THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours **Friday 03 May 2002 a.m.**

Instructions

1. This paper consists of sections A and B.
2. Answer **ALL** questions in section A and any **FOUR** (4) questions from section B.
3. All answers must be written in the answer booklet provided.
4. All work done in answering each question must be clearly shown.
5. Mathematical tables, mathematical formulae, slide rules and unprogrammable scientific pocket calculators may be used.
6. Cellular phones are **not** allowed in the examination room.
7. Write your Examination Number on every page of your answer booklet.

SECTION A (60 Marks)

Answer ALL questions in this section, showing all necessary steps and answers.

1. (a) Simplify giving reasons the statement $(A-B)-(A \cup B^{C})$ (2 marks)

   (b) One poultry farm in Dar es salaam which produces three types of chicks, had its six-month report which revealed that out of 126 of its regular customers, 65 bought broilers, 80 bought layers and 75 bought cocks. 45 bought layers and cocks, 35 bought broilers and cocks, 10 bought broilers only, 15 bought layers only and 6 bought cocks only. 6 of the customers did not show up.

   (i) How many customers bought all three products? (2 marks)

   (ii) How many customers bought exactly two of the farm's products? (2 marks)

2. (a) Find the equations of the lines through the point (2,3) which make an angle 45° with the line $2x - y + 3 = 0$. (3 marks)

   (b) Show that the radical axis of two circles $x^2 + y^2 + 2gx + 2fy + c = 0$ and $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ is perpendicular to the line joining their centres. (3 marks)

3. (a) Given that $f(x) = 4x - 1$ and $g(x) = \frac{1}{3X-2}$ verify that $(f_o g)^{-1} = g_o^{-1} f^{-1}$ (3 marks)

   (b) Find the relation connecting the constants a, b and c if one root of the equation $ax^2 + bx + c = 0$ is three times the other. (3 marks)

4. (a) The sum of the first n terms of a series is $(2^n - 1)$. Find the general term of the series. (3 marks)

   (b) Prove using Mathematical Induction that $6^n + 8^n$ is divisible by 7 for all positive odd numbers n. (3 marks)

5. (a) Write $\sin(\sin^{-1} x + \cos^{-1} x)$ in its most simplified form. (2 marks)

   (b) Use the definition $t = \tan \frac{\theta}{2}$ to solve $2\cos\theta + 3\sin\theta - 2 = 0$ for θ between 0° and 360° inclusive. (4 marks)

6. (a) If $R = ar^n$ where a is a constant and an error of x % is made in measuring r, prove that the resulting error in R is nx %. (3 marks)

   (b) A curve is defined by parametric equations $x = \frac{t}{1+t^2}$ and $y = \frac{1}{(t-1)}$. Find $\frac{d^2y}{dx^2}$. (3 marks)

7. (a) Find the cartesian equation of the plane containing the vector $r_o = i - 2j + k$ which is perpendicular to $c = 2i - j + 3k$. (2 marks)

(b) Find the cartesian equation of the line ℓ which passes through the point (2, −1, −3) and parallel to the vector perpendicular to the vectors $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{k}$. (4 marks)

8. (a) Evaluate $\int \frac{dx}{3 + 5\cos 3x}$ (3 marks)

(b) Find the length of the spiral $r = ae^{k\theta}$ from $\theta = 0$ to $\theta = 2\pi$. (3 marks)

9. (a) A function $f(x) = cx^2$ defines a continuous probability function for a random variable x from $x = 0$ to $x = 3$. Find the value of c. (2 marks)

(b) Calculate the mean and variance of the distribution in (a) above. (4 marks)

10. The marks of 200 form six students in the Physics terminal examination at a certain school were recorded as follows:-

Marks	11 - 20	21 - 30	31 - 40	41 - 50	51 - 60	61 - 70	71 - 80	81 - 90
Frequency	8	20	32	48	44	24	15	9

(a) Construct the histogram representing the data. (2 marks)

(b) Calculate (i) Mode (2 marks)
(ii) Median. (2 marks)

SECTION B (40 Marks)

Answer any **FOUR (4)** questions from this section, showing all necessary steps and answers.

11. A small furniture company has two workshops which produce timber used in the manufacture of tables and chairs. In one day operation, workshop A can produce timber required to manufacture 20 tables and 60 chairs and workshop B can produce the timber required to manufacture 25 tables and 50 chairs. The company needs enough timber to manufacture at least 200 tables and 500 chairs.

If it costs 100,000/= to operate workshop A for one day and 90,000/= to operate workshop B for one day, how many days should each workshop be operated in order to produce a sufficient amount of timber at a minimum cost? What is the minimum cost? (10 marks)

12. (a) Locate the conjugates of the following complex numbers in the same complex plane.

(i) $(1 + i)(2 + 3i)$ (ii) $(-2 + 2i) + (5 - i)$ (3 marks)

(b) If $\frac{2z + i}{z - i}$ is an imaginary part of a complex number z, what is the nature of the locus of the complex number? (4 marks)

(c) Solve $z^2 - 1 - \sqrt{3}\,i = 0$ giving the values of z in the form of $a + bi$ where a and b are real numbers. (3 marks)

13. (a) If $\log(1.96) = 0.2923$, $\log(1.97) = 0.2945$, use linear interpolation to find the number x for which $\log x = 0.2936$. (3 marks)

(b) Find the value of $\int_0^{0.8} e^{x^2}\, dx$ with 5 ordinates using :

(i) The Trapezium rule

(ii) Simpson's rule. (7 marks)

14. (a) Form a differential equation whose solution is $y^2 = ax + b$. ($2^1/_2$ marks)

(b) Solve the following differential equations:

(i) $(2x - y) \frac{dy}{dx} = 2x - y + 2$ given that $y = 1$ when $x = 2$. (4 marks)

(ii) $\frac{d^2x}{dt^2} + 9x - 36 = 0$ given that $x = 6$, $\frac{dx}{dt} = 9$ when $t = 0$ ($3^1/_2$ marks)

15. (a) 7 seeds, each with a probability of germinating of 0.2 are planted in each of the 80 pots. How many of these pots may be expected to have 2 or less seedlings? (5 marks)

(b) A continuous random variable x has a probability density function f(x) where

$$f(x) = \begin{cases} kx \; ; \; 0 \leq x \leq 2 \\ k(4 - x) \; ; \; 2 \leq x \leq 4 \\ 0 \text{ otherwise.} \end{cases}$$

(i) Find the value of the constant k. (1 mark)

(ii) Sketch $y = f(x)$ (2 marks)

(iii) Find $p(^1/_2 \leq x \leq 2^1/_2)$ (2 marks)

16. The position vector r of a particle of mass 5 kg moving in space at any time (t) seconds is given by

$$r(t) = \left(2t^2 - 7t + \frac{65}{8}\right) i + 4j + 3k.$$

(a) Verify that the acceleration of the particle is a constant. (3 marks)

(b) Calculate:

(i) the time and distance of the particle from the origin when it is temporarily at rest. (4 marks)

(ii) the momentum and force on the particle at $t = 5$ sec. (3 marks)