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Background: Eberhard and the 'Philosophisches Magazin'

It would be no exaggeration to claim that, by the end of the penultimate decade of the eighteenth century, Kant's Critical philosophy, and his *Critique of Pure Reason* (henceforth *CPR*)[1] in particular, had brought about a revolution in German intellectual life. Inevitably, such a change was bound to be resisted, and the resistance from the dominant Wolffian school of metaphysics in the Leibnizian tradition was chiefly led by Johann August Eberhard (1739–1809). Eberhard took it upon himself, after the publication of the second edition of Kant's *CPR* in 1787, to organize a response to the spread of the new Critical philosophy. In 1788, he launched a new philosophical journal, the *Philosophisches Magazin* (*PM*), to which several Wolffians contributed.[2] This journal was primarily designed to publish papers criticizing Kant's Critical philosophy from a Wolffian angle, and specifically aimed at opposing the views published in the *Allgemeine Literatur-Zeitung* (*ALZ*). The latter was a journal started in 1785 by C. G. Schütz, which was committed to the propagation and defence of the new Critical philosophy. Among its contributors were some of Kant's most prominent followers, especially Johann Friedrich Schultz (1739–1805)[3] and Karl Leonhard Reinhold (1757–1823),[4] who were keen to spare him the need to get involved in polemics, and thus allow him to concentrate fully upon the completion of the Critical system.[5] They therefore took up the gauntlet, and assumed the responsibility of coordinating a Kantian response to the criticisms that were aimed at his philosophical system from various directions.[6]

Over a short period of time, a number of papers were published in *ALZ* that, even if not written by Kant himself, expressed Kantian rejoinders to the criticisms published by the faction around Eberhard. Only once did Kant himself put pen to paper to respond to articles by Eberhard that were published in the first volume of *PM* (1788–9). Kant deemed that the author was not only fundamentally mistaken about the meaning of his writings, but also had been particularly dishonest in dealing with them. Kant's response was published as a separate work titled *On a Discovery According to which Any New Critique of Pure Reason Has Been Made Superfluous by an Earlier One* (1790) (AA 8: 185–251). The main aim of this work was to counter Eberhard's attacks on two fronts, namely on the issue of the limits of knowledge, with particular emphasis on the problem of synthetic *a priori* judgements, and on the very distinction these limits rest upon, namely that between analytic and synthetic judgements.[7]

For Kant's responses to other attacks from the Eberhard camp, we have to rely upon *ALZ* and Kant's correspondence with Schultz and Reinhold in particular, which indicates to what extent Kant's voice is speaking through them. One case in which the notes that Kant included as attachment in his reply to Schultz (2 August 1790; AA 11: 184) are, at Kant's suggestion, reproduced practically verbatim by Schultz in the *ALZ* less than two months later,[8] is in his discussion of three essays by the mathematician Abraham Gotthelf Kästner (1719–1800), professor of mathematics and physics in Göttingen, director of the old Göttingen observatory, and also a noted epigrammist (see Kästner 1797). These were published in the second volume of *PM* (1790) and bear the titles: 'What is the Meaning of "Possible" in Euclid's Geometry?' (Kästner 1790a), 'On the Mathematical Concept of Space' (Kästner 1790b) and 'On Geometric Axioms' (Kästner 1790c).

On Kästner

As Allison (1973: 13) notes, Kant treats Kästner with a great deal more respect than he does Eberhard. This is partly a strategic decision no doubt: Kästner was a very well-respected figure in mathematics, chiefly known for his textbooks which were widely used in German universities. And although Kästner's papers discuss issues in the philosophy of mathematics from a broadly Leibnizian perspective, it was shrewder to avoid an open attack and rather attempt to show that there were no grounds for dispute between them. Unlike Eberhard's very polemical writings, which questioned just about all the key features of the Critical project, Kästner's papers are aimed at a particular issue, and his style is dispassionate.

Kästner did not make a name for himself through original research in mathematics (cf. Sinaceur 1974). His main contribution was in the writing of textbooks (he published *Anfangsgründe der Mathematik* in 4 volumes, 1758–61) as well as about the applications and the history of mathematics. His interest in geometry is, however, noteworthy, and of particular interest here. Specifically, Kästner proposed an alternative axiomatization of Euclid's geometry. In so doing, he attempted to find grounds for the selection of the parallel postulate (the fifth postulate) in Euclidean geometry. However, he did not follow those (including Kant's contemporary J. H. Lambert, 1728–77[9]) who unsuccessfully tried to derive it from the other postulates. His interest in this postulate is historically relevant insofar as among Kästner's pupils we find Bolyai's father and Lobachevsky's teacher. We can thus see a direct line of influence connecting Kästner to these two great originators of the first non-Euclidean geometry to be published, namely hyperbolic geometry.

Although Kästner's philosophical sympathies were Leibnizian, Kant would have had no grounds for considering him an enemy in the same vein as Eberhard. Aside from their having mutual friends, such as the physicist and aphorist G. C. Lichtenberg (1742–99), who was Kästner's Ph.D. student, and his long-standing admiration for Kästner,[10] Kant also corresponded with Kästner, having asked him to arbitrate in his dispute with Eberhard (see AA 11: 186). Although Kästner declined to take on such a role, he advised Kant to aim for more clarity in exposing his ideas (AA 11: 214), advice which Kant apparently took heed of in writing his *Religion within the Limits of Reason alone* (AA 11: 427). As Förster and Rosen report in their edition of Kant's *Opus Postumum* (Kant 1993: 267–8), Kästner can hardly be said to have converted to the Critical philosophy, but at least he viewed Kant, the author of *Towards Eternal Peace,* as a 'wise man' (Kästner 1797: 100).[11]

Kästner's Articles

The first two topics discussed in Kästner's *PM* articles (Kästner 1790a, b and c) were of particular interest to Kant: the notion of possibility in geometry, as defined through construction, and the characterization of space as infinite. In the first article (1790a), Kästner differentiates between those propositions which Euclid takes as postulates, and those whose possibility is inferred by deriving them from the first, and voices his scepticism that any further grounding of the possibility of the postulates can be achieved (Kästner 1790a: 392–5). The notions introduced with these postulates are however sufficiently clear so that no further grounding is needed since, for Kästner, the absence of any contradiction is sufficient to establish possibility. Kästner explains that what is conceivable to 'make' (*machen*) according to well-defined and non-contradictory geometric rules, as are found in Euclid's geometry, is thereby established as possible (Kästner 1790a: 391–2), and even 'necessarily possible' (Kästner 1790a: 400). And the operation of the understanding by which I grasp the conceivability of 'making' the geometric entity, is just what 'making' in one's understanding amounts to. Together with the Leibnizian claim that what is in the understanding is real, this entails that what is possible in geometry is real (Kästner 1790a: 400). In these claims, Kästner wants to deny any essential role to any auxiliary figures which may be used (Kästner 1790a: 398), a point which Eberhard will have taken as grist to the mill of his particularly vehement attack on Kant's assigning a key role to *a priori* intuition in mathematical knowledge. Indeed, in the preceding issue of the same volume of *PM*, Eberhard published an article in which he expressly attacks the very notion of *a priori* intuition (Eberhard 1789: 84–90). Kästner's claims are: (i) that it is impossible to have an image of the infinite as all images are determinate; and, for the same reason, (ii) that no general image of space is possible. Kästner stresses the fact that certainty does not reside in any image which may correspond to a concept, but in the understanding (Kästner 1790b: 419).

But of course, Eberhard's and Kästner's claims can only be viewed as critiques of Kant if the latter's notion of intuition and image are identifiable, a point that Kant unequivocally denies in his response to Kästner (AA 20: 413). In line with his general attitude towards Kästner, Kant further seeks to defuse any other appearance of dissent from his Critical philosophy of mathematics by cleverly translating Kästner's notion of 'making' into one of 'construction' carried out by the imagination *a priori* (AA 20: 411), thus altering Kästner's operation of the understanding to one that is carried out by the imagination under a rule prescribed by the understanding.

Kästner's understanding of the infinite follows from his understanding of geometric space as abstracted from our representations of outer sense (Kästner 1790b: 407): the infinite is just that which is unlimited, in the sense that there are no limits as to how far geometric figures extend (Kästner 1790b: 407–8). In this sense, he opposes Schultz's theory of the infinite, expounded in *Versuch einer genauen Theorie des Unendlichen* (1788), which, in distinguishing different actual infinite magnitudes, prefigures Cantor's groundbreaking work on the infinite. Although Kant is aware of Kästner's views on Schultz's theory (cf. AA 11: 184), he stresses his agreement with Kästner on the issue of the geometric notion of space, which is indeed a notion of potential infinite, and differentiates it from the actual infinity of metaphysical space. In so doing, he draws unexpected support from Joseph Raphson, a seventeenth-century English mathematician, whom Kästner refers to, claiming that Raphson understands the actual infinity of space to be in the mind. However, as Michel Fichant (1997b: 42, 44–5) points out, Kant appears to make a rare interpretative mistake here, as it is most unlikely that the Newtonian Raphson would hold such a view of space.[12]

Kant's strategy is therefore two-fold in responding to Kästner: he accommodates Kästner's views on infinity through his distinction between metaphysical and geometric space, suggesting it is implicitly assumed by Kästner (AA 20: 411–12, 416); and he appeals to the difference between intuition and image to neutralize Kästner's statements about the irrelevance of images to mathematical truth (AA 20: 422), thereby cleverly emphasizing that the error is Eberhard's. But there remains a clear gulf between Kästner's claims and the Critical philosophy's outlook on mathematical truth (see Fichant 1997a: 8–9). First, Kästner sticks to the Leibnizian-Wolffian line according to which mathematical truth relies entirely upon the criterion of non-contradiction. For Kant, the non-contradictory nature of a mathematical concept does not guarantee its truth. The question of its being constructible in *a priori* intuition, rather, is the key to mathematical truth. And second, Kästner espouses the rationalist doctrine that mathematics, and geometry in particular, defines a world of objects for the understanding (Kästner 1790a: 400). Establishing a mathematical truth is a matter of exploring this world. For Kant, there are no geometric objects, only constructions in *a priori* intuition according to geometrical concepts.[13]

Contemporary Relevance of Kant's text

Kant's text on Kästner, which for the first time appears in an integral English translation below, is important for our understanding of Kant's conception of space. The key point is Kant's insistence on a clear distinction between metaphysical and geometric space. The first is a given infinite, while the second is a potential infinite. Fichant's translation into French of this text in the 1990s (Kant 1997; Fichant 1997a) came at a time when Béatrice Longuenesse's *magnum opus* on *CPR* (Longuenesse 1998a) had just established itself as a serious alternative approach to the dominant interpretative strands in the analytic tradition.[14] A key feature of Longuenesse's work is its claim that space is, in effect, a product of the faculty of productive imagination, and therefore, given that according to Kant the productive imagination is an effect of the *understanding* on sensibility (B151–2), a product of the faculty of understanding (Longuenesse 1998a: 219, 222–3). The stress Kant puts in the article on Kästner upon the independence of a notion of space from any conceptualization such as occurs in geometry certainly provided ammunition for an 'anti-conceptualist' backlash. In French Kant scholarship, this was led by Fichant's translation of Kant's reply to Kästner, as well as a paper underlining its importance for an understanding of Kant on space (Fichant 1997b). Replies followed from Longuenesse (1998b, 2005), and from philosophers upholding the stronger conceptualist stance of Marburg neo-Kantianism (Dufour 2003).

In English-speaking Kant scholarship, a related discussion has been taking place about the sense to be given to Kant's claim that constructions in spatial intuition are required to derive geometric truths. A key text in this debate is Friedman (1992). Friedman claims that Kant's requirement is, in effect, a consequence of the limitations of the monadic logic he used. Friedman (1992: 63–4) points out that, while Kant thought he needed to carry out iterative constructions to construct new points as part of a geometric proof, existential quantifiers in the setting of a polyadic logic could have been used instead, had such a logic been available to Kant. For Friedman, this shows that Kant's view of geometry as synthetic was conditioned by the limits of the logic at his disposal. Friedman's reading of the Metaphysical Exposition (1992: 68–70) has it that it is only because of its role in grounding geometry that the representation of space must be an *a priori* intuition. Carson (1997: 495–7) takes up this point and shows that the infinity of space described by Kant in the Metaphysical Exposition is not that of infinite iteration, but of an infinite given space, and that it therefore is irreducible to logic. In so doing, she is upholding an emphasis upon the phenomenological character of space first made by Parsons (1992: 72), which parallels the anti-conceptualist thrust of Fichant's position. Partly in response to Carson, Friedman revised his position (Friedman 2000, 2003, 2012) leading to some convergence with Carson's position, eventually abandoning in effect the fully conceptualist take upon space which he still held in 2000, as we shall see below, while other authors reinforced the need to take the phenomenological notion of space seriously (e.g. Kjosavik 2009).[15] It is noteworthy that Kant's response to Kästner is discussed by Friedman in the last of those three papers, and indeed its key distinction between metaphysical space that is a given infinite and space as the topic of geometry is what is at the heart of this particular debate around the importance of the phenomenology of space.

For simplicity, we shall describe all positions which uphold a key role for the faculty of the understanding in defining the unity of space as 'conceptualist', while we are aware that is, strictly speaking, a misnomer in some cases.[16]

The Response to Kästner Confronted with B160–1n in the *Critique of Pure Reason*

A key text used by the conceptualist reply is footnote B160–1n in the B-version of the Transcendental Deduction of the Categories. And this is hardly surprising given its emphasis upon the understanding in defining the unity of the formal intuition of space: B160–1n is thus often read as a reinterpretation of the Transcendental Aesthetic in the light of the results of the Analytic (e.g. Longuenesse 1998a: 214ff.). On the conceptualist reading, the unity of space discussed in the Aesthetic is now to be assigned to the unity of the understanding. It is certainly the case that some statements in B160–1n lend themselves to this interpretation. For example, Kant writes: 'In the Aesthetic, I ascribed this unity merely to sensibility . . . though to be sure it presupposes a synthesis, which does not belong to the senses but through which all concepts of space and time first become possible.' The conceptualist reading according to which the unity of space requires the understanding is given further support towards the end of B160–1n when Kant, ambiguously, talks of space and time being '*given* as intuitions', 'as the understanding determines sensibility'.

These statements appear to be flatly contradicted by Kant's assertions in *On Kästner's Treatises*. There, Kant insists upon a notion of 'metaphysically, i.e. originally, nonetheless *merely subjectively given* space, which (because there is no plurality thereof) cannot be brought under any concept which would be constructible, but to be sure contains the ground of the construction of all possible geometrical concepts' (AA 20: 420, our emphasis). This space is described as existing in 'the pure form of the sensible mode of representation of the subject' (AA 20: 421). As such, it is defined independently of any synthesis. In the response to Kästner, this is contrasted with space as the topic of geometry, which is discussed in B160–1n, where Kant considers '[s]pace, represented as *object* (as is really required in geometry)'.

As a result, we have a contrast between, on the one hand, a metaphysical space that is subjectively given and, on the other, geometric spaces defined by conceptual determinations of regions of this space. This reading of the response to Kästner provides direct support to an anti-conceptualist understanding of the representation of space, such as

Fichant's for instance (see Fichant 1997b: 36–7, 42–3; cf. 1997b: 24), a position that could be taken to be in tension with the claims made in B160–1n.[17]

Indeed, what is striking in Kant's response to Kästner, as Fichant quite rightly emphasizes, is the importance accorded to a notion of metaphysical space as a subjective form of sensibility. While footnote B160–1n in the Transcendental Deduction appears to play down the role of the form of intuition as it 'merely gives the manifold', *On Kästner's Treatises* repeatedly emphasizes the key notion of metaphysical space as 'space . . . considered in the way it is *given*, before all determination of it'.[18] To be sure, this reflects the different purposes of these two texts, but it certainly represents a worry for conceptualist readings. On the latter, the representation of space is a product of the understanding on its own (Dufour 2003), or on Longuenesse's reading, of the understanding together with some unknown ground of spatiality providing the innate seed for the process of epigenesis of the form of intuition of space. Additionally, an unfortunate consequence of such interpretative stances is that they end up deleting any meaningful distinction between the notions of 'form of intuition' and 'formal intuition', as Fichant (1997b: 36) points out, whereas the point of the footnote B160–1n is precisely to draw a clear distinction here.

But, most importantly, as far as the response to Kästner is concerned, the issue is not simply one of a difference in emphasis: the biggest stumbling block to a conceptualist reading of Kant's response must be the topic which is the focus of Kästner's article, namely the infinity of space (see Fichant 1997b: 25–7). Eberhard saw Kästner's article as providing support from the mathematical establishment for his Leibnizian take upon the issue of the infinity of space. Leibniz's space, which is relational, is not infinite, and indeed Leibniz argues against the notion of an infinite space in his correspondence with Samuel Clarke. The motives for the avoidance of such a notion were partly theological: there can only be one infinite, God. The Leibnizian view of mathematical truth, moreover, does not require any appeal to infinite space. For Leibniz – and Eberhard later takes this up to attack Kant – the mathematician proceeds merely on the basis of the principle of non-contradiction (cf. AA 20: 413–14) or '*mediately* through inferences' (AA 20: 411), hence purely conceptually, without beforehand ascertaining, through construction in intuition, the reality of the object of a geometric concept. But in *On Kästner's Treatises* Kant responds that the geometer *already presupposes* the pure forms of sensibility by means of which an object is in fact first constructed, and that this guarantees its reality, which no amount of conceptual analysis can bring about. Kant is therefore keen to suggest that this is something Kästner himself, as mathematician, effectively already presupposes (AA 20: 411–12, 416). He thereby identifies the actual infinity of space as a requirement for the possibility for the mathematician of constructing, to infinity, any geometric figure, so that the mathematician need only ever concern himself with the potential infinity of his constructions (AA 20: 419–20). For instance, considering Euclid's second postulate which states that any straight line can be extended indefinitely, an infinite space must first be given for such a construction to be possible. It is hard to see how any conceptualist understanding of space could do justice to this notion of the given infinity of space.[19]

Footnote B160–1n also crucially introduces the notions of 'form of intuition' and 'formal intuition'. The proposed reading of the response to Kästner might suggest an identification of metaphysical space with the form of intuition, and of the representation of the space of geometry with the formal intuition of space. But we must be careful here, because Kant talks of geometrical spaces in the plural (AA 20: 419), so that equating these spaces with space as object that is the topic of B160–1n, as such an identification would imply, is not straightforward. In his response to Kästner, Kant does not suggest that these geometric spaces are somehow dependent upon one geometric space that would be space considered as object; rather, he clearly states of metaphysical space that 'hence in this, as singular representation, the possibility of all spaces, which goes to infinity, is *given*' (AA 20: 421), and 'there are (many) spaces, of which the geometer however, in accord with the metaphysician, must admit as a consequence of the foundational representation of space, that they can only be thought as parts of the single original space' (AA 20: 419). The conceptually determinate geometric *spaces* are therefore parts of *metaphysical* space, not of one geometric space. Conceptually determinate geometric spaces are 'a proper subset of the determinations of the single metaphysical space' (Patton 2011: 282–3). Such statements as in Kant's response to Kästner suggest that metaphysical space could then also be identified with what B160–1n describes as '[s]pace, represented as *object* (as is really required in geometry)' and which is identified as the formal intuition of space. In view of these issues, it would seem that what the response to Kästner describes as metaphysical space is represented, in B160–1n, in terms of *both* notions of 'form of intuition' and 'formal intuition'. In other words, Kant's distinction in the Kästner piece between metaphysical and geometric space does not separate out 'form of intuition' and 'formal intuition': both refer to metaphysical space and the distinction between the senses of these expressions is not directly relevant to the Kästner text.

That this is a plausible interpretation of the relations between the two texts should, moreover, come as no surprise. The topic in the response to Kästner is very much that of the Transcendental Aesthetic: metaphysical space is essentially space as it is described in the Metaphysical Exposition (see also Friedman 2012: 241–3), whereas the Transcendental Exposition identifies the role space plays in grounding geometry.[20] The discussion of the Transcendental Deduction, in which the issue is that of how objects in space and time stand under the categories, is a different topic which Kant would certainly have been wise not to bring up in the discussion of Kästner's views about geometric space if he wanted to achieve the kind of clarity that Kästner himself advised Kant to aim for in his writings, as we saw above. As Kant explains in the footnote B160–1n, he had ascribed the unity of space 'merely to sensibility' in the Transcendental Aesthetic because of the nature of the investigation in that particular chapter, namely establishing the distinctly non-conceptual nature of space, i.e. 'in order to note that it precedes all concepts'. The account in the Transcendental Deduction demands however that the role of the understanding in the *grasp* of space as a unity be investigated further, and this gives rise to footnote B160–1n, in which two distinct notions of the representation of space are differentiated. As this would have been of no use in responding to Kästner, it therefore seems reasonable to view the way the Transcendental Aesthetic refers to space as indifferent between its characterization as form of intuition or as formal intuition, and this perhaps explains the simple claim that the representation of space is an '*a priori* intuition' (AA 20: 421), exactly as in the Transcendental Aesthetic. What the response to Kästner and the Transcendental Aesthetic do not address, therefore, is the full story of how it is that the subjectively given metaphysical space can indeed provide a grounding for the objective science of geometry: such a full story must explain the role of the understanding (and imagination), and this is where the footnote B160–1n comes into play.

Outlining an Interpretation of B160n in the Light of the Response to Kästner

Even though the topic of the present article is not footnote B160–1n, to which we have devoted a comprehensive study (Onof and Schulting, forthcoming), our reading of Kant's response to Kästner must, nevertheless, be able to give some account of how the notions of 'form of intuition' and 'formal intuition' are distinguished, insofar as this completes the account of how subjectively given metaphysical space functions as ground of the objectivity of geometry by focusing upon the role of faculties other than sensibility in this grounding. This interpretation will serve the purpose of shedding further light upon our proposed reading of *On Kästner's Treatises* as not introducing any distinction corresponding to that between 'form of' and 'formal' intuition in B160–1n. The space of our pure form of outer sensible intuition has a unity which Kant describes in the Metaphysical Exposition of space. It is not a *conceptual* unity because it is not a unity *of a multiplicity of representations* that are contained *under a higher one* (A69/B94; A78/B104),[21] and there are no grounds for viewing the understanding as involved in defining this unity. Rather, it is the internal unity of the space *in which* manifolds in intuition are first represented.[22] The pure receptivity of our faculty of sensibility defines a unity which Kant describes at A25/B39 as the 'single all-encompassing space' whose parts are 'only thought in it'.[23] This characterizes the metaphysical space of *On Kästner's Treatises* in its original givenness. *Pace* Friedman (2012), it is not the set of all possible spatial perspectives (what he calls 'perspectival space'),[24] but rather a precondition for the very possibility of considering such a multiplicity of perspectives as belonging to one space. The unity of this unitary space is the unity that Kant refers to when he claims that a single representation is always a unity (A99): i.e. a single representation is always an 'absolute unity', which is different from the unity that the understanding brings to the manifold.

So far, we have identified a purely subjective receptivity to manifolds of representations of outer sense, of which the necessary, subjective form is space with its sui generis unity. If cognition is to be possible, as Kant explains in A99–100, this manifold must be apprehended, i.e. I must take the manifold *as manifold* for my cognition. And he explicitly indicates that this brings a unity to my intuition 'as, say, in the representation of space' (A99). The manifold is now regarded as objective (although the object is not as yet[25] thereby determined), i.e. as independent of any particular subjective perspective. The space in question here is of course still the same metaphysical space that was subjectively given, but now grasped as a conceptually differentiated (rather than merely absolute, undifferentiated) unity, through a synthetic act of the understanding. Kant adds that there is a '*pure* synthesis of apprehension' (A100) involved in grasping space as a unity, but later indicates that it is in fact the imagination 'whose action exercised immediately upon perceptions

I call apprehension' (A120), in line with his assertion that '[t]he synthesis of apprehension is . . . inseparably combined with the synthesis of reproduction', i.e. 'the transcendental faculty of the imagination' (A102). Focusing upon the B-Deduction, Friedman (2012: 248) correctly indicates that it is the transcendental synthesis of the imagination which is at stake here, also in line with Longuenesse's (1998a) assigning a central role to *synthesis speciosa*. But importantly, in contrast to Longuenesse's reading, the unity brought about by the imagination is not equivalent to the unity and singularity of space. Rather, it is a grasp of space, with its already independently characterized singularity and unity, *as a unity for my cognition*: in other words, we have moved from a mere subjective reception of a manifold in space (with its internal sui generis unity) to the consideration of this manifold *as* objective, which involves unifying all possible perspectives upon it.

What is thus obtained is the formal intuition of space. It does indeed precede all concepts insofar as the apprehension is *logically* prior to bringing the manifold under a concept (in the terms of the A-Deduction: synthesis of apprehension is logically prior to, or a necessary condition of, the synthesis of recognition). However, *pace* Longuenesse, there are no grounds for considering the latter synthesis as involving a separate act of the understanding. That is, the act of apprehension is just one aspect of the synthesis which will bring the manifold under a certain concept, and thus it also requires the synthesis of reproduction and finally the synthesis of recognition; all of the different aspects of synthesis are equiprimordially and simultaneously involved. This can be seen by considering that apprehension is merely taking the manifold *as manifold* for my cognition. But a manifold is never taken as manifold for my cognition without also being determined in some way, i.e. taken *as manifold of representations of some object*. To suppose otherwise would be equivalent to claiming that an 'I think' could accompany my representations, without it being an 'I think that . . .', i.e. an 'I think' which is necessarily followed by a clause defining a fully fledged epistemic claim (see A102–4).

The role of the categories in the synthesis that brings about the formal intuition of space is to ensure that space is taken as a unity across representations, as explained above. While the manifold in space will thereby be determined under some concept (by means of the synthesis of recognition), this does not amount to a conceptualization of space *itself* as indeed there is only one space, which, given Kant's denial of the possibility of a singular concept, cannot as a unique individual be brought under a concept. Rather, what the synthesis achieves is to bring space under the categories, thereby making conceptual determinations of any manifold in space possible.

Kant does not shed much light on how a synthesis gives rise to the unity of space, but we can look at the case of time in the chapter on Schematism: 'The schema of a pure concept of the understanding . . . is a transcendental product of the imagination, which concerns the determination of the inner sense in general, in accordance with conditions of its form (time) in regard to all representations, insofar as these are to be connected together *a priori* in one concept in accord with the unity of apperception' (B181/A142). Here, as explained above, we note how the determination of inner sense *must accord* with the conditions of its form, time, while at the same time this determination is *in accord* with the unity of apperception. The two sets of conditions of accordance are only possible insofar as the unity of apperception is in the driving seat as it were: this requires that the form of time be brought under the categories. These determinations of time are essentially ways of enabling what is in time to be determined under the categories. The same applies mutatis mutandis to the determinations of space (as in geometric constructions): they have *to accord* with both the form of space whose unity has been examined in the Metaphysical Exposition of the Transcendental Aesthetic, *and* the unity of apperception. And this is achieved through the determination of the unity of the form of space under the categories. Kant does not provide us with further insights into this, but one can infer what this might amount to from the analysis of the form of space in the Aesthetic; this issue is discussed in further detail in Onof and Schulting (forthcoming).

With this interpretation, we can therefore explain how metaphysical space is presented by Kant both as subjectively given, and as conforming to a concept of object to the extent that space is categorially determinable: it is the same space that is considered, first as mere form of receptivity, i.e. as a receptacle for manifolds of outer sense, and second, in the grasp of its unity by means of synthesis (through the imagination, and hence involving an application of the categories) through which any such manifolds are taken as manifolds for my cognition. Through this grasp, that which is a sui generis unity inherent to the structure of space[26] is grasped as a unity for the understanding.

This might still leave it unclear why the intuition of space is said in B160–1n to require a synthesis, i.e. insofar as Kant says that it is 'as the understanding determines sensibility' that 'space and time are first *given* as intuitions' (Kant's emphasis). Of course, it is the notion of formal intuition which Kant means here, but it now seems that this is somewhat different from the notion of 'intuition' as it was originally presented in the Transcendental Aesthetic. This apparent ambiguity around the way the term 'intuition' is used is not restricted to this passage, as Kant says, for instance in the *Stufenleiter* at A320/B376–7, that there are two types of 'objective perception', intuitions and concepts, which is *prima facie* puzzling for, on Kant's theory of cognition (cf. A51–2/B75–6), if an intuition is objective, then it must be united with concepts and therefore involve some role for synthesis by means of the transcendental unity of apperception. However, there is in fact no ambiguity in such terminology. Rather, there is a clear distinction between the mere intuition as the *a priori* form in which the manifold of representations is received ('form of intuition'), and the intuition of an object, namely of an appearance (B34/A20), which is determined by means of *a priori* synthesis ('formal intuition'). When considering our mere receptivity for manifolds, i.e. when Kant considers manifolds in intuition (e.g. A97), the intuition involves only sensibility, i.e. 'form of intuition'. When the intuition is that of an object, this assumes that, although the intuition provides us with the *immediate* relation to the object (A68/B93), it has been synthesized by means of the transcendental unity of apperception, in order for such an intuition to amount to *knowledge* of an object (i.e. intuition as 'formal intuition'). As indicated in the discussion above, in the Transcendental Aesthetic such distinctions are not yet relevant as the role of the understanding has not yet been examined.

Final Remarks

In the case of space, just as in the case of time, the conceptualist must explain how the determination of the form of intuition, which is at the heart of the chapter on Schematism, should warrant such attention if the form of intuition were nothing but a 'projection' of the understanding. Why then, in the case of time, would this determination be described by Kant as 'a hidden art in the depths of the human soul' (B180–1/A141)? On the contrary, this issue becomes central to the success of the Critical project if space (and time), as metaphysically given infinite (cf. AA 20: 420–1), has its own essential structure that is to be grasped by the faculty of understanding. It is part of the goal, in fact, of the so-called second step of the B-Deduction to explain, by means of a deduction of the 'concept of space' (B120–1/A88), how we can conceive of the unity of space without the latter being thereby reducible to a *conceptual* or purely formal *analytical* unity, since space (and time), as Kant had explicitly asserted in the Aesthetic (A24–5/B39), is not itself a concept. As we explain in Onof and Schulting (forthcoming), this grasp of the form of space by the understanding is possible because, as Kant shows in the Metaphysical Exposition's analysis of the form of space, we have an intimate phenomenological acquaintance with space as a given infinite magnitude, with what in the response to Kästner is called metaphysical space. It is for this reason that, in the Metaphysical Exposition, it is possible to describe (necessarily in conceptual terms) the essential properties of what is *not* conceptual, namely space as *a priori* form of intuition, and therefore that it is possible to bring the unity of this form under the unity of apperception.[27]

Notes

1 The *Critique of Pure Reason* (*CPR*) is cited according to the standard manner of giving reference to the original pagination of the A and B editions (A/B). All other works of Kant are referred to as AA = *Kants gesammelte Schriften*. Ed. Königlich Preußische (later Deutsche) Akademie der Wissenschaften, Berlin: de Gruyter, 1900–), followed by volume and page numbers, and (where relevant) line numbers. The translation we used for *CPR* is by P. Guyer and A. Wood in *The Cambridge Edition of the Works of Immanuel Kant* (Kant 1998).

2 Among the Wolffians who contributed frequently were J. G. E. Maass, who also wrote the anti-Kantian *Briefe über die Antinomie der Vernunft* (1788), J. F. Flatt, author of *Fragmentarische Beyträge* (1788), J. C. Schwab, and J. A. Ulrich, whose *Institutiones logicae et metaphysicae* (1785) and especially *Eleutheriologie* (1788) were specifically directed against Kant's philosophy. See further F. Beiser, 'The Revenge of the Wolffians', in Beiser (1987: ch. 7). On Ulrich, see di Giovanni (2005: 108–18).

3 Schultz's *Erläuterungen über des Herrn Professor Kant Critik der reinen Vernunft* (1784), an exposition of Kant's *CPR* that was initially intended as a review but became a book, was for many, for a long time, the only substantial source of Kant's thought. Most people, especially early on, did not have first-hand access to Kant's *CPR*. In a later work, *Prüfung der Kantischen Critik der reinen Vernunft* (1792), Schultz continued to respond specifically to Eberhard's criticisms in a broadly Kantian spirit, to which the Wolffians in turn replied in editions of the *Philosophisches Archiv*, the successor journal to *PM*.

4 Reinhold published an important exposition of the Critical philosophy, the *Briefe über die Kantische Philosophie*, first in a series of articles in *Der Teutsche Merkur* from Aug. 1786 to Sept. 1787 and later expanded in book form in 1790, which was probably even more effective than Kant's own *Prolegomena to Any Future Metaphysics* (1783) in propagating the central ideas of the new Critical philosophy (see Reinhold 2005 for an English translation of some of the letters and a helpful introduction by Karl Ameriks).

5 Another defender of Kant against attack from the Wolffians was A. W. Rehberg (1757–1836), who in the late 1780s wrote a widely read critical review of Kant's second *Critique*, and also was an important interlocutor with Kant on the topic of the philosophy of mathematics, specifically the implications for arithmetic of a passage at B188 in *CPR*, where Kant asserts that 'mathematical principles . . . are drawn only from intuition, not from the pure concept of the understanding'; Rehberg believes that while this may be true for geometry, it does not apply to arithmetical truth (see e.g. AA 11: 205–10). On Rehberg, see further di Giovanni (2005: 125–36).

6 There were other critical voices around apart from the Wolffians, in particular F. H. Jacobi with his *David Hume über den Glauben, oder Idealismus und Realismus* (1787) and J. G. Herder, with whom Kant was personally heavily embroiled in a dispute in the aftermath of Kant's critical review, also in *ALZ*, of Herder's *Ideen zur Philosophie der Geschichte der Menschheit* (1784). Before he became a proponent of Kantianism, Reinhold, already a frequent contributor to *ALZ* and closely acquainted with Herder through his father-in-law C. M. Wieland, attempted to arbitrate in this dispute. Later in the 1790s, Herder hit back with his *Verstand und Erfahrung: Eine Metakritik zur Kritik der reinen Vernunft* (1799). J. G. Hamann's criticism of his friend Kant's work, contained in *Metakritik über den Purism der Vernunft* (1784), was not published but enjoyed a large readership. On Hamann's reaction to the *Critique*, see Kuehn (2001: 252–4). On Herder's quarrel with Kant, see Beiser (1987: 149–53) and Kuehn (2001: 292ff.). On Jacobi's attack on Kant, see also Beiser (1987: 122–6) and di Giovanni (2005: 16–24, 77–91). For early critics of Kant's philosophy from the empiricist camp, such as Johann Feder, Christian Garve, and Hermann Pistorius, see Sassen (2000).

7 For discussion, see Allison (1973), Gawlina (1996) and La Rocca (1994).

8 Schultz reviewed a volume of the *Philosophisches Magazin* in four consecutive issues of vol. 3 of *ALZ* (1790). The text provided to him by Kant appears in issue nos. 283 and 284 (dated 26 and 27 Sept. 1790), 807–13.

9 In non-published work, it appears that Lambert understood that the fifth postulate is not derivable from the others. Lambert also proved the non-Euclidean result that the sum of the angles of a triangle increases as the area decreases.

10 Kant owned some of Kästner's works (Warda 1922: 39), and esteemed Kästner's work from early on (cf. the references to Kästner in *The Only Possible Argument in Support of a Demonstration of the Existence of God* (1763), AA 2: 130, *Attempt to Introduce the Concept of Negative Magnitudes into Philosophy* (1763), AA 2: 170, and in the 'Inaugural Dissertation' (1770), AA 2: 400). Among other things, as Förster and Rosen (Kant 1993: 263 n. 34) note, Kant credited Kästner 'with the first mathematically satisfactory demonstration of the lever'. See also the reference to Kästner in the Mrongovius lecture notes on metaphysics from the 1780s (AA 29: 921).

11 It is noteworthy that in his review of Kant's *Metaphysical Foundations of Natural Science* (1786) in the *Göttingische Anzeigen von gelehrten Sachen*, 191 (2 Dec. 1786), 1914–18, Kästner ends his rather discursive review by noting that, for reasons of space and because Kant's exposition deserves detailed attention, he holds back 'where, *unlike in many other instances*, he is not in complete agreement with Mr K[ant]' (p. 1918; our emphasis and translation). For more on the biography of Kästner, see Baasner (1991).

12 See further below n. 16 to the translation.

13 See further Fichant's excellent introduction to the French translation (1997a).

14 Longuenesse's *Kant and the Capacity to Judge* was first published in French in 1993 under the title *Kant et le pouvoir de juger: Sensibilité et discursivité dans l'Analytique transcendantale de la Critique de la raison pure* (Paris: PUF).

15 Friedman (2012) addresses the issue of the use of diagrams in mathematics, which has recently been the focus of renewed interest (see Manders 2008) and which has led Shabel (2003) to draw upon this as support for Kant's claim that spatial intuition is required in geometry. Friedman rightly emphasizes the need to distinguish between actual constructions in empirical intuition and constructions in pure intuition.

16 Longuenesse (1998a) points out in particular that her claim that the faculty of the understanding is responsible for the unity of space is not a conceptualist claim, as it

concerns a pre-discursive employment of this faculty. Friedman (2012) also comes to a similar conclusion about space. However, since Longuenesse argues that the understanding is responsible for the unity of space in its guiding function, which according to her account means the application, in some sense, of the *a priori* concepts (categories), we think it justified to call her interpretation 'conceptualist', as differentiated from what Grüne (2009) calls 'judgement-theoretical', where the latter would be the much stronger conceptualist reading that intuitions, and thus space, are only formed in the context of judgements. In the context of a discussion about non-conceptual content, Grüne also puts Longuenesse under the rubric of conceptualism (in the non-judgement-theoretical sense).

17 Cf. Allison's critical discussion of 'conceptualist' positions on the B160–1 note, e.g. Waxman's and especially Longuenesse's, in Allison (2004: 192–3, 483 n. 67; 2012a: 37–41; 2012b: 46–8). By contrast, by explicitly referring to Kant's response to Kästner, Heidegger (1995: 137–8) highlights the anti-conceptualist reading of the footnote. See further Onof and Schulting (forthcoming).

18 Note that there is what might be viewed as an ambiguity in this sentence where Kant describes the givenness of space, which conceptualists might be tempted to use for their purposes. Indeed, the German (AA 20: 419. 4–5) could be read to imply that 'in conformity with a certain concept of object' is a clause qualifying how space 'is given', rather than the 'determination of it'. This would however imply a contradiction as space would both be subjectively given and conform to the concept of an object. The principle of charity leads us to exclude such a reading.

19 As noted earlier, Friedman (1992) argues that this is however a consequence of the limits of Kant's monadic logic. Following Parsons, Carson (1997) makes a strong case for viewing the objectivity of geometric truths to lie in constructability in pure intuition (while Friedman views the objectivity of geometric truths as only confirmed by empirical intuition), which defines a notion of possibility that is distinct from logical possibility. With this understanding of geometric truth, constructability in space would still be required even with the availability of polyadic quantification. See also Patton (2011), who raises some difficulties with Friedman's later reading of metaphysical space and the role of imaginative construction, specifically how Friedman can account for the necessary singularity of space (Friedman 2000).

20 While the response to Kästner emphasizes a distinction between two ways of considering space, it is also clear that it is the same representation of space that is at stake, since metaphysical space provides the foundation for geometry: '[T]he geometer however, in accord with the metaphysician, must admit [the possibility of constructing many geometrical spaces] as a consequence of the foundational representation of space' (AA 20: 419).

21 That spatial unity is not a conceptual unity is also confirmed by Kant's observation in the response to Kästner that 'there is no plurality' of the 'metaphysically, i.e. originally . . . given space' (AA 20: 420). In other words, the representation of space is just the representation of *one* single space, and not a representation of one among *many* others that would equally be subsumable under the same concept.

22 This is, arguably, the 'synopsis' of which Kant speaks in the A-deduction ('the *synopsis* of the manifold *a priori* through sense', A94). As we shall see below, this mere receptivity of the manifold is only cognitively relevant insofar as there is a synthesis, i.e. a grasping of the manifold as manifold (apprehension through the imagination), and as conceptually determined (recognition through the understanding). And indeed, Kant reminds us of the correspondence between the synopsis of sense and the required synthetic activity at A97.

23 Since Kant prefers to use the word 'unity' in the *Critique* when the understanding is involved, we do not find many references to the unity of space independently of the understanding, though it is instructive that in the *Critique of the Power of Judgement* Kant talks of 'the unity that constitutes the ground of the possibility of natural formations [which] would be merely the unity of space' (AA 5: 409; Kant 2001: 278) in a thought-experiment he proposes. In this context, it is clear that this 'unity of space' cannot be a product of the understanding as the natural formations in question are things-in-themselves.

24 Friedman (2012) correctly identifies this set as involving a synthesis, hence a role for the understanding. But he overlooks the fact that a plurality of spatial perspectives is only possible if (metaphysical) space is first given, in which changes of perspective can be defined.

25 'As yet' is of course not meant temporally here, but refers to what the transcendental investigation identifies as different logical moments of the determination of an object, as Kant identifies them in the A-Deduction.

26 In Onof and Schulting (forthcoming) we call this unity that is inherent to space and independent of the unity of the understanding 'unicity', to distinguish it from the term 'unity' which Kant mostly reserves for the unity that the understanding brings about by means of its act of original synthesis.

27 We would like to thank Marcel Quarfood for his helpful comments on an earlier draft of the article.

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