SECTION A (60 Marks)

Answer ALL questions in this section, showing all necessary steps and answers.

1. (a) Use the appropriate laws to simplify $(A \cap (A \cup B)')'$.

(02 marks)

(b) A certain farmer who produces 3 types of food crops: maize, beans and millet conducted a survey of 205 families. The following were the findings: 82 use maize, 110 use beans, 73 use millet, 59 use beans and millet, 32 use maize and millet and 20 use all the products. How many of the families interviewed use none of these products.

(04 marks)

2. (a) Show that C(7, −2) and D(1, 6) are all equidistant from the line $3x - 4y - 4 = 0$.

(03 marks)

(b) Prove that $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the mid point of the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

(03 marks)

3. (a) Given that $f(x) = 10x$, $g(x) = x + 3$ find $fg(x)$ and $(fg)^{-1}(x)$. Verify that if $b = fg(a)$ then $(fg)^{-1}(b) = a$.

(03 marks)

(b) Find the set of values of p for which $f(x) = x^2 + 3px + p$ is greater than zero for all real values of x.

(03 marks)

4. (a) Show that the term of $\sum_{r=1}^{n} \ln 2^r$ are in arithmetic progression. Fnd the sum of the first n terms.

(02 marks)

(b) The third term of a convergent geometric progression is the arithmetic mean of the first and second terms. Find the common ratio and the sum to infinity if the first term is 1.

(04 marks)

5. (a) Prove that $\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$.

(02 marks)

(b) Find the general solution of the equation $3\sin\theta + 4\cos\theta = -2.5$ for $0° < \theta < 360°$.

(02 marks)

(c) Parametric equations of a curve are $x = \cos\theta$ and $y = \sin\theta$. Find the equation of the tangent to the curve at the point $(\cos\theta, \sin\theta)$ on the curve.

(02 marks)

6. (a) Find the derivative of $y = x^{\sin x}$.

(02 marks)

(b) Show that every function f(x) defined by $f(x) = Ae^{4x} + Be^{-2x}$ where A and B are arbitrary constants, is a solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{2dy}{dx} - 8y = 0.$$

(04 marks)

7. (a) Find a vector equation of the line passing through points A(3, –2) and B(–1, 4).
(02 marks)

(b) Find the vector and cartesian equation of the plane through the points A(2, 0, –2), B(–1, 1, 3) and C(2, 1, –1).

(02 marks)

8. (a) Integrate with respect to x.

(i) $\int \frac{4x^2 - 2x + 3}{(x^2 + 1)(x - 2)} dx.$

(ii) $\int \sqrt{(x^2 + 2x - 1)} \; dx.$

(04 marks)

(b) Evaluate

$$\int_0^1 \frac{dx}{(1 + x^2)^2}$$

(02 marks)

9. (a) From the probability distribution below find

(i) E(x). (ii) $E(x^2)$. (iii) $E(x - \bar{x})^2$.

x	8	12	16	20	24
P(x)	$^1/_8$	$^1/_6$	$^3/_8$	$^1/_4$	$^1/_{12}$

(03 marks)

(b) In a certain mathematics examination, one student estimates that his chances of getting A is 10%, B is 40%, C is 35%, D is 10%, E is 4% and S is 1%. By obtaining an A he gets 5 points, B he gets 4 points, C he gets 3 points, D he gets 2 points, E he gets 1 point and S he gets ½ point. Find his expectation.

(03 marks)

10. A zoologist weighs 200 eggs and records the weights in the following grouped frequency table.

Weight (g)	24 – 29	30 – 35	36 – 41	42 – 47	48 – 54
Number of eggs	22	45	72	43	18

Find the mean and standard deviation correct to 2 decimal places.

(02 marks)

SECTION B (40 Marks)

Answer any FOUR (4) questions from this section showing all necessary steps and answers.

11. A farmer has two godowns A and B for storing his groundnuts. He stored 80 bags in A and 70 bags in B. Two customers C and D place orders for 35 and 60 bags respectively. The transport costs per bag from each godown to each of the two customers are as tabulated below.

Godown	Transport costs	
	Customer C	Customer D
A	8/= x	12/= y
B	10/=	13/=

$x = \sin t$

$y = \sin t$

How many bags of the groundnuts should the farmer deliver to each customer from each godown in order to minimize the total transport costs? (10 marks)

1 6

1 1 1

1 2 1 2

1 3 3 1 3

1 4 6 4 1 4

12. (a) Find the roots of the equation $Z^3 - 8i = 0$. (03 marks)

(b) Using binomial theorem, expand $(\cos\beta + i\sin\beta)^4$. With the help of the Demoivres theorem, show that

$\cos t$

$\cos t$

$$\tan 4\beta = \frac{4\tan\beta - 4\tan^3\beta}{1 - 6\tan^2\beta + \tan^4\beta}.$$

$$\frac{+2 \pm \sqrt{4-(4\times1\times-1)}}{2} \qquad 2t$$

(05 marks)

(c) Find the equation in terms of x and y of the locus represented by $|Z - 1| = |Z - i|$. (02 marks)

√13. (a) Show that the equation $x^2 - 2x - 1 = 0$ has a root lying between $x = 2$ and $x = 3$. Apply the method of bisection in four iterations to obtain an approximate root. (05 marks)

(b) Use Simpson's rule with 7 ordinates to find an approximate value of

$$\int_0^6 x\, e^{-x}\, dx$$

(05 marks)

√14. (a) If $y = (A + Bx)e^{-2x}$, prove that

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 0.$$

$x + y \geq 35$

$x + y \geq 71$

x

(03 marks)

(b) Solve the following differential equations.

(i) $\dfrac{dy}{dx} = \dfrac{y + \sqrt{x^2 + y^2}}{x}$ if $y(1) = 0$.

$$\frac{dy}{dx} = \frac{y + x\sqrt{1+u^2}}{x}$$

$$\frac{dy}{dx} = u + \sqrt{1+u}$$

$x + y \geq 35$

$x + y \geq 71$

$x + y \geq 65$

$x + y \geq 4$

(ii) $\frac{xdy}{dx} - xy = y \quad (x > 0,\ y > 0)$ given that $y = 1$ when $x = 1$.

(07 marks)

15. (a) Write down four conditions required for an experiment to be a binomial experiment.

(02 marks)

(b) Let p be the probability of an event to be successful and q be the propability of an event to be unsuccessful. If n independent trials are performed in a binomial experiment

(i) show that the mean value of the random variable x which represents the number of successes in the binomial experiment is np.

(ii) find the mean and standard deviation for the distribution of defective dry cells in a total of 400, given that the probability of a defective dry cell is 0.1

(08 marks)

16. 15. A projectile, fired with speed v and at an elevation θ from point A on the ground, reaches the ground at a horizontal distance d metres from A.

(a) Prove that $v = \sqrt{gd \operatorname{cosec} 2\theta}$ and that the greatest height attained by the projectile is $\frac{1}{4} d \tan\theta$.

(b) If the projectile just clears an obstacle of height h metres at a horizontal distance a metres from A, show that $\tan\theta = \frac{hd}{a(d-a)}$.

(10 marks)

$\binom{n-1}{k-1}$

$y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$