts reduce the incentives of firms to design contracts that induce managers to exert effort. Such subsidy distortions are present in both the one-period model and the dynamic model. The size externality is a new type of externality that arises from a free-rider problem generated by the sustainability requirement. When firms in the aggregate increase their size, the resources lost to bankruptcy increase and the bailout authority is more tempted to intervene ex post. Each individual firm is made better off if all firms reduce their size below privately optimal levels so that the government does not intervene, but no individual firm will agree to do so.

The ex post costs of intervention are critical in generating this externality. To understand the role of these costs, note that our one-period model has no such costs so that the bailout authority eliminates all bankruptcies regardless of the size of firms. Hence, the size of firms does not affect the extent of intervention and the model has no size externalities. In this sense, the one-period model lacks a critical force that is present in our dynamic model.

We model the orderly resolution provision by introducing an orderly resolution authority that can force debtholders of firms facing impending bankruptcy to accept lower debt payments. We show that this authority eliminates the subsidy distortions by effectively making each firm pay for its own bailouts. It does not, however, address the size externalities and hence the resulting outcomes are sustainably inefficient. Optimal regulation addresses both the subsidy distortions and the size externalities. It eliminates the subsidies and addresses the free-rider problem by constraining all firms to maintain their debt levels and size below privately optimal levels.

The general implication of our analysis is that regulation should be most stringent when the bailout authorities have the strongest incentive to intervene. In an extension of our model, we use this implication to analyze which industries should face the most stringent regulation. We think of industries as differing in the severity of their incentive problems. We show that industries with more severe incentive problems have higher debt to value ratios and must be regulated to a greater extent. In this sense, our model is consistent with the view that industries with high debt to value ratios, such as banking, are most in need of regulation.

To highlight the role of time inconsistency we have purposefully abstracted from the inefficiencies that are generated by spillover effects—say, from fire sales. A standard story is that when the financial sector undergoes severe stress these stresses spill over to other sectors in a way that is not internalized by the market. Such spillovers generate externalities that can be mitigated by regulation. Clearly, these spillover externalities have nothing to do with the externalities generated by time inconsistency; regulation is needed even under commitment. Obviously, we can add such spillovers to our model. Our analysis can be thought of as describing regulation that is needed to address the externalities arising from time inconsistency problems over and above those needed to cure standard spillover externalities.

We have also purposefully abstracted from the inefficiencies that arise from coordination problems by focusing on the best equilibrium in each policy regime. We have abstracted from coordination problems for two reasons. The first is that we want to focus on the role of regulation in mitigating distortions and externalities arising from a time inconsistency problem. The second is that our dynamic model has multiple Pareto-ranked equilibria, and regulation cannot cure this multiplicity.

In focusing on the best equilibrium, we follow a long tradition in public finance and mechanism design that looks for policies and mechanisms that *weakly implement* desired outcomes. That is, under the constructed policies and mechanisms, the desired outcome is one equilibrium among many possible equilibria. The idea is that society can somehow find a way to coordinate the desired equilibrium outcome. Note, however, that even if we follow this tradition, a classic *time inconsistency problem* remains and regulation is desirable. (See Lucas and Stokey (1983) for the classic definition.[1])

*Related Literature.* In interesting related work, Farhi and Tirole (2012) analyze the

[1] Recall from Lucas and Stokey (1983) that a Ramsey plan starting at date 0, thought of as the best equilibrium with commitment, is *time consistent* if, taking as given the history of policies and private allocations from the date 0 plan, a Ramsey planner at any future date $t \geq 1$ would choose to continue with the original Ramsey policies. If the Ramsey plan does not have this property, then the economy has a *time inconsistency problem.*

role of regulation in addressing a coordination problem. Absent regulation, without commitment their model has multiple equilibria, one of which coincides with the equilibrium under commitment. Regulation can uniquely implement the commitment equilibrium. Since the commitment equilibrium is also an equilibrium without commitment, their model does not have a classic time inconsistency problem. Hence, if we applied the traditional weak implementation approach to their model, we would conclude that regulation is unnecessary.

More important, their model does not have bailouts that involve purchases of debt by the government accompanied by forgiveness. Thus, they cannot analyze the role of policy interventions, such as orderly resolution, that force unsecured creditors to absorb losses.

Other interesting related work is that by Keister (2014), who studies the roles of bailouts in a Diamond and Dybvig (1983) model. Keister's model also has multiple equilibria, and the best equilibrium without commitment coincides with the commitment equilibrium. In this sense, Keister also does not have a time consistency problem, and the commitment outcome can be weakly implemented without regulation. A key feature of Keister's model that generates multiple equilibria is that the financial intermediaries that operate the technology in the Diamond-Dybvig model can alter the payments to withdrawing depositors in ways that make some depositors strictly worse off than under the original contract. If instead, as we require, contracts can be renegotiated only if all parties agree, there would be a unique equilibrium, namely, the equilibrium under commitment. In this sense, Keister studies coordination problems generated by the power given to some private agents to unilaterally change private contracts.

Our paper builds on the insightful analysis of bailouts in Stern and Feldman (2004) and is related to a literature that emphasizes the expectations of bailouts played in exacerbating financial crises in developing and emerging economies. See, among others, Burnside, Eichenbaum, and Rebelo (2001), Conesa and Kehoe (2012), Gertler, Kiyotaki, and Queralto (2012), Nicolini and Teles (2014), and Schneider and Tornell (2004).

A burgeoning recent literature gives a prominent role to regulation as the way to correct subtle externalities arising either from lack of commitment by private agents or from hidden trading. (See, for example, the work of Lorenzoni (2008); Farhi, Golosov, and Tsyvinski (2009); Bianchi and Mendoza (2010); and Bianchi (2011).) In contrast, in our work, a subtle externality arises because of lack of commitment by the government.

Finally, a recent literature has also examined the quantitative effects of policy interventions like bailouts on the risk-taking decisions of financial institutions. (See, for example, the work of Gertler, Kiyotaki, and Queralto (2012).)

## 1. The One-Period Economy

We begin with a one-period economy that sets the stage for our infinite-horizon analysis. We describe the key frictions in the economy, solve for the optimal contracts, and show that the competitive equilibrium is efficient. We then introduce a government and show that without commitment, governments engage in bailouts and these bailouts are sustainably inefficient. Finally, we show that an orderly resolution authority implements sustainably efficient outcomes.

### A. With Only Private Agents

We begin by considering a one-period economy without a government. In this economy, managers and investors design optimal contracts that are intended to induce effort and share output in the face of three key frictions. First, the effort of the manager is privately observed by the manager. Second, although the manager costlessly observes a component of productivity, labeled *private*, investors can observe this component only by putting the firm through bankruptcy. We assume that bankruptcy is costly in that it reduces the productivity of firms proportionately. We think of this output reduction as arising from a variety of sources, including replacing incumbent managers with new managers with fewer firm-specific skills.[2] Third, the manager and the investors cannot commit to the terms of their contracts; that is, if the manager and investors agree to renegotiate, they can renegotiate the terms of a contract after the manager chooses effort. This lack of commitment implies that any contract must be immune to renegotiation and that such contracts are *ex post efficient*. We allow for private renegotiation to ensure that the ex post gains from bailouts do not arise from a desire to improve ex post efficiency.

We allow investors and managers to sign any contracts as long as they respect the

[2] Alternatively, the reduction in output could arise because bankruptcy leads specialized forms of capital to be sold for less suitable uses.

informational and renegotiation constraints of the environment. We show that the resulting optimal contracts are *debt-equity* contracts. These contracts specify a fixed payment to debt holders unless the firm declares bankruptcy. In the event of bankruptcy, debt holders receive all of the proportionately reduced output of the firm, managers are replaced, and equity holders receive nothing. Equity holders receive the residual output of the firm after payments to debt holders and managers in the event of no bankruptcy. We show that the resulting competitive equilibrium is efficient in that a planner, confronted with the same information and renegotiation frictions, would choose the same outcomes.

In the model, decisions are made in two stages: a first stage at the beginning of the period and a second stage at the end. The economy has two types of agents, called *managers* and *investors,* both of whom are risk neutral and consume at the end of the period. The economy has a measure 1 of managers and a measure 1 of investors.

The technology uses two inputs in the first stage, an investment of $k$ unit of goods per manager and effort $p$ by the manager, to produce capital goods at the second stage. The capital goods can then be used to make consumption goods. The effort level $p$ of managers is unobserved by investors.

The amount of capital goods produced in the second stage stochastically depends on the effort level $p$, the amount of investment $k$, and two idiosyncratic shocks denoted $A_s$ and $\varepsilon$. The shock $A_s$, $s \in \{H, L\}$, determines the average level of productivity and is called the health status. It is publicly observed at no cost. We refer to $A_H$ as the *healthy state* and $A_L$ as the *distressed state*. These shocks satisfy $A_H > A_L$. With probability $p_H = p$ the healthy state is realized, and with complementary probability $p_L = 1 - p$ the distressed state is realized.

The shock $\varepsilon$ is privately observed by the manager and is made public only if the firm declares bankruptcy, as described below. We assume that $\varepsilon$ has density $h_s(\varepsilon)$ and distribution $H_s(\varepsilon)$ with mean 1 and support $[\underline{\varepsilon}_s, \bar{\varepsilon}_s]$. The idiosyncratic shocks $A_s$ and $\varepsilon$ are realized after the effort level is chosen and are independently and identically distributed across firms.

After a manager has exerted effort and a certain amount of capital has been produced, the firm can choose to continue the project under the incumbent manager, or it can declare bankruptcy. If the project continues, then it produces $A_s \varepsilon g(k)$ for $s \in \{H, L\}$ so that $A_s \varepsilon$ is the productivity of the project relative to its size $g(k)$. We assume that $g(k)$ is an increasing concave function. If the firm declares bankruptcy, then the incumbent manager is removed, the firm is monitored, and the idiosyncratic shock $\varepsilon$ becomes publicly known. The replacement manager is less efficient and produces consumption goods from the given capital according to $RA_s\varepsilon g(k)$, where $R < 1$.

For each health state $s$ and shock $\varepsilon$, let $\phi_s(\varepsilon) = 0$ denote that the firm declares bankruptcy with health state $s$ and shock $\varepsilon$, and let $\phi_s(\varepsilon) = 1$ denote no bankruptcy.[3]

Managers have no endowments of goods but do have the specialized skills needed to operate the technology. Investors have $\omega$ units of endowments but do not have these specialized skills. Investors choose how much to invest in the technology and can store the rest of their endowments at a one-for-one rate. The only role of storage is to pin down the opportunity costs of funds to be 1. We assume that $\omega$ is sufficiently large so that some amount of the endowment is always stored. We also assume that the technology is sufficiently attractive so that it is always active.

We think of a firm as consisting of a manager and a contract with investors. This firm should be thought of an entity that consolidates financial firms, such as banks, with nonfinancial firms into a single entity. In doing so, we follow a long tradition in financial economics, including the work of Diamond and Dybvig (1983).

A contract specifies the size of the firm, the recommended effort level for the manager, the consumption level of the manager, payments to investors, and bankruptcy decisions. Let $C_s(\varepsilon)$ denote the consumption of the managers when the health state is $s \in \{H, L\}$ and the idiosyncratic shock is $\varepsilon$. Let $B_s$ denote the *bankruptcy set*, namely, the set of idiosyncratic shocks $\varepsilon$ such that the firms declare bankruptcy when the health state is $s \in \{H, L\}$. The complementary set in which no bankruptcy occurs is denoted $N_s$. Managers are risk neutral over consumption. Their disutility from effort depends on $p_H = p$, is proportional to the size of the project $g(k)$, and is given by $v(p_H)g(k)$ where $v(p_H)$ is an increasing convex function.

[3] We assume that monitoring is deterministic. For analyses with stochastic monitoring, see the work of Townsend (1979) and Mookherjee and Png (1989).

Thus, the manager's utility function is given by

$$(1) \quad \sum_s p_s \int C_s(\varepsilon)\ dH_s(\varepsilon) - v(p_H)g(k).$$

The consumption of the managers must satisfy a *nonnegativity constraint*:

$$(2) \quad C_s(\varepsilon) \geq 0.$$

Let $D_s(\varepsilon)$ denote the payments the firm makes to the investors when the shocks are $s$ and $\varepsilon$. Investors invest $k$ units of their endowment with the managers and store $\omega - k$ units, so their utility is given by

$$(3) \quad \sum_s p_s \int D_s(\varepsilon)\ dH_s(\varepsilon) + \omega - k.$$

When the firm does not declare bankruptcy, the firm's resource constraint is

$$(4) \quad C_s(\varepsilon) + D_s(\varepsilon) = A_s \varepsilon g(k),$$

and when the firm does declare bankruptcy, the firm's resource constraint is

$$(5) \quad C_s(\varepsilon) + D_s(\varepsilon) = R A_s \varepsilon g(k).$$

The total consumption of investors $C_s^I(\varepsilon)$ is the sum of the payments $D_s(\varepsilon)$ from the production technology and $\omega - k$ from storage. Thus, the *overall resource constraint* is

$$(6) \quad \sum_s p_s \left[ \int C_s(\varepsilon)\ dH_s(\varepsilon) + \int C_s^I(\varepsilon)\ dH_s(\varepsilon) \right]$$

$$\leq \sum_s p_s \left[ \int_{\phi_s(\varepsilon)=1} A_s \varepsilon\ dH_s(\varepsilon) + \int_{\phi_s(\varepsilon)=0} R A_s \varepsilon\ dH_s(\varepsilon) \right] g(k) + \omega - k.$$

An allocation, or a *contract*, consists of $x = \{k, p, C_s(\varepsilon), D_s(\varepsilon), \phi_s(\varepsilon)\}$. Here effort is not observable so it is not directly contractible, but the contract will be designed to ensure that the manager exerts the agreed-upon level of effort.

The timing is as follows. The investors and managers first agree to a contract, and then the managers choose their effort levels $p$. After the effort level is chosen, the health status of each firm $s$ is publicly realized. Investors and managers then renegotiate the contract. Finally, the idiosyncratic shocks $\varepsilon$ are realized, and the bankruptcy decisions are made according to the contract.

To be part of a competitive equilibrium, a contract has to satisfy various conditions. One is that any contract must be *incentive compatible*; that is, a manager must prefer to report the idiosyncratic shock $\varepsilon$ truthfully rather than misreport it. A manager with a shock $\varepsilon$ in the nonbankruptcy set must not have an incentive to misreport any other shock $\hat{\varepsilon}$ in this nonbankruptcy set, so that

$$(7) \quad C_s(\varepsilon) = A_s \varepsilon g(k) - D_s(\varepsilon) \geq A_s \varepsilon g(k) - D_s(\hat{\varepsilon}) \text{ for all } \varepsilon \in N_s, \hat{\varepsilon} \in N_s.$$

This constraint implies that for all $\varepsilon \in N_s$, payments to investors $D_s(\varepsilon)$ are constant in the nonbankruptcy set at some level, denoted $D_s$. Also, a manager with a shock $\varepsilon$ in the bankruptcy set must not have an incentive to misreport any $\hat{\varepsilon}$ in the nonbankruptcy set, so that

$$(8) \quad C_s(\varepsilon) = RA_s \varepsilon g(k) - D_s(\varepsilon) \geq A_s \varepsilon g(k) - D_s \text{ for all } \varepsilon \in B_s, \hat{\varepsilon} \in N_s,$$

where we have imposed that $D_s(\hat{\varepsilon})$ is constant at $D_s$ in nonbankruptcy sets. We will say that a contract is *incentive feasible* if it is incentive compatible in that it satisfies (7) and (8), and resource feasible in that it satisfies the resource constraints (4) and (5) and the nonnegativity constraint (2).

We also require that neither managers nor investors have an incentive to renegotiate the contract. Before renegotiation begins, a particular contract $x$ has been agreed to, effort $p$ has been chosen, and a health shock $s$ has been realized. We say that a contract $x$ is *immune to renegotiation* given $k$ and $p$ at $s$ if it is incentive feasible and no alternative incentive feasible contract exists that makes the managers and the investors strictly better off at $s$. Specifically, given $k$ and $p$, an alternative allocation $\left\{\hat{C}_s(\varepsilon), \hat{D}_s(\varepsilon), \hat{\phi}_s(\varepsilon)\right\}$ cannot exist that satisfies the resource and incentive constraints (4)–(8) and makes both the manager and the investors better off:

$$(9) \quad \int \hat{C}_s(\varepsilon)\ dH_s(\varepsilon) \geq \int C_s(\varepsilon)\ dH_s(\varepsilon) \equiv \bar{C}_s$$

$$(10) \quad \int \hat{D}_s(\varepsilon)\ dH_s(\varepsilon) \geq \int D_s(\varepsilon)\ dH_s(\varepsilon) \equiv \bar{D}_s$$

with at least one of the two inequalities strict.

We now turn to the ex ante optimal contract in our economy. We think of managers as offering contracts $x = \{k, p, C_s(\varepsilon), D_s(\varepsilon), \phi_s(\varepsilon)\}$.[4] Such investors will accept the contract as long as the expected rate of return on their investment is at least 1. Thus, any contract must satisfy the *participation constraint*

$$(11) \quad \sum_s p_s \int D_s(\varepsilon)\ dH_s(\varepsilon) \geq k$$

as well as the resource constraints (4) and (5). The contract must also give the manager incentive to exert the intended level of effort $p$ and thus satisfy

$$(12) \quad p = p_H \in \arg\max_{p_H} \sum_s p_s \int C_s(\varepsilon)\ dH_s(\varepsilon) - v(p_H)g(k).$$

Since all contracts can be renegotiated after the manager has chosen effort, when defining an equilibrium, it suffices to consider contracts that are immune to renegotiation.

We say that an contract is *implementable* if it satisfies the participation constraint, (11), satisfies the manager's incentive constraint, (12), and is immune to renegotiation. Note that in this definition, the requirement that contracts be immune to renegotiation incorporates incentive feasibility, so we do not need to have incentive feasibility as a separate constraint.

A *competitive equilibrium* consists of a contract $x$ that maximizes the manager's utility over the set of implementable contracts. Here, we have defined a competitive equilibrium by having managers offer contracts to investors. An alternative way of setting up the equilibrium is to have investors offer contracts to managers—contracts that maximize expected profits

[4]Here, we abstract from contracts with randomized effort. For analysis of such contracts, see the work of Fudenberg and Tirole (1990).

subject to the incentive constraints, feasibility constraints, and participation constraints on managers. By duality, the two definitions are equivalent.

We now turn to the efficiency of a competitive equilibrium. A contract $x$ is *efficient* if it is implementable and no alternative implementable contract $x'$ exists that has higher utility levels for the manager and the investor, with at least one being strictly higher. The following proposition is immediate.

**Proposition 1.** The competitive equilibrium is efficient.

We now turn to characterizing the competitive equilibrium. We begin by showing that a contract is immune to renegotiation if and only if it has a simple form, labeled a *debt-equity contract* for reasons discussed below. Consider a contract with $U_s = (\bar{C}_s, \bar{D}_s)$ defined by the right sides of (9) and (10). The debt-equity contract has three key features. First, if the expected payments $\bar{D}_s$ to investors are sufficiently small, in that $\bar{D}_s \leq A_s \underline{\varepsilon}_s g(k)$, then the contract has no bankruptcy in health state $s$, whereas if $\bar{D}_s > A_s \underline{\varepsilon}_s g(k)$ the contract specifies a bankruptcy cutoff level $\varepsilon_s^*$ that depends on $U_s$ such that the firm continues for $\varepsilon_s > \varepsilon_s^*$ and declares bankruptcy for $\varepsilon_s \leq \varepsilon_s^*$. Second, the payments to investors are constant in the nonbankruptcy set, bankruptcy occurs when the firm is unable to make this constant payment, and investors receive all the profits of the firm in bankruptcy. Third, in the event of some bankruptcy, the cutoff level $\varepsilon_s^*$ is the smallest value of the bankruptcy cutoff that gives the required expected payments $\bar{D}_s$ to investors.

Specifically, if there is no bankruptcy in health state $s$, the consumption of the manager is given $C_s(\varepsilon) = A_s \varepsilon g(k) - \bar{D}_s$. If instead there is bankruptcy in health state $s$, a cutoff $\varepsilon_s^*$ exists such that the contract has bankruptcy for $\varepsilon \leq \varepsilon_s^*$ and the payments to the investors are given by

$$(13) \quad D_s(\varepsilon, k) = \left\{ \begin{array}{c} D_s = A_s \varepsilon_s^* g(k) \text{ for } \varepsilon \geq \varepsilon_s^*, \\ R A_s \varepsilon g(k) \text{ for } \varepsilon < \varepsilon_s^* \end{array} \right\},$$

and the consumption of the manager is given by $C_s(\varepsilon) = A_s \varepsilon g(k) - D_s$ for $\varepsilon > \varepsilon_s^*$ and $C_s(\varepsilon) = 0$ for $\varepsilon \leq \varepsilon_s^*$. Given a cutoff $\varepsilon_s^*$, any such contract induces expected payments to investors given by

$$\bar{D}_s(\varepsilon_s^*, k) = \left[ R \int_{\underline{\varepsilon}}^{\varepsilon_s^*} A_s \varepsilon_s^* dH_s(\varepsilon) + \int_{\varepsilon_s^*}^{\bar{\varepsilon}} A_s \varepsilon_s^* dH_s(\varepsilon) \right] g(k).$$

In order to understand how the cutoff $\varepsilon_s^*$ is determined by the expected payments to investors $\bar{D}_s$, consider the scaled payments $d_s(\varepsilon_s^*) = \bar{D}_s(\varepsilon_s^*, k)/g(k)$. It is straightforward to show that $d_s'(\bar{\varepsilon}_s)$ is negative so that any contract that is immune to renegotiation cannot have bankruptcy in all idiosyncratic states $\varepsilon_s$. We will assume that $d_s(\varepsilon_s^*)$ is *single-peaked* in $\varepsilon_s^*$. This assumption holds for a wide variety of distribution functions $H_s(\varepsilon)$ including the uniform distribution. This assumption along with the observation that $d_s'(\bar{\varepsilon}_s) < 0$ implies either that $d_s(\varepsilon_s^*)$ at first increases and then decreases with $\varepsilon_s^*$ or that it decreases everywhere. Let $\varepsilon_{s,\max}$ be the value of $\varepsilon$ that maximizes $d_s(\varepsilon_s^*)$. The following proposition and all others are proved in the Appendix.

**Proposition 2.** A contract is immune to renegotiation if and only if it is a debt-equity contract, in that it has the form given in (13), where $\varepsilon_s^* = \varepsilon_s^*(U_s)$ is the cutoff for bankruptcy and $\varepsilon_s^* \leq \varepsilon_{s,\max}$.

The proof is similar to that in Townsend (1979). This proposition says that without loss of generality, we can restrict attention to such debt-equity contracts. In what follows, we will restrict attention to such contracts that satisfy $\varepsilon_s^* \leq \varepsilon_{s,\max}$.

Here we have assumed that investors and managers can renegotiate their contracts immediately before the idiosyncratic shock $\varepsilon$ is realized. It should be clear that if we allowed a second renegotiation phase after the idiosyncratic shock is realized, incentive compatibility implies that no renegotiation will take place.

As with our scaling of our payments to investors, we find it convenient to let $y_s(\varepsilon_s^*)$ and $c_s(\varepsilon_s^*)$ denote the values of expected output and consumption scaled by the size of the project $g(k)$ from a contract with cutoffs $\{\varepsilon_s^*\}$. Thus, the scaled expected output is

$$y_s(\varepsilon_s^*) = RA_s \int_{\underline{\varepsilon}}^{\varepsilon_s^*} \varepsilon \; dH_s(\varepsilon) + A_s \int_{\varepsilon_s^*}^{\bar{\varepsilon}} \varepsilon \; dH_s(\varepsilon),$$

and the scaled expected consumption is

$$(14)\quad c_s(\varepsilon_s^*) = A_s \int_{\varepsilon_s^*}^{\bar{\varepsilon}} (\varepsilon - \varepsilon_s^*)\ dH_s(\varepsilon)$$

if $\varepsilon_s^* > \underline{\varepsilon}$ and is simply some constant, say $c_s$, otherwise. There should be no confusion if, for brevity, we simply refer to $y_s(\varepsilon_s^*)$, $c_s(\varepsilon_s^*)$, and $d_s(\varepsilon_s^*)$ as output, consumption, and payments to investors, realizing that they are all scaled. The scaled variables in the event of no bankruptcy are defined analogously. For future use note that $y_s(\varepsilon_s^*)$ and $c_s(\varepsilon_s^*)$ are both decreasing in $\varepsilon_s^*$.

For simplicity only, we will assume throughout that, in any contract that is immune to renegotiation, no bankruptcy occurs in the healthy state. This assumption is satisfied if $d_s(\varepsilon_s^*)$ decreases everywhere in $\varepsilon_s^*$. It is easy to show that this function is decreasing if $\bar{\varepsilon}_H/\underline{\varepsilon}_H$ is sufficiently close to one. Under this assumption, it follows that output in the healthy state is simply $A_H$, and we denote the manager's consumption in this state by $c_H$. Since under this assumption idiosyncratic shocks in the healthy state play no role, we will suppress these shocks from now on and let $h(\varepsilon)$ and $H(\varepsilon)$ denote the density and the distribution in the distressed state. We will also conserve on notation by letting $\varepsilon^* = \varepsilon_L^*$, $d(\varepsilon^*) = d_L(\varepsilon_L^*)$, and $\varepsilon_{\max} = \varepsilon_{L,\max}$

Proposition 2 implies that a competitive equilibrium consists of a debt-equity contract that maximizes the manager's utility subject to the participation constraint, (11), and the manager's incentive constraint, (12). Given Proposition 2 and our assumption of no bankruptcy in the healthy state, the contract in a competitive equilibrium solves

$$(15)\quad \max_{c_H,p,k,\varepsilon^*} \left[pc_H + (1-p)c_L\left(\varepsilon^*\right) - v(p)\right] g(k)$$

subject to $\varepsilon^* \leq \varepsilon_{\max}$,

$$(16)\quad c_H - c_L(\varepsilon^*) = v'(p)$$

$$(17)\quad \left[p(A_H - c_H) + (1-p)(y_L(\varepsilon^*) - c_L(\varepsilon^*))\right] g(k) \geq k,$$

where we have assumed that the first-order approach is valid so that we can replace the global incentive constraint (12) by its local counterpart (16).

This contracting problem sheds light on the desirability of having bankruptcy in the distressed state. To see why bankruptcy is desirable, notice from (16) that since the marginal disutility of providing effort $v'(p)$ is increasing in the level of effort $p$, the manager's effort decision $p$ is increasing in the spread in expected consumption $c_H - c_L(\varepsilon^*)$. Since the manager's expected consumption in the distressed state is decreasing in the amount of bankruptcy in that state, in that $c_L(\varepsilon^*)$ is decreasing in $\varepsilon^*$, it follows that incentives to exert effort are improved by having bankruptcies in the distressed state. Bankruptcies, of course, reduce output. Thus, bankruptcies in the distressed state trade off improved incentives for the manager against reduced output.

Note that this kind of reasoning adds an additional force that makes bankruptcy in the healthy state undesirable. Such bankruptcies reduce output and, unlike bankruptcies in the distressed state, worsen incentives. Thus, even if $d_H(\varepsilon_H^*)$ is not decreasing everywhere, if the incentive effects are sufficiently strong, the optimal contract will have no bankruptcy in the healthy state.

We now turn to simplifying the contracting problem. First, we substitute out for $c_H$ and combine (16) and (17) into a single constraint called the *implementability* constraint:

(18) $f(p, \varepsilon^*)g(k) \geq k$

where $f(p, \varepsilon^*) = p\left[A_H - c_L(\varepsilon^*) - v'(p)\right] + (1-p)\left[y_L(\varepsilon^*) - c_L(\varepsilon^*)\right]$. Note that an outcome is implementable if and only if it satisfies the implementability constraint. Next, using a duality argument instead of maximizing the welfare of the manager, we can equivalently maximize the sum of the utilities of the managers and the investors, referred to as the total *surplus*, denoted $U(p, \varepsilon^*)g(k) + \omega - k$ where

(19) $U(p, \varepsilon^*) = pA_H + (1-p)y_L\left(\varepsilon^*\right) - v(p).$

We then have the following lemma, which combines these results.

*Lemma 1.* The contracting problem (15) reduces to

$$\max_{p,\varepsilon^*,k} U(p,\varepsilon^*)g(k)+\omega-k \tag{20}$$

subject to $\varepsilon^* \leq \varepsilon_{\max}$ and the implementability constraint (18).

It is straightforward to show that if the resources lost to bankruptcy from raising the bankruptcy cutoff above its minimum value are small, then in any solution to the contracting problem, there is bankruptcy in the distressed state in that $\varepsilon^* > \underline{\varepsilon}$. These lost resources are small if $R$ is sufficiently close to 1 or if $\underline{\varepsilon}$ is sufficiently close to zero. In what follows, we will focus on economies in which there is bankruptcy in the distressed state. In light of these results, we can refer to a contract $x$ as a three-tuple $(p,\varepsilon^*,k)$ where the corresponding consumption allocation $c_L(\varepsilon^*)$ is obtained from (14) and $c_H(\underline{\varepsilon})$ from (16).

To understand why we label our contracts debt-equity contracts, consider the following implementation under the assumption that $A_H$ is sufficiently large so that $d_H > A_L\varepsilon^*$. This implementation uses a combination of debt, outside equity, and inside equity thought of as managerial compensation. Let $A_L\varepsilon^*$ be the *face value* of debt in both the healthy and distressed states. In the healthy state, debt holders receive the face value of their debt, whereas in the distressed state, they receive the face value in the event of no bankruptcy and all of output $RA_L\varepsilon$ in the event of bankruptcy. The *payments to outside equity* are given by $d_H - A_L\varepsilon^*$ in the healthy state and are 0 in the distressed state. The *payments to inside equity* are given by managerial compensation. Given this implementation, for future use note that the market value of the debt in the distressed state before the idiosyncratic shock $\varepsilon$ is realized is $d(\varepsilon^*)$.

In setting up this implementation, we have in mind that the outside equity holders control the firm and have entered into binding contracts with inside equity holders and debt holders. Here the contract specifies that if the firm cannot make the face value payments to debt holders, the firm is forced into bankruptcy, inside equity holders receive zero, the manager is replaced, and the debt holders become the residual claimants.

In sum, in the competitive equilibrium, optimal contracts imply that many firms undergo bankruptcies. These bankruptcies are necessary in order to provide optimal incentives for managers to exert effort. Once the manager has exerted effort, these bankruptcies play no useful social role. Nevertheless, private agents do not renegotiate away these bankruptcies because there is no alternative contract that can make investors and managers in individual firms better off.

**B. Adding a Bailout Authority**

Here, we introduce a benevolent government in the form of a bailout authority and show that with commitment the bailout authority does not intervene, but without commitment this authority finds it optimal to eliminate all bankruptcies. The difference between these policies with and without commitment implies that the bailout authority faces a time inconsistency problem.

The bailout authority is in many ways symmetric to private agents. It can participate in negotiating alternative contracts, and it faces the same informational constraints as the private agents. We require that such negotiations be voluntary. The key difference between private agents and this authority is that this authority has the power to levy taxes on healthy firms without their consent. We suppose first that the bailout authority chooses its policies at the beginning of the period and can commit to them. As we have shown in Proposition 1, since the competitive equilibrium is efficient, it follows that a bailout authority with commitment will choose not to intervene. Thus, the *commitment equilibrium* outcomes, denoted $p_{CE}, \varepsilon^*_{CE}, k_{CE}$ with associated utility level $U_{CE}$, are those that solve the contracting problem (20).

We model lack of commitment by having the bailout authority choose its policies after the manager's effort choice has been made and all the shocks have been realized. Formally, the timing in the period is that in the first stage, each firm chooses a contract $x$. Next, each manager chooses a probability $p$. Let $x_R = (p_R, \varepsilon_R, k_R)$ denote the representative contract. Then the health shock $s$ for each firm is realized. After that, the private agents renegotiate the contract. Then the idiosyncratic shocks $\varepsilon$ are realized. Finally, the bailout authority observes the representative contract and the health state of each firm and uses the optimal decision rules of managers to infer their effort level. The bailout authority then chooses its policy $\pi$, which has four parts: a decision to undertake a bailout, denoted $\delta = 1$ for a bailout and $\delta = 0$ for no bailout, a (scaled) debt purchase offer $d_b$, a renegotiated debt level indexed by $\varepsilon_b$, and a tax rate $\tau$. Hence, $\pi = (\delta, d_b, \varepsilon_b, \tau)$. Individual firms decide whether to accept or reject the bailout offer. Both investors and managers must agree to accept the offer in order for the firm to accept it. If the firms accept the offer, the investors receive $d_b$ and the bankruptcy cutoff is set at $\varepsilon_b$. If they reject the offer, the original contract is implemented.

Under this policy, if the bailout authority undertakes a bailout, it offers to purchase debt from investors for an amount $d_b$ and offers a new debt contract for managers, summarized by a bankruptcy cutoff $\varepsilon_b$. Given a representative contract $x_R$, we require that

(21) $d_b = A_L \varepsilon_R$ and $\varepsilon_b \leq \varepsilon_R$.

These requirements capture the idea that if the government undertakes a bailout, it must make a *serious offer* that the representative firm will accept. Since the face value of debt $A_L \varepsilon_R$ is always greater than the market value of debt $d(\varepsilon_R)$, investors with a representative contract will clearly accept such an offer. Given that a lower bankruptcy cutoff (weakly) raises the manager's consumption for all $\varepsilon$, we assume that a manager with a representative contract will always accept such an offer. Finally, the tax policy is a uniform tax $\tau$ on receipts of investors scaled by the size of the project $g(k)$.

Note that a bailout policy implies that the bailout authority pays $d_b$ to each firm and receives payments $d(\varepsilon_b)$. Thus each firm that accepts the offer can be thought of as receiving a subsidy given by $d_b - d(\varepsilon_b)$. This subsidy is financed by taxes on all firms. We will show that this subsidy induces a distortion.

Here we have assumed that the bailout authority must pay at least the face value of debt. An alternative assumption is that the bailout authority must pay some amount in between the market value of debt $d(\varepsilon_R)$ and the face value of debt $A_L \varepsilon_R$. All the results in this paper go through under this alternative assumption.

We have also assumed that the bailout authority intervenes after the idiosyncratic shocks $\varepsilon$ are realized. We did so to allow for the possibility that the bailout authority seeks to bail out only those firms that would declare bankruptcy without its intervention. It is possible to show both here and in the dynamic version of the model that the results are identical if we instead assume that the bailout authority intervenes before these shocks are realized.

An equilibrium here consists of strategies that are functions of relevant histories and are optimal for the bailout authority and for private agents. The strategy for an individual firm consists of a contract $x$. When the bailout authority makes its decision, the history is $x_R$ and a strategy for the bailout authority consists of a policy $\pi(x_R) = \big(\delta(x_R), d_b(x_R), \varepsilon_b(x_R), \tau(x_R)\big)$.

Taking as given the representative contract and the acceptance strategies of private agents, the bailout authority chooses its policy to maximize the sum of utilities of the manager and the investor given by

$$(22)\quad [\delta U(p_R, \varepsilon_b) + (1-\delta)U(p_R, \varepsilon_R)]\, g(k_R) + \omega - k_R$$

subject to the budget constraint

$$(23)\quad \delta\tau g(k_R) = \delta\,(1-p_R)\,[A_L\varepsilon_R - d(\varepsilon_b)]\, g(k_R),$$

where $\delta = \delta(x_R)$ and the function $U$ is defined in (19).

Consider the problem the contract $x$ solves for an individual firm. If the strategy specifies a bailout with policies $(d_b, \varepsilon_b)$, then investors and managers need to think through what outcomes will occur for different original contracts they specify, including those that differ from the representative contract. They understand that for certain choices of $x$, they will accept the bailout and the investors will receive $d_b$ and the bankruptcy cutoff $\varepsilon_b$ will be implemented, whereas for other choices, they will reject the bailout and the original contract will be implemented.

In order to describe the strategies of the individual firms, a useful approach is to break the contracting problem into two parts. In the first part the firm chooses effort $p$ and size $k$ given that it accepts the bailout. The contracting problem in this case is given by

$$(24)\quad \max_{p,k} \left[\tilde{U}(p, \varepsilon_b, d_b) - \tau\right] g(k) + \omega - k,$$

where $\tilde{U}(p, \varepsilon_b, d_b) = pA_H + (1-p)\,(d_b + c_L(\varepsilon_b)) - v(p)$ subject to the implementability constraint

$$(25)\quad \left[\tilde{f}(p,\varepsilon_b,d_b)-\tau\right]g(k)\geq k,$$

where $\tilde{f}(p,\varepsilon_b,d_b)=p\left[A_H-c_L(\varepsilon_b)-v'(p)\right]+(1-p)d_b$. Here $\varepsilon_b$ and $d_b$ are obtained from the policy $\pi$ induced by the strategy $\pi(x_R)$ given the representative contract $x_R$. Note, for later, that the two first-order conditions for $p$ and $k$ can be summarized in a single condition:

$$(26)\quad \left[\tilde{U}(p,\varepsilon_b,d_b)-\tau\right]g'(k)-1-\frac{\tilde{U}_p(p,\varepsilon_b,d_b)}{\tilde{f}_p(p,\varepsilon_b,d_b)}\left[\left(\tilde{f}(p,\varepsilon_b,d_b)-\tau\right)g'(k)-1\right]=0.$$

In the second part, the firm determines the best contract among the set of contracts such that the firm will reject the bailout. Such contracts specify either $d(\varepsilon^*)>d_b$ so that investors will reject the bailout or $\varepsilon^*<\varepsilon_b$ and let $X(\varepsilon_b,d_b)$ denote the set of such contracts. Here managers anticipate that if their idiosyncratic shock $\varepsilon>\varepsilon^*$, they strictly prefer to reject the bailout. If $\varepsilon\leq\varepsilon_b$, they are indifferent between accepting or rejecting the bailout since their consumption will be zero in either case. We break this indifference by assuming that the managers reject the bailout. This way of breaking the indifference does not affect our results but allows us to save on notation.

The contracting problem in this case is then given by

$$(27)\quad W(\pi)=\sup_{x\in X(\varepsilon_b,d_b)}\left[U(p,\varepsilon^*)-\tau\right]g(k)+\omega-k$$

subject to the associated implementability constraint

$$(28)\quad \left[f(p,\varepsilon^*)-\tau\right]g(k)\geq k,$$

where $U(p,\varepsilon^*)$ is given in (19) and $f(p,\varepsilon^*)$ is given in (18). The individual firm's problem is to choose the contract that gives the larger surplus of the contracting problems (24) and (27) and we assume that if these surpluses are tied the firm chooses (24). We refer to this combined problem as the *bailout contracting* problem.

Finally, if the strategy of the bailout authority specifies no bailout, then the contract solves the standard problem (20).

A *bailout equilibrium* consists of strategies for managers, investors, and the bailout authority that satisfy $(i)$ representativeness in that the solution to the individual contracting problem $x$ coincides with the representative contract $x_R$; $(ii)$ given the policy $\pi$ and the representative contract $x_R$, the individual contract $x$ solves the contracting problem at $\pi = \pi(x_R)$; $(iii)$ for every representative contract $x_R$, the policy $\pi$ maximizes the bailout authority's objective subject to its budget constraint. Let $(x_R, \tau_b)$ denote an equilibrium outcome.

This economy has a time inconsistency problem in that any bailout equilibrium yields worse outcomes than those in the commitment equilibrium. To see why, note that at the time the bailout authority makes its decision, the managers have already made their effort decision and bankruptcies are a pure deadweight loss. Hence, the bailout authority will intervene by purchasing all the debt and canceling all bankruptcies. Given this policy by the bailout authority, private agents will not find it optimal to choose the same contract as they would under the commitment equilibrium.

In addition to the time inconsistency problem, the economy has a coordination problem in that it has multiple equilibria. In all of them, firms accept a bailout offer that has no bankruptcies and taxes are positive. What differs across these equilibria is the level of the taxes and the payments to investors $A_L \varepsilon_R$. In particular, the economy has an equilibrium in which $\varepsilon_R$ is relatively low so that bailout payments and associated taxes are relatively low, and an equilibrium in which $\varepsilon_R = \varepsilon_{\max}$ so that bailout payments are at their maximal feasible level and taxes are relatively high. Indeed, any value of $\varepsilon_R$ is part of an equilibrium if the value of accepting the bailout at $d_b = A_L \varepsilon_R$ is at least as large as the value of rejecting the bailout, that is,

$$(29) \quad \left[\tilde{U}(p_R, \underline{\varepsilon}, d_b) - \tau_b\right] g(k_R) + \omega - k_R \geq W(\pi),$$

where $\pi = (\underline{\varepsilon}, d_b, \tau_b)$. We refer to this inequality as the *no deviation* constraint. Any outcome $x_R, \tau_b$ such that $x_R$ solves (24) with $\varepsilon_b = \underline{\varepsilon}$, satisfies the government budget constraint, and satisfies (29) is a bailout equilibrium outcome. Let $\varepsilon_{\min}$ denote the smallest value of $\varepsilon_R$ that satisfies (29). For simplicity, we assume that the market value of debt under commitment

$d(\varepsilon_{CE})$ is greater than the face value of debt at the lowest point in the support, namely, $A_L\underline{\varepsilon}$. We then have the following proposition.

**Proposition 3.** Any bailout equilibrium in the one-period model has no bankruptcy, has positive taxes, and is inefficient. A continuum of equilibria exist. Each has an $\varepsilon_R$ that satisfies $\varepsilon_{\min} \leq \varepsilon_R \leq \varepsilon_{\max}$ where $\varepsilon_{\min} > \underline{\varepsilon}$.

Proposition 3 makes clear that our economy has a classic time inconsistency problem in that the best equilibrium without commitment has strictly lower welfare than does the best equilibrium with commitment. In addition, without commitment the economy has a coordination problem in that there are a continuum of equilibria with differing levels of welfare. In contrast, Farhi and Tirole (2012) do not have a classic time inconsistency problem but rather have only a coordination problem. In their economy, the best equilibrium without commitment coincides with the best equilibrium with commitment.

In our economy, the lack of commitment leads to lower welfare for two reasons. The first is that a benevolent bailout authority always finds it optimal to buy up all the debt and eliminate all bankruptcies. Doing so distorts effort and investment. The second is that buying up this debt forces the bailout authority to levy taxes, which also distort effort and investment.

In Figure 1 we display the surplus and taxes in a bailout equilibrium as a function of $\varepsilon_R$. The figure demonstrates our result that $\varepsilon_{\min} > \underline{\varepsilon}$, taxes are increasing in $\varepsilon_R$, and surplus is decreasing in $\varepsilon_R$. Taxes are increasing in $\varepsilon_R$ because the face value of the debt $A_L\varepsilon_R$ is increasing in $\varepsilon_R$. The surplus is decreasing in $\varepsilon_R$ because the higher taxes distort both the effort and size decisions. (This figure and those that follow are qualitative representations of what emerges from numerical examples.)

In our dynamic model it turns out that the best equilibrium is sustained by reversion to the worst equilibrium of the one-period model. We refer to this worst equilibrium that we now characterize as the *full bailout* equilibrium. In this equilibrium, the representative firm chooses its debt level to satisfy $d_{FB} = \max_{\varepsilon^*} d(\varepsilon^*)$. The outcomes in a full bailout equilibrium are $(p_{FB}, \underline{\varepsilon}, k_{FB})$ and the policies are $\pi_{FB} = (1, d_{FB}, \underline{\varepsilon}, \tau_{FB})$.

Thus far we have considered bailout policies in which the bailout authority directly buys all the debt of distressed firms and renegotiates contracts with managers. These policies can be interpreted as ones in which the bailout authority provides funds to distressed banks under the implicit or explicit condition that these funds be used to renegotiate the terms of bank loans with firms. To do so, consider a slight variant of our model in which households invest their endowments with banks, which then provide funds to firms. Suppose that banks do not hold a completely diversified portfolio. Some banks will then be confronted with situations in which a large fraction of their funds have been lent to distressed firms. Such banks may well be threatened with the possibility of default. The bailout authority will then find it optimal to bail out such banks under the condition that the banks renegotiate the terms of their loans with the distressed firms. In this sense, our model is consistent with bailouts, in practice, being primarily directed at banks and similar financial institutions. (See also Section 2D for an alternative interpretation.)

### C. Improving on Bailout Equilibria

Taking as given that the bailout authority is always present and will intervene if it finds it optimal to do so, we ask whether there are policies that can improve upon bailout equilibria. To answer this question, we begin by developing a notion of constrained efficiency for our environment without commitment. This notion serves as a benchmark against which policies can be evaluated. One set of constraints arises from the idea that an allocation must respect the incentives of private agents and resource constraints and is summarized by the implementability constraint. Another set of constraints arises from the idea that an allocation must respect the incentives of the bailout authority and is captured by a sustainability constraint as described below. The relevant notion of constrained efficiency is the best outcome that satisfies both the sustainability constraint and the implementability constraint. We label such outcomes *sustainably efficient*.

Confronted with any allocation $(p, \varepsilon^*, k)$ with surplus $U(p, \varepsilon^*)g(k)$, the bailout authority has the option of intervening by bailing out firms and attaining $U(p, \underline{\varepsilon})g(k)$. Thus, to respect the incentives of this authority to intervene, an allocation must satisfy

(30) $U(p, \varepsilon^*)g(k) \geq U(p, \underline{\varepsilon})g(k)$,

which we refer to as the *sustainability* constraint. Thus, a sustainably efficient outcome maximizes surplus subject to the implementability constraint (18) and the sustainability constraint (30).

Clearly, since bankruptcies are a deadweight loss, the constraint (30) is equivalent to $\varepsilon^* = \underline{\varepsilon}$. Thus, in both a sustainably efficient outcome and in all bailout equilibria, there are no bankruptcies. Hence, the sustainably efficient outcome solves a problem similar to the contracting problem in a bailout equilibrium (24), except that subsidies and taxes are zero. Since subsidy distortions are eliminated in a sustainably efficient outcome, ex ante welfare in such an outcome is strictly higher than in any of the continuum of bailout equilibria. Thus, all bailout equilibria are sustainably inefficient.

We now turn to analyzing sustainable efficiency under certain policies. Consider first a benevolent *orderly resolution* authority that can impose losses on debt holders. Specifically, after the manager has exerted effort and all shocks have been realized, this authority can reduce the bankruptcy cutoff from $\varepsilon_R$ to a lower level, say, $\varepsilon_O$ and, rather than levy taxes, can force the investors to accept the lower associated payments. After the orderly resolution authority has chosen its policy, the bailout authority chooses its policy. This formulation formalizes the idea that the bailout authority is always present and can engage in bailouts if it desires to do so.

We now show that the sustainably efficient outcome can be implemented as an orderly resolution equilibrium. To do so, consider an orderly resolution authority that eliminates all bankruptcies by setting $\varepsilon_O = \underline{\varepsilon}$ and forces investors to accept the lower payments. Given this policy, the bailout authority will not intervene. Since private agents anticipate these policies, the contracting problem faced by private agents is (20) with the additional restriction that $\varepsilon^* = \underline{\varepsilon}$. Clearly, the resulting equilibrium is sustainably efficient and, in this sense, introducing an orderly resolution authority is desirable.

The economy with an orderly resolution authority also has a continuum of equilibria in which the orderly resolution eliminates only some of the bankruptcies and the bailout authority intervenes to eliminate the rest and levies taxes for debt payments. The reason is that after the effort and size choices have been made, taxes are not distorting ex post. Thus, the orderly resolution authority is indifferent about whether bankruptcies are eliminated by its policies or by the intervention of the bailout authority. In such equilibria, the taxes distort ex ante decisions and hence lead to sustainably inefficient outcomes. Nonetheless, the orderly resolution authority weakly implements the sustainably efficient outcome.

Consider next a regulatory authority that chooses its policies before private contracts are set. Specifically, the authority can set upper bounds on the ratio of the face value of the debt relative to the value of the firm and on capital that no firm can exceed. It is immediate that by setting the upper bound of the debt to value ratio at $A_L\underline{\varepsilon}g(k_{SE})/k_{SE}$ and by setting the upper bound on $k_{SE}$ where $k_{SE}$ is the sustainably efficient size, this authority can uniquely implement the sustainably efficient outcomes.

Of course, given our focus on weak implementation, adding a regulatory authority does not improve upon outcomes if an orderly resolution authority is present. We argue that this result holds only because our one-period model lacks a critical feature present in our dynamic model. In our one-period model, all sustainable outcomes are at a corner in that they have no bankruptcies. In our dynamic model, reputation effects ensure that sustainable outcomes can have bankruptcies. This feature ensures that the extent of intervention by the orderly resolution authority is influenced by the sizes of firms. This influence induces a subtle size externality that orderly resolution cannot cure but regulation can.

## 2. The Dynamic Model

As we argued above, in practice, we observe partial but not complete bailouts and typically only during crises. Our one-period model has complete bailouts and makes no distinction between normal and crisis times. Here we develop a dynamic stochastic model with partial bailouts that occur only during crises. In it the economy can be either in *normal* times or in *crisis* times. We model normal times so that in such times, output is higher and the resources lost to bankruptcy are smaller than in crisis times. The dynamic model generates partial bailouts because in it reputational considerations generate ex post costs that depend on the size of the bailout. Given the greater lost resources during bankruptcy, the bailout authority has stronger incentives to intervene in crisis times than in normal times.

We start by showing that if the bailout authority is not too patient, the equilibrium is sustainably inefficient. The inefficiency arises from two sources: subsidy distortions and size externalities. The first source of inefficiency arises for the same reason as it did in the one-period model: the subsidies associated with bailouts distort both the effort decision of the manager and the size choice of the firm.

The second source of inefficiency is more subtle and does not arise in the one-period model. In the one-period model, the sustainably efficient outcome is at a corner in which all bankruptcies are eliminated, and variations in the debt or size of firms do not affect this outcome. In the dynamic model, a sustainably efficient outcome typically has a strictly positive amount of bankruptcies, and variations in firms' debt and size decisions in the aggregate affect the amount of intervention by the bailout authority. In particular, as firms in the aggregate increase their debt and size, they increase the extent of intervention by the bailout authority. Since no individual firm internalizes the effect of its decisions on the bailout authority's decisions, this dynamic economy has a size externality that is not present in the one-period economy.

We show that an orderly resolution authority improves welfare relative to bailout outcomes but does not attain sustainable efficiency. Unlike a bailout authority, an orderly resolution authority does not have to induce firms voluntarily to reduce their debt levels and therefore does not create subsidy distortions. This authority does not, however, have a policy instrument that would make firms internalize their size externalities.

We also show that a regulatory authority equipped with the power to decree maximal debt to value ratios and maximal sizes of firms can achieve the sustainably efficient outcomes. Briefly, by specifying such maximal levels of debt and size, the regulatory authority can ensure that the bailout authority does not intervene in equilibrium, and hence it eliminates the subsidy distortions. Also, by specifying these maximal levels to be those that occur in the sustainably efficient outcome, this authority can make the firms internalize the externalities from their decisions.

### A. Bailouts

Except for the addition of aggregate shocks, each period of the dynamic model is identical to the one period in the static model. In particular, no physical state variables link periods.

*Setup and Definition of Bailout Equilibrium.* At the beginning of each period, an aggregate shock $S$ is realized. For ease of notation only, we let this shock take on two values $S \in \{S_N, S_C\}$ with probabilities $\mu_N$ and $\mu_C$. We refer to the state $S_N$ as *normal* times and the state $S_C$ as *crisis* times. This shock affects the probability of the healthy idiosyncratic state. Specifically, if the manager chooses $p$, then in crisis times the probability is $p$, but in normal times the probability of the healthy state is $p + \gamma$ for some positive $\gamma$ (with the understanding that this probability is one if $p + \gamma > 1$).

After the aggregate shock is realized, investors choose a contract $x_t$. Next the manager chooses $p_t$, and then the idiosyncratic shocks $(s_t, \varepsilon_t)$ for each firm are realized. Finally, the bailout authority chooses its policy $\pi_t$.

To focus attention on the dynamic incentive problem of the bailout authority, we make an anonymity assumption. We assume that managers are anonymous in the sense that their identities cannot be recorded from period to period. Hence, current contracts cannot be conditioned on the past track record of individual managers, and long-term contracts are infeasible. To allow the bailout authority to have dynamic incentives, we assume that past aggregates, including the policies of the bailout authority, are observable. These assumptions imply that the only links between periods are strategic ones in which the bailout authority forecasts the responses of private agents in the future to its current actions. To capture these strategic links, we allow strategies to depend on the histories faced by agents when they choose actions. (Technically, we focus attention on perfect, public equilibria.)

The histories needed to describe strategies evolve as follows. Let $H_t$ be the history at the beginning of period $t$ after the current aggregate shock $S_t$ has been realized. Let $H_{Bt} = (H_t, x_{Rt})$ denote the history faced by the bailout authority where $x_{Rt}$ is the representative contract chosen at $t$, and let $H_{t+1} = (H_{Bt}, \pi_t, S_{t+1})$.

The strategies are functions of the histories and are denoted by $x_t(H_t)$ and $x_{Rt}(H_t)$ for an individual and the representative contract and by $\pi_t(H_{Bt})$ for the bailout authority.

The payoffs of the bailout authority given a history $H_{Bt}$ are the sum of its period payoffs and continuation values and are given by

$$(31) \quad [\delta U(p, \varepsilon_b, S) + (1-\delta)U(p, \varepsilon^*, S)]\, g(k) + \omega - k + \beta V_{t+1}(H_{Bt}, \pi_t).$$

Here $U(p,\varepsilon,S_N) = (p+\gamma)A_H + (1-p-\gamma)y_L(\varepsilon) - v(p)$, $U(p,\varepsilon,S_C) = pA_H + (1-p)y_L(\varepsilon) - v(p)$, and the continuation payoff $V_{t+1}(H_{Bt},\pi_t)$ is given by the present expected value of period payoffs for the bailout authority starting from period $t+1$ induced by the strategies, and $\beta < 1$ is the discount factor.

Given the anonymity assumption, the contracting problem is static and solves the analog of our earlier contracting problem for all histories. Specifically, given a history $H_t$, firms use the strategy $x_{Rt}(H_t)$ to construct the induced history $H_{Bt} = (H_t, x_{Rt}(H_t))$ and thus predict a bailout policy from the bailout authority's strategy $\pi_t(H_{Bt})$. If the predicted policy is a bailout then the individual contract is the solution to the bailout contracting problem, namely the larger of (24) and (27), whereas if the predicted policy is no bailout, then the individual contract is the solution to the contracting problem with no intervention, namely (20).

A *bailout equilibrium* is a collection of strategies $\{x_t(H_t), x_{Rt}(H_t), \pi_t(H_{Bt})\}$ for private agents and the bailout authority such that for all histories $H_t$, *(i)* the individual and representative contracts coincide; *(ii)* given the history $H_t$, the contract $x_t(H_t)$ solves the contracting problem; and *(iii)* given the strategies of the private agents, the policy $\pi_t(H_{Bt})$ maximizes the payoff for the bailout authority (31) subject to its budget constraint given by the analog of (23).

The outcomes associated with a bailout equilibrium are stochastic processes $\{x_t, p_t, \pi_t\}$ and associated continuation utilities for the bailout authority $\{V_t\}$.

*Characterization of Bailout Equilibrium Outcomes.* To characterize the bailout equilibrium outcomes, note that the optimality of private agents is captured by the requirement that the contracts solve the contracting problem for firms in each period given the policies of the bailout authority. Next, we show that the optimality of the bailout authority is captured by a sustainability constraint.

In our infinite horizon model, we focus attention on equilibria that can be supported by trigger-type strategies that specify reversion to outcomes that are no worse than the full bailout outcomes. This set of equilibrium outcomes is analogous to the set of equilibrium outcomes in repeated games that are supported by reversion to the one-shot Nash equilibria. (Of course, following the work of Abreu (1988), more sophisticated strategies could possibly support a larger set of equilibria. The results are similar, but the analysis is more cumbersome.) Specifically, we focus on equilibria in which for every history, even those after deviations by the bailout authority from a given policy plan, the continuation values of the bailout authority satisfy

(32) $V_{t+1}(H_{Bt}, \pi_t) \geq V_{FB}$,

where

$$V_{FB} = \frac{\mu_N \left(U_{FB}(S_N)g(k_{FB}(S_N) + \omega - k_{FB}(S_N)\right) + \mu_C \left(U_{FB}(S_C)g(k_{FB}(S_C) + \omega - k_{FB}(S_C)\right)}{1-\beta}$$

is the expected discounted value of the full bailout outcome in both states and the utility level associated with the full bailout outcome is $U_{FB}(S)g(k_{FB}(S)) + \omega - k_{FB}(S)$. This condition restricts the severity of the trigger strategies to be no worse than that of the strategies implicit in the infinite reversion to the full bailout equilibrium.

To set up our sustainability constraint, we use a standard result that outcomes are sustainable if and only if the payoff is at least as large as the payoff from the best one-shot deviation in the current period, followed by infinite reversion to the worst outcome. (See, for example, Chari and Kehoe 1990.) To that end, consider a period $t$ in which a contract $x_t$ and probability of healthy state $p$ have been chosen. The best one-shot deviation for the bailout authority is to buy all the debt in the distressed state and then renegotiate with the managers so as to eliminate all bankruptcies by setting $\varepsilon_b = \underline{\varepsilon}$. This deviation yields payoffs given by $U(p_t, \underline{\varepsilon}, S)g(k_t) - k_t$ in the current period followed by the full bailout outcomes from period $t+1$ onward. The *sustainability constraint* of the bailout authority is then given by

(33) $U(p_t, \varepsilon_t^*, S)g(k_t) + \beta V_{t+1} \geq U(p_t, \underline{\varepsilon}, S)g(k_t) + \beta V_{FB}$,

where $V_{t+1}$ is the continuation payoff for such outcomes where we have canceled the $k_t$ term on each side. Substituting for the surplus function and simplifying we can rewrite this constraint as

$$(34)\quad \left[(1-p_t)(1-R)A_L\int_0^{\varepsilon_t^*}\varepsilon dH(\varepsilon)\right]g(k_t)\leq\beta(V_{t+1}-V_{FB}).$$

At an intuitive level this constraint requires that the value of the resources saved by cancelling all bankruptcies in the current period are smaller than the present value of losses induced by the switch to the full bailout outcome in all future periods.

We then have the following proposition.

**Proposition 4.** Under our reversion assumption (32), a stochastic process $\{x_t, p_t, \pi_t\}$ is the outcome of a bailout equilibrium in a dynamic model if and only if the outcomes (*i*) solve the contracting problem for normal and crisis times, (*ii*) satisfy the bailout authority's budget constraint (23), and (*iii*) satisfy the sustainability constraint (33).

Purely for ease of exposition, in what follows we assume that $\gamma$ is sufficiently high so that the contracting problem under commitment in normal times has no bankruptcies. Given this assumption, it immediately follows that in normal times the efficient outcome is sustainable. Our results below hold even with bankruptcies in normal times under appropriate sufficient conditions. We denote the surplus and the project size associated with the efficient outcome in normal times by $U_N$ and $k_N$.

We now show that if the bailout authority is sufficiently impatient, the equilibrium has bailouts in crisis times. To do so, we show that if the discount factor is above a critical level, then the efficient outcome is sustainable, and if the discount factor is below this critical level, then the equilibrium necessarily has bailouts. That is, at the equilibrium, the tax rate $\tau$ is positive, the bailout authority buys debt in the distressed state, and it renegotiates the terms of the contract of the manager in order to have less bankruptcy. The critical level of the discount factor $\bar{\beta}$ is defined as the discount factor such that the sustainability constraint holds with equality at the efficient outcome; that is,

$$(35)\quad U(p_{CE},\varepsilon_{CE},S_C)g(k_{CE})+\bar{\beta}V_{CE}=U(p_{CE},\underline{\varepsilon},S_C)g(k_{CE})+\bar{\beta}V_{FB},$$

where $p_{CE}$, $\varepsilon_{CE}$, and $k_{CE}$ are the commitment equilibrium outcomes in the crisis state and $V_{CE}$ is the continuation equilibrium payoff from the commitment equilibrium. Clearly $\bar{\beta}<1$

since the continuation losses from a deviation, $V_{CE} - V_{FB}$, converge to infinity as $\beta$ goes to 1. In contrast, the gains from a one-shot deviation, namely $U(p_{CE}, \underline{\varepsilon}, S_C) - U(p_{CE}, \varepsilon_{CE}, S_C)$ given by

$$\left[(1-p)(1-R)A_L \int_0^{\varepsilon^*} \varepsilon dH(\varepsilon)\right] g(k)$$

evaluated at the commitment equilibrium outcome are bounded.

Next, for $\beta > \bar{\beta}$ the left side of (35) is greater than the right side and the sustainability constraint is slack. In this case, the commitment outcomes are equilibrium outcomes in the dynamic model without commitment by the bailout authority. In contrast, if $\beta < \bar{\beta}$ the reverse inequality holds, the sustainability constraint is violated, and the commitment outcomes are not equilibrium outcomes in the dynamic model.

Furthermore, if $\beta < \bar{\beta}$, then in any sustainable outcome, the surplus $U(p_t, \varepsilon_t^*, S_C)$ is strictly less than the corresponding surplus under commitment. This result follows because the commitment outcome is not sustainable. We have the following proposition.

**Proposition 5.** If $\beta \geq \bar{\beta}$, then the commitment outcome is sustainable, and if $\beta < \bar{\beta}$, then any equilibrium allocation has bailouts in that $\delta_t = 1$ and $\tau_t > 0$ in all crisis times.

Next, we show that the best bailout outcomes $(x; \pi) = (p, \varepsilon_R, k; \delta, \varepsilon_b, d_b, \tau)$ solve a programming problem referred to as the *best bailout* problem. Clearly, optimality by private agents requires that an equilibrium outcome solve problem (24) and satisfy the no deviation constraint (29). An equilibrium outcome must also satisfy the serious offer requirement (21), the government budget constraint (23), and the sustainability constraint (33). The best bailout outcomes can be obtained by choosing these outcomes to maximize surplus subject to all of these constraints.

*Inefficiency of Bailout Equilibria.* Clearly, if the sustainability constraint is binding, any bailout equilibrium yields worse outcomes than those in an equilibrium with commitment by the bailout authority. Here we show that bailout equilibria are sustainably inefficient.

Again, in our environments, the bailout authority is always present and will intervene if it finds it optimal to do so. The incentives of the bailout authority are captured by the sustainability constraint (33). Since an attainable outcome must also satisfy the implementability constraint (18), we say that an outcome is *sustainably efficient* if it maximizes social surplus subject to both of these constraints. Let $(p_{SE}(S), \varepsilon_{SE}(S), k_{SE}(S))$ where $S \in \{S_N, S_C\}$ denote such an outcome.

The following proposition is the first of our three main results.

**Proposition 6.** If $\beta < \bar{\beta}$, then any bailout equilibrium is sustainably inefficient.

The proof of this proposition is an immediate consequence of Propositions 7 and 8 below. The bailout equilibrium has two distortions relative to a sustainably efficient outcome, a subsidy distortion similar to that in the one-period model, and a size externality that does not arise in that model. Ignoring subsidies for a moment, the only difference between the contracting problem in a bailout equilibrium and the sustainably efficient outcome is that the contracting problem lacks the sustainability constraint. This difference induces an externality because firms do not internalize that their choices of size affect the sustainability constraint.

To understand the nature of this externality, imagine starting at a bailout equilibrium and suppose that the size of all firms is reduced. The fall in aggregate size reduces the resources lost to bankruptcy and, hence, reduces the temptation of the bailout authority to undertake a full bailout. Hence, the sustainability constraint is now slack. All firms can now increase their bankruptcy cutoffs to some extent and induce managers to exert greater effort without inducing the bailout authority to undertake a full bailout. An individual firm is made even better off if it does not participate in the joint size reduction. These incentives to not participate lead to a free-rider problem which manifests itself as an externality.

## B. Orderly Resolution

Here we study an orderly resolution authority equipped with powers motivated by a key provision of the Dodd-Frank Act. This provision allows regulators to impose losses on creditors without going through bankruptcy. We capture this provision in our model by introducing an orderly resolution authority that has the power to reduce debt payments to investors but not to levy taxes. After the orderly resolution authority has chosen its policies, the bailout authority can intervene if it chooses to do so. Again, the idea here is that the bailout authority cannot commit to not intervene ex post, so that if the orderly resolution authority attempts to implement allocations that do not satisfy the sustainability constraint, the bailout authority will intervene ex post. We show that the best equilibrium with orderly resolution removes the subsidy distortion associated with a bailout equilibrium but does not remove the size externality.

Here the timing of intervention by the orderly resolution authority is similar to that for the bailout authority. Briefly, after private agents have entered into a contract $x = (p, \varepsilon^*, k)$ with a representative contract $x_R = (p_R, \varepsilon_R, k_R)$ and the idiosyncratic shocks have been realized, the orderly resolution authority intervenes by setting a bankruptcy cutoff $\varepsilon_{OR}$ and an associated face value of debt $A_L \varepsilon_{OR}$ subject to the *debt reduction* constraint

$$(36) \quad \varepsilon_{OR} \leq \varepsilon_R$$

which requires that the authority can reduce the face value of the debt but cannot increase it. For a particular firm, if $\varepsilon^* > \varepsilon_{OR}$ then the bankruptcy cutoff is reduced to $\varepsilon_{OR}$ while if $\varepsilon^* < \varepsilon_{OR}$ then its bankruptcy cutoff is left unaffected. After the orderly resolution authority has intervened, the bailout authority can purchase the debt at new face value $A_L \varepsilon_{OR}$, reduce the bankruptcy cutoff to $\varepsilon_b$, and levy the appropriate taxes. If the bailout authority does not intervene, the bankruptcy cutoff $\varepsilon_{OR}$ is implemented. It is notationally convenient and without loss of generality to assume that the orderly resolution authority always intervenes, perhaps trivially by setting $\varepsilon_{OR} = \varepsilon_R$. Thus, a policy here consists of the policy of the orderly resolution authority $\varepsilon_{OR}$ together with the policy of the bailout authority $\pi = (\delta, d_b, \varepsilon_b, \tau)$.

An *orderly resolution equilibrium* is defined analogously to a bailout equilibrium. In particular, any such equilibrium must satisfy the sustainability constraint (33) so that neither the orderly resolution authority nor the bailout authority has an incentive to deviate from its policy.[5]

The best orderly resolution outcome solves a programming problem referred to as the *best orderly resolution* problem. This problem is to choose a contract and a policy to

[5] The sustainability constraint here is the same as in a bailout equilibrium because the worst equilibrium here coincides with the worst bailout equilibrium. To see why these worst equilibria coincide, consider the payoffs of an orderly resolution authority in some period $t$, knowing that regardless of the period $t$ policies of this authority, the future outcomes will be the full bailout outcomes. The current payoffs of this authority are the same if it cancels all bankruptcies and it does not intervene and lets the bailout authority cancel all bankruptcies. Thus, in the worst equilibrium, the orderly resolution authority is indifferent between intervening and not, and so the worst equilibria in the two economies coincide.

maximize surplus subject to the constraints that these allocations solve problem (24), satisfy the serious offer requirement (21), the debt reduction constraint (36), the government budget constraint (23), and the sustainability constraint.

Note that this problem is identical to the best bailout problem except that we have added the debt reduction constraint and have dropped the no-deviation constraint (29). To see why the no-deviation constraint can be dropped, recall that in a bailout equilibrium, firms have the option of choosing debt levels so high that, ex post, investors will reject the bailout. Since the orderly resolution authority involuntarily reduces debt levels we do not have to consider deviations that raise the debt level about the representative debt level $A_L\varepsilon_{OR}$. Since we are focusing on the best outcomes we do not have to consider deviations that reduce the debt level below the representative debt level.

Since the orderly resolution authority has the option to trivially intervene, the best orderly resolution outcome weakly dominates the best bailout outcome. Under fairly general conditions, it is possible to show that this dominance is strict. Here we provide sufficient conditions under which in the best orderly resolution outcome the bailout authority does not intervene so that taxes are zero. This result is more subtle for reasons having to do with the theory of the second best. Our environment has both subsidy distortions and size externalities. In such an environment it is not generally true that removing one distortion alone improves welfare. Our sufficient conditions are

(37) $g(k) = k^\alpha$ and $v(p) = vp^{1+a}$ where $a \geq 1$.

**Proposition 7**. Under (37), if $\beta < \bar{\beta}$, then the best orderly resolution outcome has strictly higher surplus than the best bailout outcome. Morever, the best orderly resolution outcome has zero taxes.

In the proof we show that the best orderly resolution outcome has zero taxes. This result immediately implies that the best orderly resolution outcome yields strictly higher surplus than the best bailout outcome.

We turn now to demonstrating that if $\beta < \bar{\beta}$ the best orderly resolution outcome is sustainably inefficient. To do so note that since taxes are zero, the first-order conditions for the contract can be summarized by the analog of (26),

$$(38)\quad U(p,\varepsilon_{OR})g'(k) - 1 - \frac{U_p(p,\varepsilon_{OR})}{f_p(p,\varepsilon_{OR})}\left[f(p,\varepsilon_{OR})g'(k) - 1\right] = 0.$$

The best orderly resolution problem thus reduces to choosing a contract $(p, \varepsilon_{OR}, k)$ to maximize surplus subject to the implementability constraint (18), the sustainability constraint (33), and the first order condition (38). Note that the only difference between the problem that defines the sustainably efficient outcome and the best orderly resolution problem is that the best orderly resolution problem has an additional constraint, (38). This extra constraint is present because the orderly resolution authority cannot directly control the size of firms.

We then have the following proposition, which is our second main result.

**Proposition 8**. For $\beta \in (0, \bar{\beta})$, the best orderly resolution equilibrium is sustainably inefficient.

Technically, if the sustainability constraint binds and the bankruptcy cutoff is interior then the combined first order condition in the sustainably efficient outcome does not satisfy (38) which implies that the orderly resolution outcomes are sustainably inefficient. Clearly, if $\beta < \bar{\beta}$ the sustainability constraint binds. In the proof we show that if $\beta > 0$, then any sustainably efficient outcome must have an interior bankruptcy cutoff.

Intuitively, this inefficiency arises from the size externality discussed above. Here the externality arises from the free-rider problem which is encapsulated by the constraint (38) which is present in the orderly resolution problem but not in the sustainably efficient problem. Specifically, when firms choose their size freely, as they do in the orderly resolution equilibrium, they do not incorporate the effect of their size choices on the sustainability constraint. That is, individual firms' size choices have an external effect on other firms by tightening the sustainability constraint and inducing the orderly resolution authority to change the bankruptcy cutoff on all firms.

Next, if $\beta = 0$, then the bankruptcy cutoff is no longer interior and our dynamic model effectively collapses to our one-period model. Here the best orderly resolution equilibrium is sustainably efficient. The reason is that here the sustainability constraint is equivalent to $\varepsilon^* = \underline{\varepsilon}$, so that altering the size does not change the incentives of the orderly resolution authority to intervene. Hence, in this case there is no size externality, and the best orderly resolution equilibrium is sustainably efficient.

Finally, if $\beta \geq \bar{\beta}$ the sustainability constraint is slack and the commitment outcome is sustainable. Since the commitment outcome satisfies (38) this constraint is redundant and the best orderly resolution outcome coincides with the commitment outcome.

In Figure 2 we compare the properties of the best bailout equilibrium, the best orderly resolution equilibrium, the sustainably efficient outcome, and the commitment outcome. Panel A illustrates our theoretical ranking of welfare. The order from highest to lowest is commitment, then sustainably efficient, then best orderly resolution, then best bailout. Panels B and D illustrate that the rankings for the bankruptcy cutoff and effort follow that of welfare. Panel C shows that although size is largest under commitment, the rankings of size for the three economies without commitment are in reverse order compared with the rankings for welfare. That is, in the best bailout equilibria, firms are the largest; in the sustainably efficient outcome, firms are the smallest.

To understand the nature of the size externality compare the best orderly resolution equilibrium with the sustainably efficient outcome. Starting at the orderly resolution equilibrium, suppose that all firms could somehow agree to reduce their size. Then the sustainability constraint would be relaxed, and each firm could increase its bankruptcy cutoff. Of course, any individual firm has no incentive to agree to reduce its own size. This free-rider problem leads to an inefficiency: firms are inefficiently large in an orderly resolution equilibrium because they do not internalize the effects of their decisions on the sustainability constraint. Since the sustainability constraint must be satisfied, the inefficiently large size of firms implies that bankruptcy cutoffs must be inefficiently low.

To understand the nature of the subsidy distortion compare the best bailout equilibrium with the best orderly resolution equilibrium. Starting at the best bailout equilibrium suppose that subsidies and taxes are eliminated. As we have discussed eliminating the subsidy distortion raises effort and reduces size thereby relaxing the sustainability constraint and allowing the bankruptcy cutoff to be set at a higher level. Figure 2 shows that at the best orderly resolution equilibrium, effort continues to be higher and size smaller than in the best bailout equilibrium.

### C. Optimal Regulation

Here we show that a sustainably efficient outcome can be achieved by regulation. We model regulation by adding a regulatory authority that chooses policies at the beginning of the period after the aggregate shock is realized. These policies consist of upper bounds on the amount of debt and capital that each firm can choose. Here the regulatory authority moves before private agents choose their contracts and, because there are no state variables that connect one period to the next, the regulatory authority can be thought of as committing to a sequence of policies at date zero. We show that, armed with these policies, the authority can implement a sustainably efficient outcome.

Specifically, consider setting the upper bounds on debt and capital in crisis times to be equal to the sustainably efficient levels of debt and capital, $d(\varepsilon_{SE}(S_C))$ and $k_{SE}(S_C)$. In the contracting problem, if firms choose their debt and capital levels to be at these upper bounds, these policies implement the sustainably efficient outcome.

Consider the contracting problem in crisis times, referred to as the *regulatory problem*,

$$(39)\quad \max_{p,\varepsilon^*,k} U(p,\varepsilon^*,S_C)g(k)+\omega-k$$

subject to

$$(40)\quad f(p,\varepsilon^*,S_C)g(k)\geq k$$

$$(41)\quad d(\varepsilon^*)\leq d(\varepsilon_{SE}(S_C)) \text{ and } k\leq k_{SE}(S_C).$$

In the next proposition, which is the third main result in the paper, we give sufficient conditions under which the upper bounds in (41) bind so that regulation achieves sustainable efficiency.

**Proposition 9.** If productivity in the healthy state, $A_H$, is sufficiently large, the probability of crisis times, $\mu_C$, is sufficiently small, $g(k)=k^\alpha$, and $v''(p)$ is bounded above, the sustainably efficient outcomes can be implemented as a regulatory equilibrium.

Notice that the regulatory problem is identical to the problem of finding the sustainable efficient allocation except that we replace the sustainability constraint (33) by the upper bounds in (41). Clearly, if the regulatory problem attains these bounds, then the problems are the same. We have provided sufficient conditions for these bounds to be attained. The key idea is that by imposing upper bounds on debt and size, the regulatory authority can address both the subsidy distortion and the size externality in a bailout equilibrium.

Here we have considered regulation that imposes quantity restrictions on debt and size. Alternative regulations that would be equally effective would impose Pigouvian taxes on debt and size.

### D. Other Policies

Here we briefly discuss two other policies: *risk-based premium* policies and *too big to fail* policies.

For the risk-based policy imagine a variant of the bailout policy in which the tax paid by an individual firm depends on the contract it chooses. Specifically, if the firm chooses a contract $x$, let the taxes the firm pays be given by $\tau(x)$. Suppose also that the bailout authority's budget constraint is balanced firm by firm so that

$$\delta\tau(x) = \delta(1-p)\left[A_L\varepsilon^* - d(\varepsilon_b)\right],$$

for every value of $x = (p, \varepsilon^*, k)$. (This policy is in the spirit of risk-based premiums in the literature on deposit insurance.) It is easy to show that the best risk-based outcome coincides with the best orderly resolution outcome. The intuition for this result is that as a firm increases its debt level and hence its subsidy in the distressed state its taxes increase by an amount that leaves its implementability constraint unaffected. Hence this policy eliminates the subsidy distortion. In effect, each firm internalizes that it is paying for its own bailouts.

For the too big to fail policy, imagine a variant of our regulatory policy in which the regulator chooses only the limit on size and the bailout authority can intervene if it chooses to do so. Here firms choose effort $p$ and a bankruptcy cutoff $\varepsilon^*$ given a value of $k$. This choice implies a combined first order condition for $p$ and $\varepsilon^*$ which plays an analogous role to (38). The best too big to fail outcome maximizes surplus subject to the implementability constraint, the sustainability constraint, and this combined first order condition. It is possible to show that, under suitable sufficient conditions, the sustainably efficient outcome violates the combined first order condition. It then follows that the too big to fail policy leads to a sustainably inefficient outcome. The intuition for this result is that this policy addresses the size externality but not the subsidy distortion.

**E. Which Industries Should Be Regulated?**

The general implication of our analysis is that regulation should be most stringent when the bailout authorities have the strongest incentive to intervene. For example, in our model the bailout authority has stronger incentives to intervene in crisis times than in normal times because the losses due to bankruptcy are larger in crisis times. This general implication extends to an analysis of which industries should face the most stringent regulation.

We capture heterogeneity across industries by allowing the severity of the incentive problem to differ across industries. The idea is that when incentive problems are more severe, optimal contracts imply higher bankruptcy cutoffs as way of providing incentives. Such bankruptcy cutoffs are typically associated with higher debt to value ratios. Of course, when bankruptcy cutoffs are higher, the losses due to bankruptcy are typically higher, and the incentives of a bailout authority to intervene are also higher. This reasoning suggests that regulation may be most desirable in industries in which firms have the highest debt to value ratios.

We capture heterogeneity in the incentive problem by allowing the curvature of the disutility of effort function $v(p)$ to vary across industries. From the first order condition for a manager's effort (16) we see that the elasticity of the manager's effort in response to a change in the spread $c_H - c_L(\varepsilon^*)$ falls as $v(p)$ becomes more convex, so that industries with more convex disutility of effort functions face more severe incentive problems.

Specifically, we let $v(p) = vp^{1+a}$ and increase the curvature parameter $a$ to make the incentive problems more severe. We use the face value of the debt relative to the value of the firm as our measure of the debt to value ratio. Thus, this ratio is given by $A_L \varepsilon_R g(k)/k$. With bailouts, the bailout authority purchases the outstanding debt and pays the face value $A_L \varepsilon_R g(k)$.

In Panels A and B of Figure 3 we focus on the equilibrium outcomes in crisis times. Let $r_D$ denote the debt to value ratio in the best bailout equibrium relative to that in the sustainably efficient outcome. Let $r_k$ denote the size in the best bailout equilibrium relative to that in the sustainably efficient outcome. We think of these two measures as capturing the extent of regulation needed to implement the sustainably efficient outcome. Panels A and B of Figure 3 plot $r_D$ and $r_k$ as we vary the curvature parameter $a$. These figures show that if the curvature parameter is less than a threshold, $\hat{a}$, the best bailout outcome is sustainably efficient in that $r_D = r_k = 1$. Thus for industries with $a \leq \hat{a}$, no regulation is needed. The figures show that $r_D$ and $r_k$ increase as $a$ increases above $\hat{a}$ so that industries with a high values of $a$ need more stringent regulation on both debt to value ratios and size.

Next, we apply this theory to analyze which industries should be regulated. If the extent of incentive problems across industries was directly measured, the theory suggests that industries with more severe incentive problems should be regulated more stringently. In practice such direct measurements are not available. Instead we use our theory to obtain an indirect measure. Our theory implies that industries with more severe incentive problems have higher debt to value ratios in normal times. In Panel C we show that debt to value ratios in normal times increase with $a$. Thus our theory implies that industries which have higher debt to value ratios in normal times should be regulated more stringently in crisis times.

In practice, firms with high debt to value ratios are disproportionately located in financial industries. The classic example, of course, is the banking industry in which firms tend to have much higher debt to value ratios than in essentially all other industries. Our analysis thus suggests that industries such as banking should be the most highly regulated and those with sufficiently low debt to value ratios should not be.

## 3. Conclusion

We have developed a framework that allows us to evaluate the consequences of policies, such as orderly resolution, that are at the center of current debates over ways to handle the time inconsistency problem of bailouts. In doing so, we have shown that bailouts do much more than create coordination problems that can completely be solved by regulation. Instead, they create fundamental time inconsistency problems.

Using this framework, we have identified a new type of externality that arises in dynamic environments in which decisions of private agents influence the actions of governments. This externality does not show up in our one-period model but does in our infinite horizon model. We think it is likely to be pervasive in dynamic environments in which governments cannot commit themselves to future policies. We have shown how granting governments additional powers can help mitigate such externalities.

A critical feature of our framework is that private agents will change the nature of their contracts as their expectations of government policy change. In particular, we do not restrict our analysis to a fixed set of contracts. Analyses of bailouts that restrict themselves to a fixed set of suboptimal contracts make it hard to distinguish between regulation that is needed to overcome inefficient behavior by private agents and inefficient behavior by governments without commitment. In our analysis, the inefficiency of bailout outcomes arises not from perverse incentives that private agents might have but rather from the incentives of well-meaning governments that cannot commit themselves.

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Figure 1: Equilibria in the one-period model

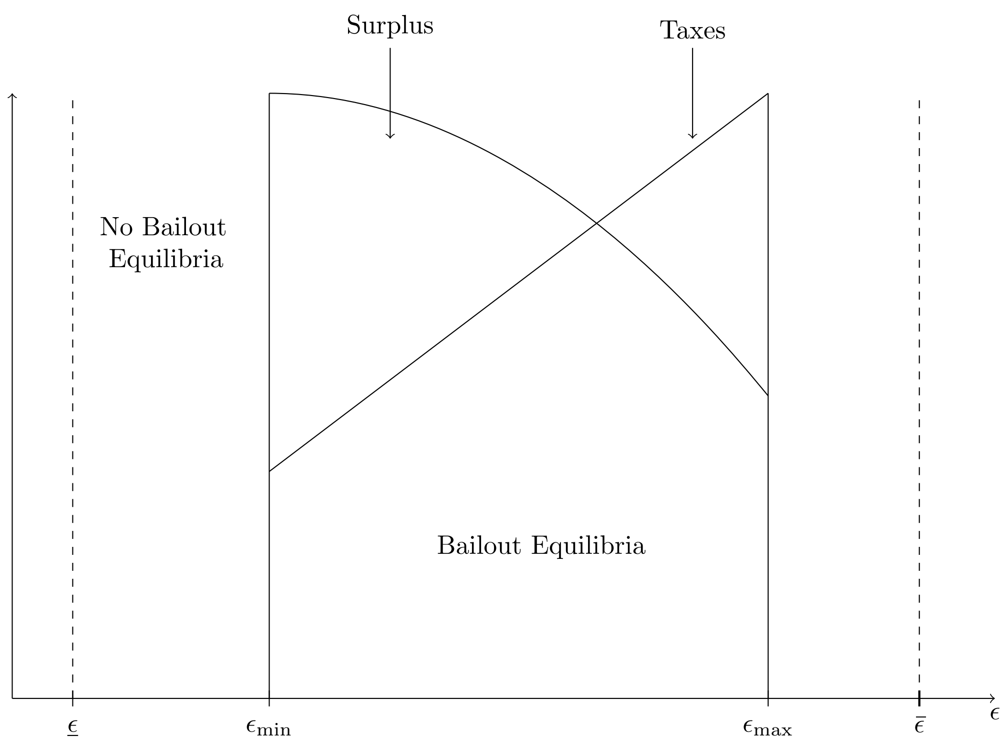


Figure 2: Comparing Equilibrium Outcomes in the Dynamic Model

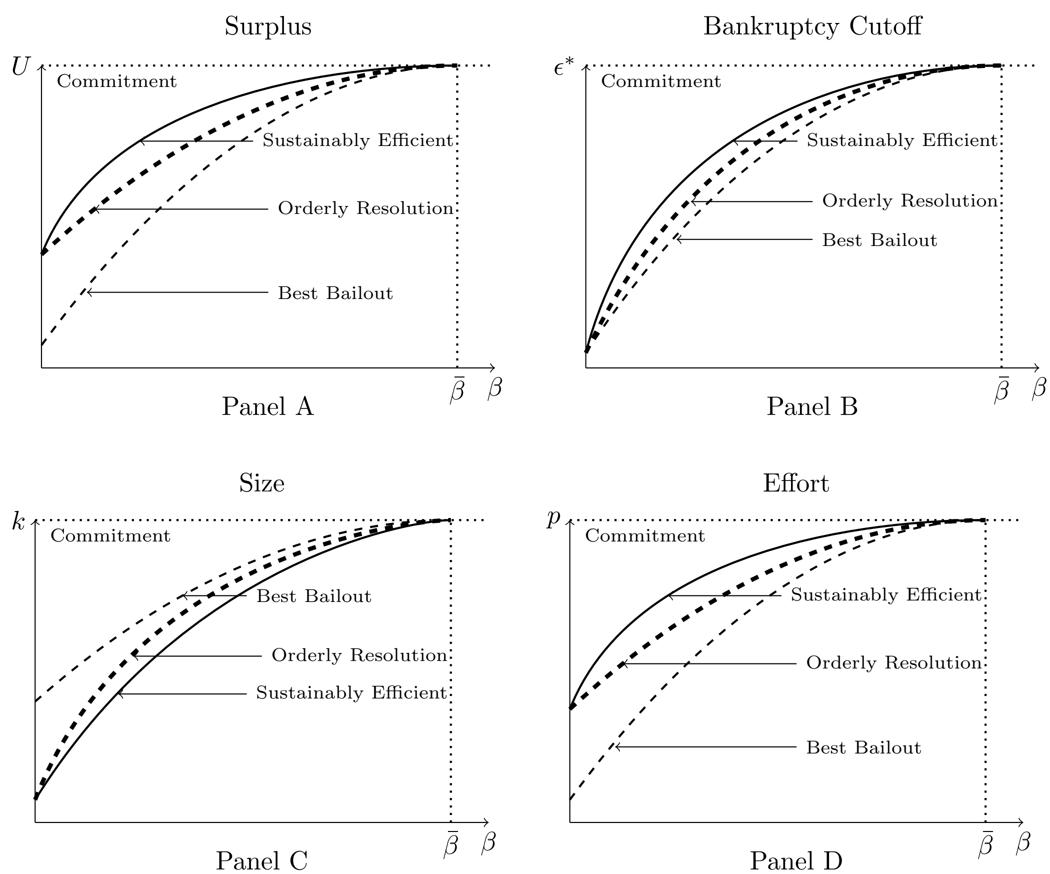
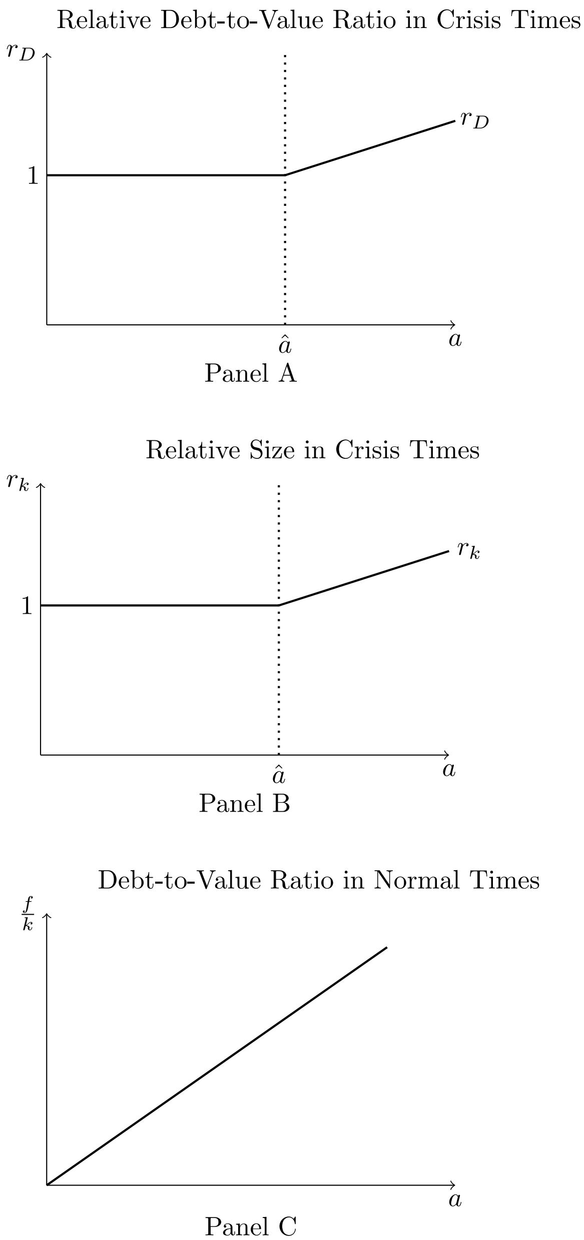


Figure 3: Regulatory Policy Across Industries




Note: Here $r_D$ is the debt-to-value ratio in the best bailout equilibrium divided by the debt-to-value ratio in the sustainably efficient outcome while $r_k$ is the corresponding ratio of sizes. The debt-to-value ratio $f/k = A_L \varepsilon^* g(k)/k$.

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# Online Appendix Not for Publication

# Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View*

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ABSTRACT

This document contains proofs of the propositions and lemmas in the paper

Keywords: Prudential regulation; Financial regulation
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Here we provide the proofs for some of our propositions and lemmas.

**A. Proving Proposition 2**

We begin with a lemma that is helpful in proving Proposition 2.

**Lemma A1**. There exist cutoffs $\varepsilon_s^*(U_s)$ such that the optimal contract has this form: continue if $\varepsilon > \varepsilon_s^*(U_s)$ and declare bankruptcy otherwise.

**Proof.** Suppose by way of contradiction that a contract that is immune to renegotiation has this form: there is a nonbankruptcy region $N = (\varepsilon_1, \varepsilon_2)$ and a bankruptcy region to the right of it, namely, $M = (\varepsilon_2, \varepsilon_3)$, where $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$.

We first develop a simple inequality that will be useful in our argument. Note that since $M$ is part of the bankruptcy region from (8) and $R < 1$, it follows that

(42) $d_s(\varepsilon) < d_s$ for all $\varepsilon \in M$.

Now consider an alternative (continuation) contract (holding fixed $k$ and $p$), denoted by $\{\hat{c}_s(\varepsilon), \hat{d}_s(\varepsilon), \hat{\phi}_s(\varepsilon)\}$. In terms of bankruptcy, this contract is the same as the original allocation except that it turns $M$ from a bankruptcy region to a nonbankruptcy region. In terms of payments to investors, it reduces the payments everywhere except the region $M$ by a constant amount $a$ and raises payments in region $M$ so as to give the investors the same expected payments as in the original contract. Finally, the manager's new consumption is defined residually from the resource constraint. Of course, since this manager is paying the same expected amount to the investor but reaps the benefit $(1 - R)A_s\varepsilon$ for all $\varepsilon \in M$, this manager's expected utility increases.

More formally, let $\hat{\phi}_s(\varepsilon) = 1$ for $\varepsilon \in M$ and coincide with $\phi_s(\varepsilon)$ for all other realizations of the idiosyncratic shocks. Let $\hat{d}_s(\varepsilon) = d_s - a$ for $\varepsilon \in M$, and for other $\varepsilon$, let $\hat{d}_s(\varepsilon) = d_s(\varepsilon) - a$, where the constant $a$ is chosen so that the payment to the investors is the same as in the original contract:

(43) $\bar{d}_s = \int_{N_s} d_s \, dH(\varepsilon) + \int_{B_s} d_s(\varepsilon) \, dH(\varepsilon) =$

$$\int_{N_s} (d_s - a)\ dH(\varepsilon) + \int_M (d_s - a)\ dH(\varepsilon) + \int_{B_s/M} [d_s(\varepsilon) - a]\ dH(\varepsilon).$$

Subtracting the left side from the right side of the second equality in (43) gives $a = \int_M [d_s - d_s(\varepsilon)]\ dH(\varepsilon)$, which we know from (42) is strictly positive. The expected consumption of the managers in the original contract is given by

$$\bar{c}_s = \int_{N_s} [A_s\varepsilon - d_s]\ dH(\varepsilon) + \int_{B_s} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon),$$

and in the alternative contract their expected consumption is given by

$$(44) \quad \int_{N_s} [A_s\varepsilon - d_s + a]\ dH(\varepsilon) + \int_M [A_s\varepsilon - d_s + a]\ dH(\varepsilon) + \int_{B_s/M} [RA_s\varepsilon - d_s(\varepsilon) + a]\ dH(\varepsilon),$$

which we know, from (43), equals $\bar{c}_s + \int_M (1 - R)A_s\varepsilon\ dH(\varepsilon)$.

Under this alternative contract, the consumption of the managers satisfies the nonnegativity constraint. To see this, note that in all states but those in $M$, we have simply added a positive number $a$ to the managers' consumption. To argue that consumption is positive for states in $M$, we note that under our contradiction hypothesis, the set $N$ is to the left of $M$. Since the consumption of the managers in the alternative contract $A_s\varepsilon - d_s + a$ satisfies nonnegativity for any $\varepsilon \in N$, this same expression clearly satisfies nonnegativity for the region $M$, which has larger idiosyncratic shocks.

This alternative contract is clearly incentive compatible. For all states besides those in $M$, we have subtracted off a constant from the repayments of the managers so that the incentive constraints are automatically satisfied. We have switched $M$ to a nonbankruptcy region, and the only incentive constraint that applies in this region is that the repayments are constant, which is satisfied by construction. Thus, we have established a contradiction. *Q.E.D.*

We now characterize the payments in the optimal contract. We let $\varepsilon_s^*$ be shorthand for $\varepsilon_s^*(U_s)$. Any contract that is immune to renegotiation must maximize, say, the payoffs to the manager subject to the constraint that investors receive at least $d_s$. Furthermore, Lemma A1 implies that any contract that is immune to renegotiation must be of the form

$c_s(\varepsilon) = A_s\varepsilon - d_s$ for $\varepsilon \geq \varepsilon_s^*$. Nonnegativity then implies that

(45) $d_s \leq A_s\varepsilon_s^*$ for $\varepsilon > \varepsilon_s^*$.

Incentive compatibility requires that

(46) $c_s(\varepsilon) = RA_s\varepsilon - d_s(\varepsilon) \geq A_s\varepsilon - d_s$ for $\varepsilon \leq \varepsilon_s^*$,

and nonnegativity requires that

(47) $d_s(\varepsilon) \leq RA_s\varepsilon$ for $\varepsilon \leq \varepsilon_s^*$.

Therefore, any contract that is immune to renegotiation must solve

$$\max_{\varepsilon_s^*, d_s(\varepsilon), d_s} \int_{\underline{\varepsilon}}^{\varepsilon_s^*} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s)\ dH(\varepsilon)$$

subject to (45), (46), (47), and

(48) $$\int_{\underline{\varepsilon}}^{\varepsilon_s^*} d_s(\varepsilon) dH(\varepsilon) + d_s[1 - H(\varepsilon_s^*)] \geq \bar{d}_s.$$

The solution to this problem depends on the size of the payments $\bar{d}_s$ owed to investors. If these payments are low enough, then there is no default, and managers pay a constant amount less than $A_s\underline{\varepsilon}$, whereas if these payments are higher, then there is default and payments are as we said. Finally, if $\bar{d}_s$ is too large, then this problem does not have a solution because there is a maximal amount of expected payments $\bar{d}_s$ that can be raised by any contract that satisfies the constraints on this problem.

We now turn to the proof of Proposition 2.

**Proof.** It is immediate that a debt-equity contract is immune to renegotiation. We now show that if a contract is immune to renegotiation, it must be a debt-equity contract. Consider the case that $\bar{d}_s > A_s\underline{\varepsilon}$. Clearly, to generate these payments to investors some bankruptcy is required, so that $\varepsilon_s^* > \underline{\varepsilon}$. We now show that the payments in the nonbankruptcy region $d_s = A_s\varepsilon_s^*$. The argument is by contradiction. Since $d_s \le A_s\varepsilon_s^*$, we need only show that $d_s < A_s\varepsilon_s^*$ leads to a contradiction.

To do so we construct an alternative contract that satisfies (45)–(48) and raises the payoffs to the manager. This alternative contract has a bankruptcy region $[\underline{\varepsilon}, \hat{\varepsilon}]$, where $A_s\hat{\varepsilon} = d_s$, so that $\hat{\varepsilon} < \varepsilon^*$. In this contract, set $\hat{d}_s(\varepsilon) = d_s(\varepsilon) - a$, where $a$ is constructed so that it satisfies (48) with equality. Hence, $a$ satisfies

$$(49)\quad \bar{d}_s = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} d_s(\varepsilon)\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} d_s(\varepsilon)\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} d_s\ dH(\varepsilon) =$$

$$\int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [d_s(\varepsilon) - a]\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} (d_s - a)\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (d_s - a)\ dH(\varepsilon).$$

Hence, $a = \int_{\hat{\varepsilon}}^{\varepsilon^*} [d_s - d_s(\varepsilon)]\ dH(\varepsilon)$, which (46) indicates is strictly positive. This alternative contract also satisfies (45)–(47) because we have simply reduced $d_s$ and $d_s(\varepsilon)$ by $a$.

We now show that in the alternative contract, the expected consumption of managers is higher than in the original contract. The consumption of the managers in the original contract is given by

$$\bar{c}_s = \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} [RA_s\varepsilon - d_s(\varepsilon)]\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s)\ dH(\varepsilon),$$

and in the alternative contract it is given by

$$(50)\quad \int_{\underline{\varepsilon}}^{\hat{\varepsilon}} [RA_s\varepsilon - d_s(\varepsilon) + a]\ dH(\varepsilon) + \int_{\hat{\varepsilon}}^{\varepsilon^*} (A_s\varepsilon - d_s + a)\ dH(\varepsilon) + \int_{\varepsilon^*}^{\bar{\varepsilon}} (A_s\varepsilon - d_s + a)\ dH(\varepsilon),$$

which, using (49), equals $\bar{c}_s + \int_{\hat{\varepsilon}}^{\varepsilon^*} (1 - R)A_s\varepsilon\ dH(\varepsilon)$. Since $R < 1$, the managers' expected payoff is strictly higher. Hence, we have proved the desired result for the case that $\bar{d}_s > A_s\underline{\varepsilon}$.

Next, consider the case that $\bar{d}_s \le A_s\underline{\varepsilon}$. Clearly, it is feasible to have no bankruptcy and repay the investors $\bar{d}_s$. Since bankruptcy simply wastes resources, it is optimal to set $\varepsilon_s^* = \underline{\varepsilon}$, and from (45), $\bar{d}_s \le A_s\underline{\varepsilon}$.

We now show that the managers' and the investors' consumption has the desired form in the bankruptcy region. Suppose, by way of contradiction, that $\int_{\underline{\varepsilon}}^{\varepsilon^*} c_s(\varepsilon)dH(\varepsilon) > 0$. Consider an alternative contract that leaves the bankruptcy set as well as the expected consumption of managers and investors unchanged. This contract reduces the managers' consumption in the bankruptcy set to zero and raises the managers' consumption in the nonbankruptcy interval by an amount that leaves overall expected consumption the same. Since the bankruptcy region is unchanged, this alternative contract gives the same expected payoffs to the investors as the original contract but has the property that $A_s\varepsilon_s^* > \bar{d}_s$. From the first step, however, we know that any such contract is strictly dominated by the optimal contract. This gives us a contradiction. *Q.E.D.*

**B. Proving Lemma 1**

**Proof.** We assume throughout the bankruptcy cutoff is interior. First we substitute out for $c_H$ using the manager's incentive constraint and drop $c_H$ as a choice variable so that the problem becomes

$$(51)\quad \max_{p,k,\varepsilon^*} \left[c_L(\varepsilon^*) + pv'(p) - v(p)\right] g(k)$$

subject to

$$(52)\quad \left[p(A_H - v'(p)) + (1-p)y_L(\varepsilon^*) - c_L(\varepsilon^*)\right] g(k) \geq k,$$

The Lagrangian for this problem is

$$\left[c_L(\varepsilon^*) + pv'(p) - v(p)\right] g(k) +$$

$$(1+\hat{\lambda})\left\{\left[p(A_H - v'(p)) + (1-p)y_L(\varepsilon^*) - c_L(\varepsilon^*)\right] g(k) - k\right\}$$

where $1+\hat{\lambda}$ denotes the multiplier on (52) which can be rewritten as

$$\left[pA_H + (1-p)y_L(\varepsilon^*) - v(p)\right] g(k) +$$

$$\hat{\lambda}\left\{\left[p(A_H - v'(p)) + (1-p)y_L(\varepsilon^*) - c_L(\varepsilon^*)\right] g(k) - k\right\}$$

The first-order condition for $\varepsilon^*$ to this modified problem is

$$-\hat{\lambda}c'_L = -(1+\hat{\lambda})(1-p)y'_L$$

Since $c'_L$ and $y'_L$ are both negative and $1+\hat{\lambda}$ is nonnegative it follows that $\hat{\lambda} > 0$. Thus the contract can be written in the desired form. *Q.E.D.*

### C. Proving Proposition 3

**Proof.** Clearly, the bailout authority's objective function (22) is maximized by setting $\varepsilon_b = \underline{\varepsilon}$ so that no bankruptcies occur. For any cutoff $\varepsilon_R$ of the representative firm, this outcome is achieved by making an offer $d_b = A_L\varepsilon_R$ and setting a new bankruptcy cutoff $\varepsilon_b = \underline{\varepsilon}$. Firms will accept such an offer since investors and managers are made better off by doing so. Thus, in any equilibrium, the outcomes differ from the efficient outcomes and are therefore inefficient.

To show that $\varepsilon_{\min} > \underline{\varepsilon}$, suppose by way of contradiction that $\varepsilon_{\min} = \underline{\varepsilon}$ so that $\tau_b = 0$. Then it is optimal for an individual firm to deviate to the efficient contract $x_{CE}$, which has $\varepsilon_{CE} > \underline{\varepsilon}$. Since, by assumption, the associated debt payments $d(\varepsilon_{CE}) > A_L\underline{\varepsilon}$, the investors will reject the bailout authority's offer and the efficient contract will be implemented, contradicting that $\varepsilon_{\min} = \underline{\varepsilon}$. Since $\varepsilon_{\min} > \underline{\varepsilon}$, taxes are positive in any equilibrium.

In order to show that our economy has a continuum of equilibria, we will show that any $\varepsilon_R \in [\underline{\varepsilon}_R, \varepsilon_{\max}]$ is part of an equilibrium where $\underline{\varepsilon}_R$ is given by $A_L\underline{\varepsilon}_R = d(\varepsilon_{\max})$. Fix an $\varepsilon_R$ in this interval and note that if a firm deviates to a lower bankruptcy cutoff than $\varepsilon_R$, this firm will accept the bailout with payments $d_b = A_L\varepsilon_R$ to investors and bankruptcy cutoff $\underline{\varepsilon}$ and therefore will receive the same payoff as under the representative contract. Thus, no such deviation is profitable. Since $A_L\varepsilon_R \geq d(\varepsilon_{\max})$, it is not feasible for any firm to deviate to a higher bankruptcy cutoff than $\varepsilon_{\max}$. Since no deviations are profitable, any $\varepsilon_R \in [\underline{\varepsilon}_R, \varepsilon_{\max}]$ is part of an equilibrium. *Q.E.D.*

### D. Proving Proposition 4

**Proof.** Suppose first that the outcomes $(x_t, p_t, \pi_t)$ are the outcomes of a bailout equilibrium. Since the contracting problem is static, these outcomes must solve the one-period contracting problem. Clearly, in any equilibrium the government budget constraint is satisfied. Next, we show that under (32), they must satisfy the sustainability constraint. To see why, suppose by way of contradiction that in equilibrium these outcomes violate (33). Then the authority, by setting the bankruptcy set to be empty in the current period, obtains current payoffs equal to the first term on the right side of (33), and under (32), its continuation payoff is at least as large as the last term. Thus, outcomes that violate (33) contradict optimality by the bailout authority.

Suppose, next, that a set of candidate equilibrium outcomes $(\hat{x}_t, \hat{p}_t, \hat{\pi}_t)$ with associated histories $\hat{H}_t$ and $\hat{H}_{Bt}$ satisfy $(i), (ii)$, and $(iii)$ of Proposition 5. We will construct revert-to-static strategies that support these outcomes as an equilibrium. For private agents, these strategies specify that if the history $H_t = \hat{H}_t$, then the contract $x_t$ equals the desired one $\hat{x}_t$; otherwise, the contract $x_t$ equals the full bailout contract $x_b$. For the bailout authority, these strategies specify that if $H_{Bt} = \hat{H}_{Bt}$, then the policies equal the desired ones $\hat{\pi}_t$; otherwise, they equal the full bailout policy of purchasing all the debt in the distressed state and eliminating all the bankruptcies.

Now consider the bailout authority. If there has been no deviation from these specified outcomes in or before period $t$, in that $H_{Bt} = \hat{H}_{Bt}$, then the payoffs associated with choosing the desired policy $\hat{\pi}_t$ are given by the left side of (33). The payoffs associated with any deviation are smaller than the right side of (33) because the first term on the right side represents the best one-shot deviation. The inequality in (33) guarantees that the desired policies are indeed optimal. If there has been a deviation in or before $t$, so that $H_{Bt} \neq \hat{H}_{Bt}$, then the continuation payoffs of the bailout authority are independent of the current policy. Hence, the bailout authority's optimal choice is the statically optimal full bailout policy.

Clearly, the private agent's strategies are optimal by construction. *Q.E.D.*

### E. Proving Proposition 5

**Proof.** We have already shown the first part of the proposition. To prove the second part, suppose by way of contradiction that $\beta < \bar{\beta}$ but no bailouts occur in a crisis time so that $\delta_t = 0$. By definition of $\bar{\beta}$, the surplus in any crisis time is strictly less than the surplus under commitment. If the strategies specify no bailouts so that $\delta_t(H_{Bt}) = 0$, the private contracts coincide with the commitment outcomes. Since $\beta < \bar{\beta}$, the bailout authority is better off by deviating by undertaking a bailout and eliminating all bankruptcies which yields a contradiction. Thus, in any equilibrium, $\delta_t$ must equal one in crisis times. The argument that $\tau_t > 0$ is then identical to that in Proposition 3. *Q.E.D.*

**F. Proving Proposition 7**

**Proof**. We will show that the best orderly resolution outcome has zero taxes. As we have argued this result immediately implies that the best orderly resolution outcome yields strictly higher surplus.

We begin by simplifying the orderly resolution problem and establish how allocations change as policies change. We then use these properties to show that if taxes are positive then we can construct a deviation that improves welfare.

The first step in simplifying the problem is to show that the orderly resolution problem is equivalent to one of choosing $(p, \varepsilon_O, k; d_O, \tau)$ subject to the first-order conditions for the contracting problem (26), the implementability constraint with taxes (25), the government budget constraint, the sustainability constraint, and

$$(53) \quad d(\varepsilon_O) \leq d_O \leq d(\varepsilon_{\max})$$

Here $\varepsilon_O$ is the implemented bankruptcy cutoff and $d_O$ is the payments received by investors. In this formulation we have dropped $\varepsilon_R$ and effectively combined $\varepsilon_{OR}$ and $\varepsilon_O$, we have also dropped $\delta$. To see this equivalence note that given these outcomes we can recover $\delta, \varepsilon_R, \varepsilon_{OR}$ and $\varepsilon_b$ as follows. If $d(\varepsilon_O) < d_O$ set $\delta = 1$ and $\varepsilon_b = \varepsilon_O$ and set $\varepsilon_R = \varepsilon_{OR} = d_O/A_L$. If $d(\varepsilon_O) = d_O$ set $\delta = 0$, $\varepsilon_b$ is irrelevant and set $\varepsilon_R = \varepsilon_{OR} = \varepsilon_O$. Note also that (26) and (25) summarizes (24).

The second step is to substitute out for taxes $\tau$ from the government budget constraint into (26) and note this constraint implicitly defines an effort function $p = p(\varepsilon_O, d_O)$. Substituting out for taxes reduces (25) to the implementability constraint without taxes (18). The resulting *simplified* problem, which we use in Lemma A2 is to choose $(\varepsilon_O, k, d_O)$ to maximize surplus subject to the implementability constraint (18), the sustainability constraint, and (53) where $p = p(\varepsilon_O, d_O)$.

Suppose by way of contradiction that in the solution to this simplified problem $d_O > d(\varepsilon_O)$ so that taxes are positive. We will show we can reduce $d_O$ and reduce $k$, holding fixed $\varepsilon_O$ and future allocations, satisfy the implementability constraint, and introduce slack in the sustainability constraint so that the conjectured solution is not optimal.

For our variation, let $\Delta d_O$ denote an infinitesimal change in $d_O$. Differentiating the implementability constraint (18) gives that the induced change in $k$, namely $\Delta k$ satisfies

$$(54)\quad f_p p_d \Delta d_O = (1-\alpha)k^{-\alpha}\Delta k.$$

Consider the sustainability constraint (34) and note the left side of that constraint, namely the resources saved by cancelling bankruptcies are proportional to $\ell = (1-p)k^{\alpha}$. Thus, the induced change in $\ell$ from this variation is

$$(55)\quad \Delta \ell = -p_d k^{\alpha}\Delta d_O + \alpha(1-p)k^{\alpha-1}\Delta k.$$

In Lemma A2 we show that the partial derivative of the effort function $p_d$ is negative and suppose, as we show below, that $f_p < 0$. Using these results, from (54) it follows that $\Delta k$ has the same sign as $\Delta d_O$ and from (55) it then follows that $\Delta \ell$ has the same sign as $\Delta d_O$. Thus, reducing $d_O$ introduces slack into the sustainability constraint.

To show that $f_p$ is negative, note that the first-order condition with respect to $d_O$ in the simplified problem is

$$U_p + \lambda f_p - \mu V_p = 0,$$

where $\lambda$ and $\mu$ are the multipliers on the implementability and the sustainability constraints, which are both positive. Here $V_p$ is the derivative of the left-hand side of (34). Since effort is below the full information level, $U_p$, is positive. Since higher effort reduces resources lost to bankruptcy, $V_p$ is negative. It follows that $f_p$ is negative. *Q.E.D.*

We turn now to our lemma.

*Lemma A2.* Under (37), the partial derivative $p_d(\varepsilon_O, d_O)$ is negative.

*Proof.* The effort function $p(\varepsilon_O, d_O)$ is implicitly defined by the equation

(56) $\alpha M(p, \varepsilon_O) + (1 - \alpha)\, N(p, \varepsilon_O, d_O) = 1.$

where $M(p, \varepsilon_O) = U(p, \varepsilon_O)/f(p, \varepsilon_O)$ and $N(p, \varepsilon_O, d_O) = \tilde{U}_p(p, \varepsilon_O, d_O)/\tilde{f}_p(p, \varepsilon_O, d_O)$. To obtain this equation use the form of $g$ to write (25) as $\left[\tilde{f}(p, \varepsilon_O, d_O) - \tau\right] = k^{1-\alpha}$ so that (26) can be written as

$$\frac{\left[\tilde{U}(p, \varepsilon_O, d_O) - \tau\right]}{\tilde{f}(p, \varepsilon_O, d_O) - \tau}\alpha - 1 = -\left(1 - \alpha\right) \frac{\tilde{U}_p(p, \varepsilon_O, d_O)}{\tilde{f}_p(p, \varepsilon_O, d_O)}.$$

Equation (56) then follows by substituting for $\tau$ from the government budget constraint.

To show that $p_d$ is negative take the total derivative of (56) to get

(57) $[\alpha M_p + (1 - \alpha)N_p]\, p_d = -(1 - \alpha)N_d.$

Next, we will show that $M_p$, $N_p$, and $N_d$ are all positive, so that $p_d < 0$. To show that $M_p$ is positive, we rewrite $M$ as

$$M(p, \varepsilon_O) = 1 + \frac{pv'(p)}{f(p, \varepsilon_O)}$$

so that $M_p$ has the same sign as

$$f\,[v'(p) + pv''(p)] - pv'(p) f_p.$$

which under (37) has the same sign as

$$[f\,(1 + a) - pf_p]\, v'(p).$$

which is positive because $f - pf_p = y_L(\varepsilon_O) - c_L(\varepsilon_O) + p^2 v''(p) > 0$ since $y_L(\varepsilon_O) > c_L(\varepsilon_O)$. To see that $N_p$ and $N_d$ are positive, note that

$$N(p, \varepsilon_O, d_O) = 1 + \frac{pv''(p)}{\tilde{f}_p(p, \varepsilon_O, d_O)}.$$

where $\tilde{f}_p = A_H - c_L - v'(p) - pv''(p) - d_O$. Next, note that since $\tilde{f}_p$ is clearly decreasing in $d_O$, it follows that $N_d$ is positive. Since $a \geq 1$, $pv''(p)$ is increasing in $p$ and since $\tilde{f}_p$ is decreasing in $p$, it follows that $N_p$ is positive. It thus follows from (57) that $p_d < 0$. *Q.E.D.*

**G. Proving Proposition 8**

**Proof**. We first show a preliminary result that we use in proving the main result: if $0 < \beta < \bar{\beta}$, the sustainably efficient outcome has bankruptcies. To see this result, suppose by way of contradiction that the sustainably efficient outcome has no bankruptcies. Then there is no static gain to canceling bankruptcies, so the first terms on the left and right sides of (33) are the same. The continuation payoffs are strictly greater than the full bailout continuation payoffs, however, so that the sustainability constraint (33) holds as a strict inequality. This is a contradiction since in any sustainably efficient outcome below commitment, the sustainability constraint binds.

We now show that the orderly resolution outcome is sustainably inefficient. Consider the outcomes of an orderly resolution equilibrium denoted $(p_O, \varepsilon_O, k_O)$ in some particular period $t$. Consider the alternative allocations that alter period $t$ outcomes but let future outcomes coincide with those of the given orderly resolution equilibrium. These alternative allocations at time $t$ maximize surplus subject to the implementability constraint and the sustainability constraint except that here the continuation surplus $V$ in the sustainability constraint is the surplus associated with the given orderly resolution equilibrium. Since the original outcomes satisfy the sustainability constraint, it is clear that $(p_O, \varepsilon_O, k_O)$ is feasible for the alternative maximization problem. We have dropped (38) at time $t$, so it is clear that surplus in the alternative allocation is weakly higher than in the orderly resolution equilibrium. Clearly, since the sustainability constraint binds and the sustainably efficient outcome has bankruptcies, the first-order conditions for this alternative allocation with respect to $\varepsilon^*$ and $k$ will not satisfy the first-order conditions with respect $\varepsilon^*$ and $k$ in an orderly resolution equilibrium. Since the sustainably efficient outcome yields even higher welfare than the alternative allocations, it follows that the orderly resolution outcome is sustainably inefficient. *Q.E.D.*

**H. Proving Proposition 9**

**Proof.** The basic idea of the proof is to show that in the solution to the regulatory problem (39), the inequalities in (41) hold with equality. We do so by using the solution to the sustainable efficiency problem to construct the multipliers for the regulatory problem and show that they are positive. To do so, we consider the first-order conditions to the this problem:

$$U_p + \lambda f_p + \mu L_p = 0 \tag{58}$$

$$U_\varepsilon + \lambda f_\varepsilon + \mu L_\varepsilon = 0 \tag{59}$$

$$U g'(k) - 1 + \lambda \left[ f g'(k) - 1 \right] + \mu L g'(k) = 0, \tag{60}$$

where $L(p, \varepsilon^*, k)$ is defined as $\left[ U(p, \varepsilon^*, S_C) - U(p, \underline{\varepsilon}, S_C) \right] g(k) + \beta V$ so that the sustainability constraint can be written as

$$L(p, \varepsilon^*, k) \geq \beta V_{FB},$$

where $\lambda$ and $\mu$ are the multipliers on the implementability and sustainability constraints. The best regulatory outcomes can be implemented if the first-order conditions to (39) evaluated at the best regulatory outcomes satisfy

$$U_p + \hat{\lambda} f_p = 0 \tag{61}$$

$$U_\varepsilon + \hat{\lambda} f_\varepsilon - \hat{\eta} d'(\varepsilon_{RE}) = 0 \tag{62}$$

$$U g'(k_{RE}) - 1 + \hat{\lambda} \left[ f g'(k_{RE}) - 1 \right] - \hat{\gamma} = 0 \tag{63}$$

for some positive multipliers $\hat{\lambda}$, $\hat{\eta}$, and $\hat{\gamma}$ on (40) and the constraints in (41). Note that (61) implies that $\hat{\lambda} > 0$ since $U_p > 0$ and $f_p < 0$. Equating the first-order conditions in the two problems yields

(64) $\hat{\lambda} = \lambda + \mu L_p / f_p$

(65) $\hat{\eta} = \left[ (\hat{\lambda} - \lambda) f_\varepsilon - \mu L_\varepsilon \right] / d'(\varepsilon_{RE})$

(66) $\hat{\gamma} = (\lambda - \hat{\lambda}) \left[ 1 - \alpha \right] - \mu L g'(k),$

where we have used that when the constraint (40) holds with equality and $g(k) = k^\alpha$, then $fg' = \alpha$.

Next we show that, under our sufficient conditions, the constructed multipliers $\hat{\eta}$, and $\hat{\gamma}$ in (65) and (66) are positive. First note from the definition of $L$ that if $\mu_C$ is sufficiently small, then $L_p > 0$, $L_\varepsilon < 0$, and $L_k < 0$. Since $f_p < 0$ and $L_p > 0$, from (64) it follows that $\hat{\lambda} < \lambda$. To do so, we note that $\hat{\lambda} < \lambda$. Since $L_k < 0$, it then follows from (66) that $\hat{\gamma} > 0$.

To show that $\hat{\eta} > 0$, we note that since $d' > 0$ from (65), it suffices to show that $(\hat{\lambda} - \lambda) f_\varepsilon + \mu L_\varepsilon > 0$. Substituting for $\hat{\lambda}$ from (61) and solving for $\lambda$ and $\mu$ from (59) and (58) gives that

$$\eta d' = -\frac{U_p}{f_p} f_\varepsilon + U_\varepsilon.$$

Substituting for $U_p, f_p, U_\varepsilon$, and $f_\varepsilon$ and simplifying gives that $\eta$ is positive if

$$\left( A_H - y_L(\varepsilon^*) - v'(p) \right) \left( 1 - H(\varepsilon^*) \right) - (1 - p) v''(p)(1 - R)\varepsilon^* h(\varepsilon^*) > 0.$$

Since $\varepsilon^* \leq \varepsilon_{\max}$, we know that $H(\varepsilon^*)$ is uniformly bounded away from 1 as $A_H$ is increased. All the other terms are also uniformly bounded. Thus, for $A_H$ sufficiently large, this inequality holds. *Q.E.D.*