AATM 2014 STATE ALGEBRA CONTEST

1. If $f(x) = 2x^3 + 4x^2 - 7x - 5$, find $f(-3)$.

   A) –26 B) 106 C) –2 D) –56

2. Solve for x: $\frac{18}{x-2} = \frac{12}{x+2}$

   A) 2 B) –2 C) –10 D) 10

3. Find the midpoint of $\overline{AB}$ if A is (–9, 1) and B is (–3, –15).

   A) (–3, 8) B) (–6, –7) C) (3, –8) D) (6, 7)

4. Give the degree of this polynomial: $2x^4y^6 - 6xy^8 + 3x^4y^7 - 2x^7y^5$.

   A) 41 B) 42 C) 8 D) 12

5. Simplify: $(-3)^2 - (6 - 2^3)^3 \div 4$

   A) 8 B) 11 C) 7 D) 4

6. Multiply: $-4x^3y^2(6x^5y^4 - 3xy^5)$

   A) $-24x^8y^6 + 12x^4y^7$ B) $-24x^{15}y^8 + 12x^3y^{10}$ C) $-24x^{15}y^8 - 12x^3y^{10}$ D) $-24x^8y^6 - 12x^4y^7$

7. Multiply: $(2x - 5y)(4x + 3y)$

   A) $8x^2 + 14xy - 15y^2$ B) $8x^2 + 26xy - 15y^2$ C) $8x^2 - 14xy - 15y^2$ D) $8x^2 - 26xy - 15y^2$

8. Identify the property that justifies this statement: $(5 + 6) + 7 = 5 + (6 + 7)$.

   A) Substitution B) Commutative C) Associative D) Symmetric

9. Give the equation of the vertical line through (– 4, 3).

A) $x = -4$ B) $y = -4$ C) $x = 3$ D) $y = 3$

10. What value of c makes this trinomial a perfect square: $x^2 + 11x + c$

A) $\frac{11}{2}$ B) $\frac{11}{4}$ C) $\frac{121}{2}$ D) $\frac{121}{4}$

11. Give all values of x for which $\frac{x - 4}{3x^2 - 27x}$ is undefined.

A) $0, \pm 3, 4$ B) $0, 9$ C) $0, \pm 3$ D) $0, 9, 4$

12. Find the slope of the line through (–9, –2) and (–6, 5).

A) $-\frac{3}{7}$ B) $\frac{3}{7}$ C) $-\frac{7}{3}$ D) $\frac{7}{3}$

13. Simplify: $7\sqrt{18} - 3\sqrt{72} - 2\sqrt{50}$

A) $2\sqrt{140}$ B) $2\sqrt{2}$ C) $-7\sqrt{2}$ D) $12\sqrt{140}$

14. Write $4x + 3y = 15$ in slope–intercept form.

A) $y = \frac{4}{3}x + 5$ B) $y = -\frac{3}{4}x + \frac{15}{4}$ C) $y = -\frac{4}{3}x + 5$ D) $y = \frac{3}{4}x + \frac{15}{4}$

15. Write the following as a mathematical expression. Two less than seven times a number is the same as five more than twice the number.

A) $7n - 2 = 5 + 2n$ B) $2 - 7n = 5 + 2n$ C) $7n - 2 = 5(n + 2)$ D) $2 - 7n = 2(n + 5)$

16. Find the distance between (–5, 2) and (–3, – 4).

A) 10 B) $2\sqrt{10}$ C) $10\sqrt{2}$ D) $2\sqrt{17}$

17. Simplify: $(-2x^5y^3)^4 (3xy^4)^2$

A) $-144x^{22}y^{20}$ B) $144x^{22}y^{20}$ C) $48x^{11}y^{13}$ D) $48x^{12}y^{13}$

18. Give the equation of the line that is perpendicular to $2x - 3y = 9$ and goes through $(-7, 5)$.

A) $3x + 2y = -31$ B) $3x - 2y = 31$ C) $3x - 2y = 11$ D) $3x + 2y = -11$

19. Rationalize the denominator and express in simplest form: $\dfrac{15}{3\sqrt{6} - 2}$

A) $\dfrac{9\sqrt{6} + 6}{10}$ B) $\dfrac{15(3\sqrt{6} - 2)}{14}$ C) $\dfrac{15(3\sqrt{6} + 2)}{14}$ D) $\dfrac{9\sqrt{6} - 6}{10}$

20. Multiply, putting the answer in scientific notation: 123.562 x .000007

A) 8.64934×10^{-4} B) 8.64934×10^{4} C) .000864934 D) 864934×10^{-4}

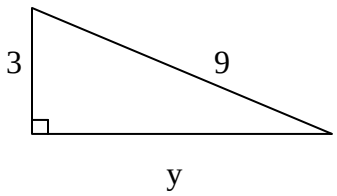
21. Evaluate $5x^3 - 3y^2 - 8z$ if $x = -2$, $y = 5$, and $z = -6$.

A) –108 B) –12 C) –67 D) –163

22. Simplify, leaving no negative exponents: $\dfrac{2^{-3}x^{-9}y^{4}}{2^{-5}x^{-5}y^{-3}}$

A) $\dfrac{x^7}{4y^4}$ B) $\dfrac{4y^7}{x^4}$ C) $\dfrac{y^7}{4x^4}$ D) $\dfrac{4x^7}{y^4}$

23. Solve for y:

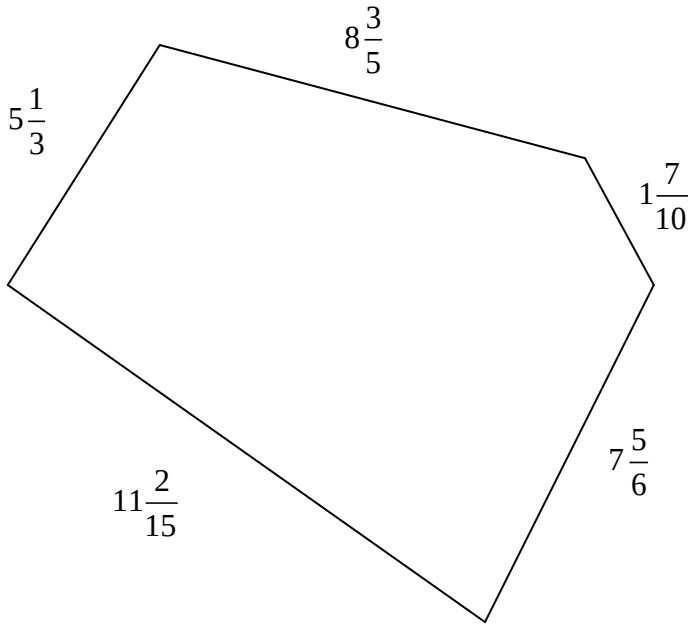


A) $2\sqrt{6}$ B) 72 C) $3\sqrt{10}$ D) $6\sqrt{2}$

24. Subtract, giving the answer in lowest terms: $\dfrac{11x + 3}{9x^2 - 64} - \dfrac{2x - 21}{9x^2 - 64}$

A) $\dfrac{1}{x + 8}$ B) $\dfrac{1}{x - 8}$ C) $\dfrac{3}{3x + 8}$ D) $\dfrac{3}{3x - 8}$

25. Find the perimeter of this pentagon:



A) $34\frac{1}{5}$ B) $34\frac{2}{5}$ C) $34\frac{3}{5}$ D) $34\frac{4}{5}$

26. Divide, giving the answer in lowest terms: $\frac{x^2 + 2x - 15}{x^2 + 3x - 18}, \frac{x^2 - 3x - 40}{x^2 + 14x + 48}$

A) $\frac{x + 8}{x - 8}$ B) -1 C) $x - 1$ D) $\frac{x - 8}{x + 8}$

27. Solve this system of equations: $3x + 2y = 8$
$5x - 3y = 45$

A) (6, 5) B) (–6,5) C) (6, –5) D) (–6, –5)

28. Find the total price, including tax, of a digital camera that costs $290 if the sales tax is 7.4%.

A) $311.46 B) $314.16 C) $297.40 D) $316.14

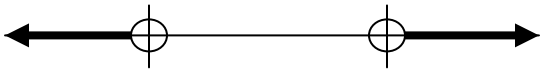
29. Find the mean, median, and mode (in that order) of this set of numbers: 19, 13, 22, 8, 13, 20, 17

A) 13, 22, 8 B) 13, 8, 22 C) 16, 17, 13 D) 16, 13, 17

30. Graph the solution to this inequality: $|2x - 5| < 13$

A)

B)

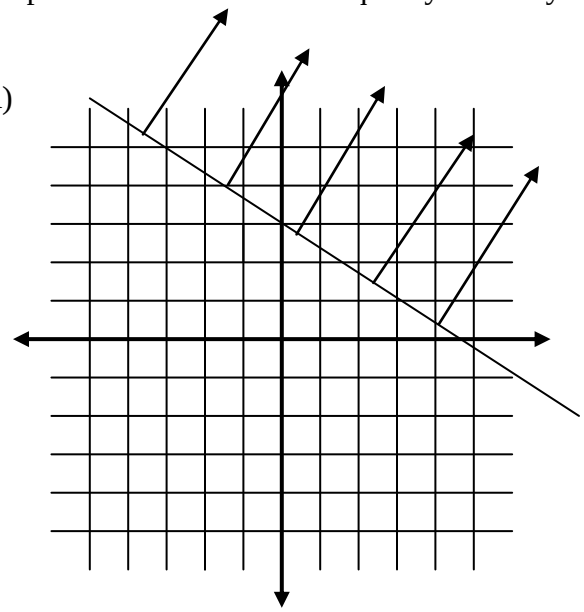


C)

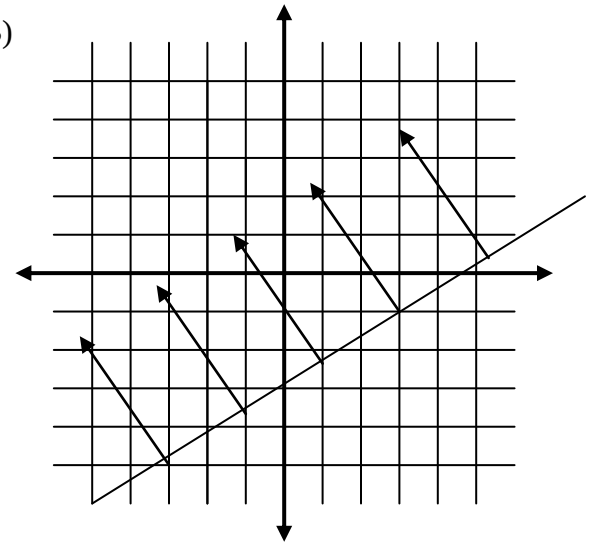
D)

31. Graph the solution to this inequality: $2x - 3y \leq 9$

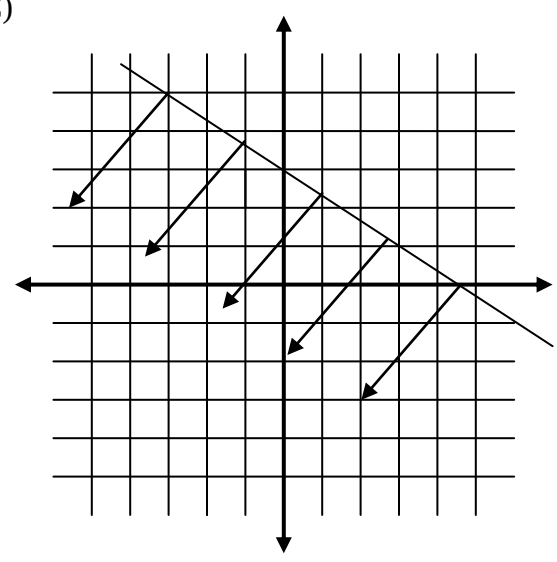
A)



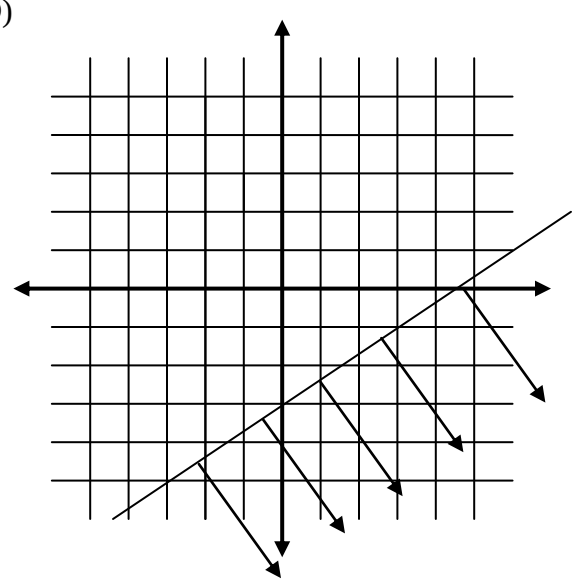
B)



C)



D)



32. Simplify: $(3x - 7y)^2$

A) $9x^2 - 42xy + 49y^2$ B) $9x^2 - 42x^2y^2 + 49y^2$ C) $9x^2 - 21x^2y^2 + 49y^2$ D) $9x^2 - 21xy + 49y^2$

33. Factor completely: $5x^3y^3 - 15x^2y^3 - 270xy^3$

A) Prime B) $5xy^3(x + 6)(x - 9)$ C) $5xy^3(x - 6)(x + 9)$ D) $xy^3(x - 9)(5x + 6)$

34. Factor completely: $7x^3 - 112x$

A) $7x(x - 4)^2$ B) $7x(x + 4)(x - 4)$ C) Prime D) $7x(x^2 - 16)$

35. Factor completely: $4x^2 + 7x - 36$

A) Prime B) $(2x + 9)(2x - 4)$ C) $(4x + 9)(x - 4)$ D) $(4x - 9)(x + 4)$

36. Factor completely: $8xy - 48x - 3y + 18$

A) Prime B) $(y + 6)(8x - 3)$ C) $(y - 6)(8x - 3)$ D) $(8x - 3)(6 - y)$

37. Solve for x: $3x - (8x - 3) = 4 - 5(2x + 1)$

A) $\varnothing$ B) all real numbers C) $-\frac{4}{5}$ D) $\frac{4}{5}$

38. Solve for x: $2x(3x + 4) + x^2 - 9 = 3x(x + 5) - 3(7 - x^2)$

A) $3, -4$ B) $-3, 4$ C) $3, 4$ D) $-3, -4$

39. Solve for x: $|4x - 7| = 15$

A) $2, -2$ B) $\frac{11}{2}, -\frac{11}{2}$ C) $\frac{11}{2}, -2$ D) $-\frac{11}{2}, 2$

40. Solve for x: $\sqrt{3x + 16} - 6 = x$

A) -5 B) $-5, -4$ C) -4 D) $\varnothing$

41. Solve for y: $\frac{ay + p}{g} = h$

A) $\frac{hg + p}{a}$ B) $a(hg + p)$ C) $\frac{hg - p}{a}$ D) $a(hg - p)$

42. Solve for x: $\frac{x + 1}{x^2 - 4} + \frac{x - 1}{x^2 + x - 2} = \frac{3}{x + 2}$

A) 1 B) Ø C) 5, 1 D) 5

43. Bill can clean the garage in 18 minutes. It takes Sue 30 minutes to do the same job. How long would it take them to clean the garage if they worked together?

A) 11.25 minutes B) 12.5 minutes C) 48 minutes D) 12 minutes

44. Using I = prt, find out how much money Tom invested at 7.5% if he earned $3750 in interest in 4 years.

A) $12,500 B) $12,000 C) $10,500 D) $15,000

45. How many grams of a 75% acid solution should be added to 36 grams of a 50% acid solution to produce a 60% acid solution?

A) 36 grams B) 24 grams C) 18 grams D) 30 grams

46. A boat that travels 13 mph is still water can make a 216 mile trip with the current in the same time that it can make a 96 mile trip against the current. Find the rate of the current.

A) 3.5 mph B) 4.5 mph C) 3 mph D) 5 mph

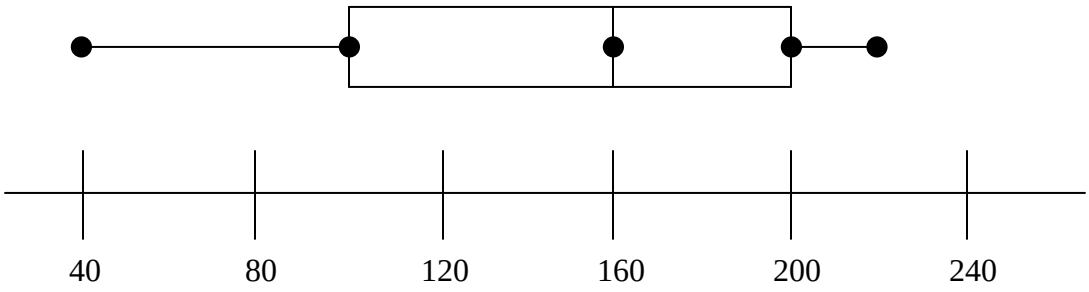
47. The length of a rectangle is 5 less than 3 times its width. If its perimeter is 78, find its area.

A) 328 B) 324 C) 308 D) 312

48. The odds of an event **not** occurring are 9:5. What is the probability that the event **will** occur?

A) $\frac{5}{14}$ B) $\frac{9}{14}$ C) $\frac{14}{5}$ D) $\frac{14}{9}$

49. From this box–and–whisker plot, identify the range, median, interquartile range, and outliers (in that order).



A) 180, 100, 160, none B) 180, 200, 160, 40 C) 100, 160, 180, 220 D) 180, 160, 100, none

50. From this stem–and–leaf plot, identify the range, median, and mode (in that order). 6 | 3 = 63

STEM	LEAF
4	1 2 5 8
5	1 2 4 4 4 8
6	3 5 8 9
7	4 5 5 6 9
8	3 6 7 7

A) 46, 54, 65 B) 42, 54, 65 C) 42, 65, 54 D) 46, 65, 54

The Euclidean Algorithm

The Euclidean algorithm can be used to solve linear Diophantine equations (one equation in two variables) by finding the greatest common factor (gcf) of the coefficients of the two variables. Finding the gcf of two numbers involves using repeated divisions until a remainder of zero is reached. An example will show the process.

Find the gcf of 1785 and 546. 1785 divided by 546 equals 3 with a remainder of 147 and is written like this: $1785 = 3(546) + 147$. Follow the arrows to complete the process.

$546 = 3(147) + 105$

$147 = 1(105) + 42$

$105 = 2(42) + 21$

$42 = 2(21) + 0$ The last non – zero remainder is the gcf. Thus, the gcf of 1785 and 546 is 21. This process is called the Euclidean Algorithm.

Solving one equation in two variables can be done by using this process as shown and then using it in reverse order, as shown in this example. Solve: $83x + 19y = 1$. First find the gcf of 83 and 19.

$83 = 4(19) + 7$ (a)

$19 = 2(7) + 5$ (b)

$7 = 1(5) + 2$ (c)

$5 = 2(2) + 1$ (d)

$2 = 2(1) + 0$ The gcf of 83 and 19 is 1.

Now go in reverse order. Solve (d) for the gcf like this: $1 = 5 – 2(2)$. Continue this process up through (a), solving each equation for the remainder and simplifying:

$1 = 5 – 2(2)$

$= 5 – 2(7 – 1(5))$

$= –2(7) + 3(5)$

$= –2(7) + 3(19 – 2(7))$

$= 3(19) + (–8)(7)$

$= 3(19) + (–8)(83 – 4(19))$

$1 = (–8)83 + 35(19)$ $x = –8$ and $y = 35$ Thus, a solution of $83x + 19y = 1$ is $(–8,35)$.

Open End Question

1. Use the Euclidean Algorithm to find the gcf of 255 and 68.

2. Use the Euclidean Algorithm to solve the equation $93x + 38y = 1$.

AATM 2014 STATE ALGEBRA CONTEST NAME – _____

SCHOOL – _____ SCORE – _____

USE CAPITAL LETTERS ONLY

1. _____	11. _____	21. _____	31. _____	41. _____
2. _____	12. _____	22. _____	32. _____	42. _____
3. _____	13. _____	23. _____	33. _____	43. _____
4. _____	14. _____	24. _____	34. _____	44. _____
5. _____	15. _____	25. _____	35. _____	45. _____
6. _____	16. _____	26. _____	36. _____	46. _____
7. _____	17. _____	27. _____	37. _____	47. _____
8. _____	18. _____	28. _____	38. _____	48. _____
9. _____	19. _____	29. _____	39. _____	49. _____
10. _____	20. _____	30. _____	40. _____	50. _____

KEY

1 C
2 C
3 B
4 D
5 B
6 A
7 C
8 C
9 A
10 D
11 B
12 D
13 C
14 C
15 A
16 B
17 B
18 D
19 A
20 A
21 C
22 B
23 D
24 D
25 C
26 A
27 C
28 A
29 C
30 D
31 B
32 A
33 B
34 B
35 D
36 C
37 C
38 C
39 C
40 B
41 C
42 D
43 A
44 A
45 B
46 D
47 C
48 A
49 D
50 D

Open end question key and grading rubric

1. $255 = 3(68) + 51$ (award 1 point)

   $68 = 1(51) + 17$ (award 1 point)

   $51 = 3(17) + 0$ (award 1 point)

   The gcf of 255 and 68 is 17. (award 1 point)

2. $93 = 2(38) + 17$

   $38 = 2(17) + 4$

   $17 = 4(4) + 1$

   $4 = 4(1) + 0$

   $1 = 17 - 4(4)$ (award 1 point)

   $= 17 - 4(38 - 2(17))$ (award 1 point)

   $= -4(38) + 9(17)$ (award 1 point)

   $= -4(38) + 9(93 - 2(38))$ (award 1 point)

   $1 = 9(93) - 22(38)$ (award 1 point)

   A solution to $93x + 38y = 1$ is $(9, -22)$ (award 1 point)