Selling with Evidence*

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Abstract

We study how to optimally sell a good in a bilateral asymmetric information monopoly setting with interdependent values when the informed seller can voluntarily and costlessly provide evidence about the good's characteristics. Equilibrium allocations are feasible and immune to deviations to any mechanism. We show that there is an ex-ante profit-maximizing selling procedure that is an equilibrium of the mechanism-proposal game. In contrast to posted price settings, information unravelling of product characteristics may fail even when all buyer types agree on the ranking of product quality.

Keywords: Informed principal; consumer heterogeneity; interdependent valuations; product information disclosure; mechanism design; certification.

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1 Introduction

Virtually all firms and businesses have more information about their products and services than their customers. In a seminal paper Akerlof (1970) shows that, in instances such as these where there is asymmetric information, perceived average quality drives the market price which leads to market failure, since sellers of high quality products are unwilling to sell at such a price. What if sellers can certify their quality? Sellers often offer hard information to their customers in the form of free samples, trial periods, review copies, third-party labels or stamps of approval. Viscusi (1978) argued that when certification is possible, it is high quality sellers who drive the market, since they have the biggest incentive to certify and to receive a high price. This market force leads to the well-known unravelling of information which–in the absence of other distortions–renders mandatory disclosure rules unnecessary.[1]

What happens when an informed seller can not only certify, but also employ more sophisticated selling procedures compared to simply posting a price? This paper answers this question. We consider a privately informed seller facing a buyer who has private information about his taste. Values are interdependent, so the seller's reservation value or cost and the buyer's valuation can depend both on the buyer's taste and on product characteristics. The seller knowing his information can provide evidence about product characteristics and can choose any selling procedure: a fixed price, an information fee followed by an acquisition fee, or any other sales contract, which we model as a mediated selling mechanism. This is the first paper to analyze an informed-principal problem where the principal has private certifiable information, and the agent has private (non-certifiable) information. We make no assumptions on how the seller's reservation value and the buyer's valuation depend on the type profile. Our model captures scenarios from Hoteling's pure horizontal differentiation model to a pure common value model where the seller's information is about the quality of the good.

Choosing a selling procedure at the interim stage (when the seller knows his type) can by itself signal some information to the buyer which can be desirable for a high-type seller but can hurt a low-type seller. So equilibrium mechanisms could potentially lead to outcomes where

[1]There is an extensive theoretical and empirical literature that studies certification and disclosure by firms. See the surveys by Dranove and Jin (2010) and Milgrom (2008) and references therein.

some seller types enjoy high profit while others do not. Equilibrium profits can be low also on average across seller types. This is a fundamental difference between this game and mechanism-selection by an uninformed seller-designer (as in Myerson, 1981) who can commit to the most profitable selling procedure. The analogue in our setup is the procedure that generates the highest expected-profit chosen by the seller at the ex-ante stage– before his private information is observed. This procedure is, at the interim stage, associated with a vector of profits, the *ex-ante optimal* ones. How do equilibrium selling procedures and profits of the informed seller mechanism selection game compare to the ex-ante optimal ones? When the seller can only post prices and has low certification costs, information unravels and there is a unique equilibrium outcome where each seller types has the same profit as if the buyer knew his information, the *the full-information* profit,[2] which is typically not *ex-ante optimal*. Is this still the case when the seller has access to certification and can choose any selling procedure?

This paper establishes that under own-type certifiability (when all seller types can be fully certified), there is always an ex-ante profit-maximizing selling procedure that is an equilibrium of the mechanism-proposal game. This result implies that the seller does not benefit from being able to commit even to the best mechanism before knowing his type. This is surprising, since, even if there is a highest quality seller type, he still does not benefit by deviating from this mechanism. Perhaps, more surprising is that we show that unravelling of information (which is associated with lower ex-ante profits for the seller compared to our equilibrium profit) may not be an equilibrium outcome, even when all buyer types agree on the ranking of seller types, when it is the *unique* equilibrium outcome under price-posting.

The ability of the seller to certify his information enlarges the set of feasible allocations since now types that cannot offer the same evidence cannot mimic each other. In other words, certifiability relaxes the seller's incentive compatibility constraints. At the same time, the ability to certify makes deviations more effective: A high quality seller, for example, can deviate from a selling procedure by providing evidence of his quality and by asking a high price. This force implies a necessary condition for equilibrium outcomes: Under own-type certifiability, no seller type can obtain an equilibrium profit below the full-information one–what he can guarantee by

[2]This terminology is introduced in Maskin and Tirole (1990) and refers only to complete information about the seller's type, not the buyer's.

providing full certification and by choosing a profit-maximizing mechanism.

The fact that certifiability relaxes the seller's incentive constraints for any belief of the buyer implies that the set of off-path feasible continuation equilibrium outcomes becomes larger. In particular, in the case of own-type certifiability any point on the Pareto frontier of all profit vectors arising from allocations that satisfy buyer-incentive and participation constraints for *some* buyer beliefs can arise as an off-path outcome. We call this frontier the set of strong Pareto optimal profit vectors, henceforth SPO.[3] This observation leads to the conjecture that if there is an SPO vector for the prior than this is an equilibrium outcome.

Relying on key ideas of Maskin and Tirole (1990), generalized by Mylovanov and Tröger (2012) for private-value setups, we establish existence of SPO for all priors in our interdependent value environment. Under own-type certifiability any SPO vector of profits is feasible for any prior and we can compare it with the ex-ante maximal profit. We establish that SPO allocations are indeed ex-ante profit-maximizing. Our main theorem establishes that if an SPO for the prior is feasible for the assumed certifiability structure, then it is an equilibrium. This, together with the previous observations, implies that there is an equilibrium of the mechanism proposal game that maximizes ex-ante expected profits.

Equilibrium outcomes are feasible *allocations* that are immune to deviations to any mechanism. We characterize all feasible selling allocations as a function of the seller's certification abilities. Then, we derive allocations that survive deviations to arbitrary mechanisms. In order to do so we formulate a canonical framework to study informed principal problems with certifiable information in transferable utility environments. The fact that mechanisms are general implies that a priori there is no belief that works for all deviations: off-path beliefs must be carefully chosen to render a deviation unprofitable. So the intricate part is to establish that for each conceivable deviation, there exist beliefs and a continuation equilibrium outcome given these beliefs that is not better for any type of the seller.

In trading environments where at an ex-ante optimal mechanism the seller can extract all surplus, we can establish that full-surplus extraction is an equilibrium by defining an appropriate zero-sum game and then relying on a min-max argument about the value of this game. The

[3]This frontier is precisely what Maskin and Tirole (1990) define as the set of SUPO (strong-unconstrained Pareto outcomes) for a private-value setup with soft information.

definition of the zero sum game "works" only if the conjectured equilibrium profit can arise at a mechanism where the good is assigned to the buyer for all type vectors. In general, to show that an SPO for the prior is an equilibrium, we rely on an appropriate belief-assigning correspondence[4] that maps the set beliefs and associated equilibrium outcomes to itself: For every conceivable deviation, the beliefs "chosen" by this correspondence assign only positive weight to types that benefit from deviating to this mechanism at a continuation equilibrium play for these same beliefs. Existence of this fixed point follows from Kakutani's Theorem. Our proof is simpler and more direct than the one in Maskin and Tirole (1990), as we just rely on the Pareto optimality property, while their argument rests in the Walrasian economy they consider.

This paper shows that flexibility in choosing the selling contract enables the seller to leverage his private information to increase profits compared with the unravelling case both from the interim and ex-ante perspective. Thus the economic nature of our results is quite distinct from results in the existing literatures on selling with certifiable information and on informed principal.

Related Literature In the papers of selling with certifiable information and posted prices (Grossman and Hart, 1980, Grossman, 1981, Milgrom, 1981, Koessler and Renault, 2012), information unravels in equilibrium and each seller type gets his full information profit. This insight has been central in growing literatures in finance and accounting. Full information profit is also the unique equilibrium outcome in informed seller games with private-values, quasilinear payoffs and type-independent outside options (see Proposition 11 in Maskin and Tirole, 1990 and the main proposition in Yilankaya, 1999). In our setup we allow for certifiable information and general mechanisms and the unravelling outcome may not be an equilibrium.

The literature on informed principal with interdependent values is quite thin. Maskin and Tirole (1992) analyze such a setup where information is soft and there is a "worst" type of principal–for example, the high-cost firm, or low-productivity worker. In Koessler and Skreta (2016) we examine a trading scenario where the seller's type affects the buyer's willingness to pay in an arbitrary way and the seller seeks to maximize revenue. We use an alternative

[4]The same one is used in Maskin and Tirole (1990).

mode of analysis, which leverages the fact that all seller-types seek higher revenue. We show that there is a continuum of Pareto-ranked equilibrium outcomes ranging from the worst-case scenario in terms of profits for the seller to a profit vector that is ex-ante profit-maximizing for the seller. Mylovanov and Tröger (2014) establish ex-ante optimality of equilibrium allocations in a private value setup under monotonicity and transferable utility. A key element of their setup is that the seller has no information that affects the agent's valuation.

The inefficiencies caused by having an informed party choose the selling procedure is the focus of De Clippel and Minelli (2004) who study a bargaining problem with bilateral asymmetric information and without transfers. The allocation (a mapping from types to a vector of payoffs) is chosen by one of the informed parties at the interim stage, but types are verifiable at the time of implementation, so it is as if the seller and the buyer are required to fully certify their type. Focusing, then, on a game protocol in which a proposed allocation is either accepted or rejected by the other party, they characterize the set of equilibrium outcomes allowing both parties to have some power to make a proposal.

To establish our results we rely on the general formulation of the informed principal problem of Myerson (1983) and extend it to allow the principal's information to be certifiable. Perhaps surprisingly, key methodological insights developed by Maskin and Tirole (1990) for a private-value setup with soft information have useful counterparts in our setup. Following the tradition of mechanism design with certifiable information (Green and Laffont, 1986, Forges and Koessler, 2005, Bull and Watson, 2007, Deneckere and Severinov, 2008, and Strausz, 2016), we take the certification structure as exogenous and taking it as a primitive, we find equilibrium selling procedures chosen by such an informed seller.[5] The advertizing literature, see, for example, Johnson and Myatt (2006) and Anderson and Renault (2006),[6] assumes that the firm is not privately informed when it designs (and commits to) its information disclosure rule. Sun (2011) and Koessler and Renault (2012) relax this and study information disclosure by an informed

[5]Most mechanism design literature assumes that the information structure is exogenous and the assumption that certification abilities are exogenous is in the same spirit. It captures well that, often, in reality the structure of available certificates is exogenous–takes the form of hygiene letter grades (A,B,C ...) for restaurants or multi-letter grades (AAA, AA+, BBB ...) for ratings of financial assets. Restaurants choose whether or not to reveal their certificate. Similarly, asset issuers can selectively disclose ratings. In this paper the seller decides what evidence to provide to the mechanism.

[6]See Renault, 2016, Section 3, for a comprehensive literature review.

firm at the interim stage, but, unlike this paper, focus on posted prices. Our results complement those, by demonstrating that when full certification of product characteristics is possible, there exists an ex-ante optimal selling procedure that is an equilibrium when the firm makes the choice knowing its type.

In broader terms, our work complements the literatures that study the interaction between an uninformed party (the principal–decision-maker) and an informed one (the agent–sender) who provides evidence to "persuade" the principal. Glazer and Rubinstein (2004) and Glazer and Rubinstein (2006) compare equilibria of a class of evidence games where the principal can only accept or reject based on evidence provided by the agent, and compare it to the situation where he can commit to the optimal acceptance rule (mechanism) before he sees the evidence. They show that is some cases commitment does not increase the principal's payoff. The usefulness of commitment in analogous evidence games is further investigated and generalized by Hart, Kremer, and Perry (2016).

Our setup differs in several key dimensions from the aforementioned ones, but a comparison is illuminating since both approaches "speak" to related economic forces. The seller-buyer relationship in the setup of Hart et al. (2016) is so that the agent (seller) has the evidence and all seller types want the buyer to pay a price that is as high as possible. The buyer–the principal in their model–wants to pay a correct price and decides what to quote either after the evidence is provided or can commit to a rule before he observes the evidence. In our paper the evidence is in the hands of the informed seller (the principal) who can choose any procedure that takes as input both evidence (and reports) from the seller and "soft" reports from the buyer. The procedure uses evidence and reports to determine the terms of trade.

Our model is simple but quite general so the results are relevant in a large number of situations, some of which are in areas of active debate on whether or not policy interventions are necessary. Clearly, sellers often use more complex contracts and selling procedures than posted prices and have access to certifiable information that can be used to attract customers. A seller giving a free sample, or a free trial period cannot damage consumers (unless the seller is giving an addictive substance!), however a pharmaceutical company selectively disclosing results of clinical trials to induce future purchases can be harmful. Likewise, keeping some evidence secret (for example, withholding an unfavorable accounting document) in company and asset sales could damage potential acquirers. The earlier papers that predict market unravelling tell us that market forces induce the revelation of all certifiable information making mandatory disclosure rules or penalties for withholding information unnecessary. Our results highlight that even if all information and evidence is of a "vertical" nature, the seller may choose selling procedures that keep consumers in the "dark" and actually could lead to ex-post regret.

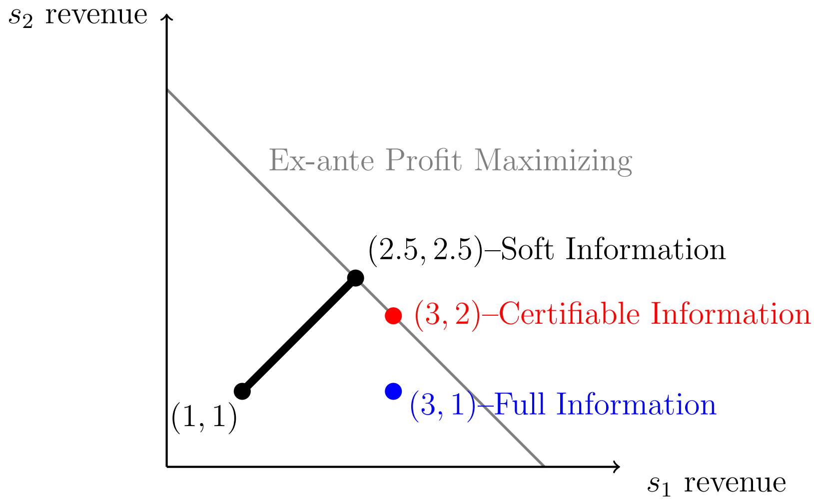
2 Motivating Example

To get a flavor of the setup and the results, consider a seller, whose good can be one of two types s_1 or s_2, facing a single buyer whose taste is equally likely to be t_1 or t_2 and whose valuation for the product is described in the following matrix:

$u(s,t) =$

	t_1	t_2
s_1	5	3
s_2	1	2

In this example the seller's reservation value is 0 (hence, only cares about revenue) and can certify quality at no cost. Observe that s_1 is the high quality product, while a t_1 consumer values quality more than t_2. When the consumer knows whether the quality is s_1 or s_2, the profit-maximizing selling procedure is for s_1 to ask a price of 3 and for s_2 to ask a price of 1 or 2, yielding interim profits $(V(s_1), V(s_2)) = (3, 1)$–this is the full-information outcome.



As mentioned in the introduction, Koessler and Renault (2012) establish that under own-type certifiability when the buyer's valuation function is "pairwise monotonic"–as is the case in this example–unravelling forces make $(V(s_1), V(s_2)) = (3, 1)$ the *unique* equilibrium profit with certifiable information and posted prices. In fact, this result holds for any certifiability structure where s_1 has a piece of evidence not available to s_2. When the seller cannot certify quality (information is "soft"), but can employ any selling procedure, Koessler and Skreta (2016) show that there is a continuum of equilibrium interim profit vectors described by the line segment along the 45-degree line between the "best safe" and the ex-ante optimal one. The "best safe" profit is 1 independent of the buyer's beliefs whereas the ex-ante optimal one depends on the prior about the seller's type (it varies from 1 when the belief is $\pi(s_1) = 0$, to 3 when $\pi(s_1) = 1$). For example, when $\pi(s_1) = \frac{1}{2}$ the set of equilibrium interim profit vectors is the line segment connecting $(1, 1)$ and $(2.5, 2, 5)$.

What are the equilibrium profits when the seller's information is certifiable and he can employ any general selling procedure? Clearly, for any certifiability structure where s_1 has a piece of evidence not available to s_2, each seller type can guarantee its full information profit. So a lower bound on equilibrium profit vectors is the vector $(3, 1)$. We show that for all such certifiability structures and for all prior buyer beliefs that assign probability greater than $\frac{1}{3}$ to the seller's type being s_1, that is $\pi(s_1) > \frac{1}{3}$, $(V(s_1), V(s_2)) = (3, 2) > (3, 1)$ is an equilibrium profit vector.

To achieve the outcome $(3, 2)$ the seller proposes an "evidence-conditional" contract: If the buyer accepts the contract, he has to pay 3 if the seller presents the evidence s_1 and otherwise 2. The buyer is willing to accept this contract because he does not know whether he will have to pay 3 or 2 and the expected payment is below the expected valuation for both of his types. This contract implements the following allocation

$$(p, x)(s, t) = \begin{array}{|c|c|c|} \hline & t_1 & t_2 \\ \hline s_1 & 1,3 & 1,3 \\ \hline s_2 & 1,2 & 1,2 \\ \hline \end{array}$$

where p is the probability of trade and x is the payment as a function of each type profile (s, t). Note that this profit-maximizing allocation is ex-post efficient.[7]

3 Setting and Definitions

3.1 The Trading Problem

Consider a monopoly seller with one indivisible good facing a single buyer with unit demand. The seller has perfect and private information about the *product's characteristics*, denoted by $s \in S$ and also called the *type of the seller*. The buyer has perfect and private information about his *taste*, denoted by $t \in T$ and also called the *type of the buyer*. The type space $S \times T$ is finite and types are independently distributed, with strictly positive probability distributions $\pi^0 \in \Delta(S)$ and $\tau \in \Delta(T)$. The buyer's valuation for the product is denoted by $u(s, t) \in \mathbb{R}$. The seller's reservation value is denoted by $v(s, t) \in \mathbb{R}$.[8]

An *allocation* is given by $p : S \times T \to [0, 1]$ and $x : S \times T \to \mathbb{R}$, where $p(s, t)$ is the probability of trade and $x(s, t)$ is the transfer from the buyer to the seller. We assume that transfers lie in

[7] If $u(s_2, t_1) = 0.999$ instead of $u(s_2, t_1) = 1$, (3,2) is still an efficient equilibrium outcome and (3,1) is still the unravelling outcome–however now s_2 strictly prefers to ask a price of 2 and sell only to t_2, which sacrifices efficiency for profit. The friction that causes this is that the buyer is also privately informed. So in the modified example, the unravelling outcome is less efficient compared to the ex-ante optimal equilibrium mechanism, however examples with the reverse feature are possible.

[8] The reservation value $v(s, t)$ could equivalently be interpreted as the constant marginal cost of the seller for delivering the good.

a compact and convex set: $x(s,t) \in [-\mathcal{X}, \mathcal{X}]$ for every s and t, where $\mathcal{X}$ is large.[9]

Both the seller and the buyer are risk-neutral. Given an allocation (p, x), the *seller's profit* and the *buyer's utility* are

$$V(s,t) = x(s,t) - p(s,t)v(s,t), \quad \text{and} \quad U(s,t) = p(s,t)u(s,t) - x(s,t).$$

The seller's interim expected profit is

$$V(s) \equiv \sum_{t \in T} \tau(t) V(s,t).$$

When writing the buyer's interim expected utility we keep track of his beliefs since, in the mechanism-proposal game we consider, they can potentially differ from the prior along and off the equilibrium path. We then let, for every $\pi \in \Delta(S)$, $U_\pi(t) \equiv \sum_{s \in S} \pi(s) U(s,t)$.

3.2 Mechanism-Proposal Game

We analyze an informed-principal game in which the seller selects a mechanism after he has learned his type. Such a mechanism specifies a probability of trade and a transfer as a function of (cheap talk) messages sent by both the seller and the buyer. In addition to these standard messages, the seller is able to certify some information by providing evidence about product characteristics at no cost. This certification ability is exogenous and represented by a *certifiability structure* $\mathcal{E} \subseteq 2^S$ which stands for the set of events that the seller is able to certify. Let $\mathcal{E}(s) = \{E \in \mathcal{E} : s \in E\}$ be the set of such events when the seller's actual type is $s \in S$. When information is not certifiable we have $\mathcal{E}(s) = \{S\}$ for every $s \in S$.[10] We assume that $\mathcal{E}(s) \neq \emptyset$ for every $s \in S$. Following Forges and Koessler (2005), we also assume that $\mathcal{E}$ is closed under intersection,[11] which means that each seller type s has the ability to certify as many events in

[9]Maskin and Tirole (1990) and Mylovanov and Tröger (2012) make an analogous assumption. It ensures that the set of allocations is compact, which is used in the existence proof of Proposition 2.

[10]Using a certifiability structure is equivalent to using any abstract message correspondence $\mathcal{Z} : S \rightrightarrows Z$ by letting $\mathcal{E}(s) = \{\mathcal{Z}^{-1}(z) : z \in \mathcal{Z}(s)\}$. The set $\mathcal{Z}^{-1}(z)$ is the set of seller types who can send message z, so $\mathcal{Z}^{-1}(z)$ is the event that message z certifies.

[11]This property is called "minimal closure condition" in Forges and Koessler (2005) and "normality" in Bull and Watson (2007).

$\mathcal{E}(s)$ as he wants. A certifiability structure satisfies *own type certifiability* if $\{s\} \in \mathcal{E}$ for every $s \in S$.

The timing of the Mechanism-Proposal Game is as follows:

1. Nature selects the seller's type, $s \in S$, according to the probability distribution $\pi^0 \in \Delta(S)$, and the buyer's type, $t \in T$, according to the probability distribution $\tau \in \Delta(T)$;

2. The seller is privately informed about $s \in S$ and the buyer is privately informed about $t \in T$;

3. The seller proposes a mechanism, consisting of (finite) sets of cheap talk messages M_S for the seller and M_T for the buyer, and a function

$$\mathcal{M} : \mathcal{E} \times M_S \times M_T \to [0,1] \times [-\mathcal{X}, \mathcal{X}],$$

   which specifies a probability of trade and a selling price as a function of the event $E \in \mathcal{E}$ certified by the seller, the cheap talk message $m_S \in M_S$ of the seller and the cheap talk message $m_T \in M_T$ of the buyer;

4. The seller and the buyer observe a uniformly distributed public signal in $[0,1]$.[12] The seller certifies an event $E \in \mathcal{E}$ and submits a cheap talk message $m_S \in M_S$ to the mechanism. Simultaneously, the buyer decides to reject or to accept the mechanism and sends a message $m_T \in M_T$ to the mechanism.

The mechanism $\mathcal{M}$ and the reporting and participation strategies implement an allocation (p, x). The default allocation of no trade and no payment arises if the buyer rejects, in which case both players' payoff is zero.

An allocation (p, x) is *feasible* if there exists a mechanism $\mathcal{M}$, and reporting and participation strategies that implement the outcome (p, x) and form a Bayes-Nash equilibrium given $\mathcal{M}$. An allocation (p, x) is an *expectational equilibrium* (Myerson, 1983) iff it is *feasible* and no type of seller can benefit from deviating to any mechanism: for every deviation to a mechanism $\tilde{\mathcal{M}}$,

[12]This is to ensure convexity of the continuation equilibrium profits given $\mathcal{M}$, which is used in the proof of Theorem 1.

there exists a belief $\tilde{\pi} \in \Delta(S)$ for the buyer, reporting and participation strategies that form a continuation Nash equilibrium given $\tilde{\mathcal{M}}$ and $\tilde{\pi}$, with outcome $(\tilde{p}, \tilde{x})$, such that $V(s) \geq \tilde{V}(s)$ for every $s \in S$.[13]

Remark 1 In our formulation evidence are submitted to the mechanism, not directly to the buyer. This is without loss of generality when transfers are available. To see this, consider an alternative formulation in which evidence can be submitted directly to the buyer. Precisely, consider a seller type s who provides evidence $E \ni s$ to the buyer before proposing a mechanism which is accepted by the buyer. This can be captured by a mechanism $\mathcal{M}$ where every type $s \in E$ is assigned the same outcome and all other seller types pay the buyer positive and constant transfers. In that way, the mechanism is still accepted and the buyer's beliefs about types $s' \notin E$ are irrelevant for incentive compatibility. This argument applies both on and off path, so the seller cannot induce new continuation equilibrium profits by offering evidence prior to proposing the mechanism.

Our goal is to characterize equilibrium profit vectors of this game. In order to do that we first characterize feasible allocations and then identify those who are immune to deviations.

3.3 Feasible Allocations

To characterize feasible allocations, we follow the revelation principle in Forges and Koessler (2005). Let $E^*(s) = \bigcap_{E \in \mathcal{E}(s)} E$ be the smallest event that the seller is able to certify when his actual type is s. The fact that $\mathcal{E}$ is closed under intersection ensures that $E^*(s)$ is certifiable by the seller when his type is s, i.e., $E^*(s) \in \mathcal{E}(s)$.

From the certifiability structure $\mathcal{E}$ we uniquely define the *reporting correspondence* of the seller as $R : S \rightrightarrows S$, with

$$R(s) \equiv \{\tilde{s} \in S : E^*(\tilde{s}) \in \mathcal{E}(s)\}.$$

The set $R(s)$ represents all seller types in S that type s is able to mimic when these types certify all information they can.

[13] By the inscrutability principle (see Myerson, 1983), an expectational equilibrium is equivalent to the strong version of Perfect Bayesian Equilibrium which imposes that all buyer types have the same off-path beliefs after a deviation (recall that we assume that types are independently distributed).

The following proposition uses the revelation principle with partially certifiable types to characterize all feasible allocations in a canonical way. It is similar to the revelation principal without certifiable information. The only difference is that both players are not only required to send a truthful cheap talk claim to the mechanism, but the seller is also required to certify as much information as he can about his type; that is, the buyer and the seller each privately make a truthful report about their type t and s respectively, and, in addition, the seller provides maximal evidence by privately certifying $E^*(s)$ to the mechanism. For a given allocation (p, x), let $V(s' \mid s) \equiv \sum_{t \in T} \tau(t)\big(x(s', t) - p(s', t)v(s, t)\big)$ be the seller's interim expected profit when his type is s but gets the allocation of s', and $U_\pi(t' \mid t) \equiv \sum_{s \in S} \pi(s)\big(p(s, t')u(s, t) - x(s, t')\big)$ be the buyer's interim expected utility when his actual type is t but gets the allocation of t'.

Proposition 1 *An allocation (p, x) is feasible for beliefs π given the certifiability structure $\mathcal{E}$ if and only if the following incentive compatibility and participation constraints are satisfied:*

$$V(s) \geq V(s' \mid s), \ \textit{for every } s \in S \textit{ and } s' \in R(s); \tag{S-IC}$$

$$V(s) \geq 0, \ \textit{for every } s \in S; \tag{S-PC}$$

$$U_\pi(t) \geq U_\pi(t' \mid t), \ \textit{for every } t, t' \in T; \tag{B-IC}$$

$$U_\pi(t) \geq 0, \ \textit{for every } t \in T. \tag{B-PC}$$

Proof. The proof directly follows the revelation principle with partially certifiable types in Forges and Koessler (2005).[14] ■

This proposition implies that the set of feasible allocations is the same for all certifiability structures that are associated with the same reporting correspondence R. This observation implies that the set of feasible allocations is the same under any certifiability structure satisfying own-type certifiability. Note that, under own-type certifiability, we have $R(s) = \{s\}$ so there is no informational incentive constraint for the seller, i.e., (S-IC) is always satisfied.

Compared to the standard setting without certifiable information, certifiability extends the

[14]See also Green and Laffont (1986), Bull and Watson (2007), Deneckere and Severinov (2008), or Strausz (2016) for similar versions of the revelation principle.

set of feasible allocations because it relaxes the seller's incentive constraints. This enlarges the set of *potential* equilibrium profit vectors. However, since it does so regardless of the buyer's beliefs π, it also enlarges the set of continuation equilibrium profit vectors. Hence, the effect of certifiability on the set of *actual* equilibrium outcomes is not a priori clear.

4 Ex-Ante, Full-Information, Pareto and Strong Pareto Optimal Allocations

Before proceeding with equilibrium analysis, we define a number benchmarks against which we compare equilibrium outcomes in terms of seller profit. We also clarify the relationship between these benchmarks.

Definition 1 An allocation (p, x) is *ex-ante optimal* for buyer beliefs π if it maximizes the ex-ante expected profit $\sum_{s \in S} \pi(s)V(s)$ under the incentive compatibility and participation constraints (S-IC), (S-PC), (B-IC) and (B-PC).

Definition 2 An allocation (p, x) is *full-information optimal* if for every $s \in S$ it maximizes the interim profit $V(s)$ under the following ex-post incentive compatibility and participation constraints of the buyer:

$$U(s,t) \geq p(s,t')u(s,t) - x(s,t'), \text{ for every } t, t' \in T; \tag{1}$$

$$U(s,t) \geq 0, \text{ for every } t \in T. \tag{2}$$

In other words, an ex-ante optimal allocation results from a profit-maximizing mechanism chosen by the seller before learning his type, while a full-information optimal allocation results from profit-maximizing mechanisms chosen by the seller when his type is commonly known. The corresponding ex-ante (interim) profit is called the *ex-ante optimal* and *full-information ex-ante (interim) profit*.

Note that a full-information optimal allocation does not depend on the certifiability structure and, in general, there is no reason for such an allocation to be feasible. However, it is clearly feasible under own-type certifiability. Note also that if $v(s,t)$ does not depend on t, then (one of) the full-information optimal allocation is simply a posted price (see Myerson, 1981; Riley and Zeckhauser, 1983). Finally, note that if a full-information optimal allocation is feasible, then it is feasible whatever the belief of the buyer: it is *safe* according to the terminology of Myerson (1983).

Fix the buyer's beliefs π and consider vectors of profits $(\hat{V}(s))_{s\in S}$ associated with allocations that satisfy (B-IC) and (B-PC)–the interim incentive and participation constraints of the buyer given π. We proceed to define the Pareto frontier of all such profit vectors when beliefs vary–the set of *strong Pareto optimal (SPO) profits*. We then establish that there is a point on this frontier for every belief.

Definition 3 An allocation (p,x) is a *Pareto optimum (PO) with belief* $\pi \in \Delta(S)$ if there exists $w \in \Delta(S)$ such that (p,x) maximizes $\sum_{s\in S} w(s)V(s)$ under (B-IC) and (B-PC).

A full-information allocation is always a PO with degenerate beliefs ($w(s) = \pi(s) = 1$ for some $s \in S$). Also, under own-type certifiability an ex-ante optimal allocation is a PO with belief $\pi = \pi^0$ and weights $w = \pi^0$.

Let $\mathcal{V}^{PO}(\pi) \subseteq \mathbb{R}^{|S|}$ be the set of Pareto optimal interim profits with belief $\pi \in \Delta(S)$ and $\mathcal{V}^{PO} = \bigcup_{\pi\in\Delta(S)} \mathcal{V}^{PO}(\pi)$ the set of all Pareto optimal interim profits.

Definition 4 The set of *strong Pareto optimal interim profits*, denoted by $\mathcal{V}^{SPO}$, is the set of Pareto optimal interim profits $V^* \in \mathcal{V}^{PO}$ such that (i) there is no $\pi \in \Delta(S)$, $V \in \mathcal{V}^{PO}(\pi)$ such that V strictly Pareto dominates V^*, and (ii) there is no $\pi \in \operatorname{int}\Delta(S)$, $V \in \mathcal{V}^{PO}(\pi)$ such that V Pareto dominates V^*.

That is, $\mathcal{V}^{SPO}$ is the outer envelope of the interim PO profits locus as beliefs vary. Let $\mathcal{V}^{SPO}(\pi) \subseteq \mathbb{R}^{|S|}$ be the set of strong Pareto optimal interim profits with belief $\pi \in \Delta(S)$.

Remark 2 The definitions of PO and SPO profits have been introduced by Maskin and Tirole (1990) (they call them (strong) unconstrained Pareto optimal profits). They showed that SPO profits are equilibrium profits under private-values and soft information.

Proposition 2 *There exists at least one SPO for every* $\pi \in \Delta(S)$.

Proof. See the online appendix. ∎

The proof relies on a key insight of Maskin and Tirole (1990). They observe that SPO profits correspond to Walrasian equilibrium outcomes of a fictitious competitive economy in which different seller types are trading slacks for the ex-post incentive compatibility and participation constraints of the buyer. Existence of a Walrasian equilibrium relative to some belief π in such an economy follows from general equilibrium theory. In Maskin and Tirole (1990) there are transfers, two possible agent types and the agent's payoff is monotonic in his type. These conditions imply at allocations associated with SPO vectors, the incentive constraint of the "high" type and the participation constraint of the low type bind. They leverage this observation to "predict" which goods are traded and hence have strictly positive prices. Then, restricting attention to these "goods," Walras' law holds and existence of an SPO for all priors follows from standard general equilibrium arguments. Mylovanov and Tröger (2012) show that, without these assumptions, SPO existence may fail.[15]

In our setup we have interdependent values so it seems even less clear whether the SPO set is useful. In addition its existence seems a priori more delicate. However the only substantive difference is that each "trader" has a different set of available goods, since different seller types deliver different utility to the buyer, so they need different slacks to achieve feasibility. This difference does not add any difficulty in the proof of existence and Maskin and Tirole (1990)'s idea of the fictitious economy can still be used. In our proof, we rely on the generalized approach of Mylovanov and Tröger (2012)–who consider private-value setups–and establish existence in our interdependent value environment. In our model there are transfers which enter linearly the buyer's payoff. This linearity allows us to establish that the "endogenous" utility function of the traders in the fictitious economy is continuous. In addition, since transfers are available, no trader is satiated, which allows us to show that Walras' law holds for all traders (seller types) although we can have several goods with zero prices. Unlike Maskin and Tirole (1990), the generality of our setup does not allow us to predict which constraints bind.

Note that the nature of SPO profits vectors differs quite dramatically between private- and interdependent value environments: Maskin and Tirole (1990) established that in private-value

[15]Mylovanov and Tröger (2012) introduce a useful concept, the one of neologism-proof allocations that always exists in their general private-value model.

setups with quasi-linear utilities, SPO profits coincide with full information profits (no benefit of information privacy for the seller). This is not the case when values are interdependent as the following example illustrates.

Example 1 In the introductory example, the PO interim profits are given by the dashed grey area in Figure 1. The SPO interim profits are the red segments going through the points $(\mathcal{X}, 1)$, $(5, 1)$, $(3, 2)$ and $(3, \mathcal{X})$. The only SPO profits with interior beliefs are points on the segment $[(5, 1) - (3, 2)]$, obtained with $\pi = 1/3$. Note that, among the SPO profits, only $(3, 2)$ is feasible for the prior. Interestingly, the full-information vector of interim profits $(3, 1)$ is not SPO, and $(4, 1)$ is an ex-ante optimal vector of interim profits, it is above the full-information profits, but it is not SPO.

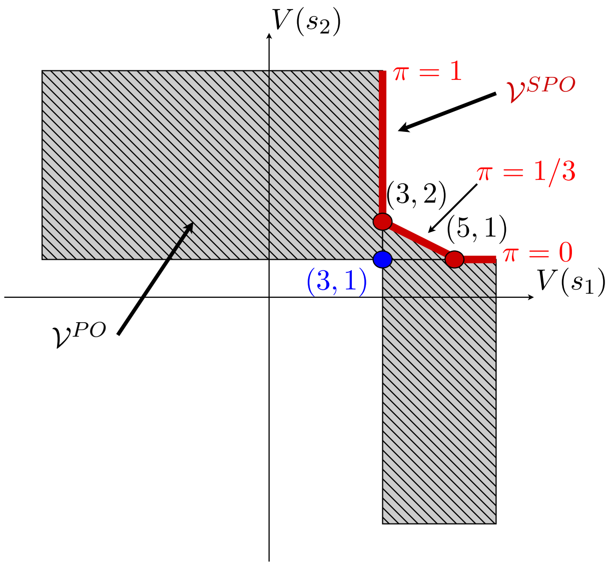


Figure 1: PO and SPO in the Example

◇

5 Finding Equilibria

5.1 Simple Illustration

To get a sense of how to show that an SPO allocation is an expectational equilibrium consider the following trivial example: The buyer has no private information and his valuation is 0 regardless of the seller's type. The seller has two equally likely types s_1 and s_2 that he must fully certify ($\mathcal{E} = \{\{s_1\}, \{s_2\}\}$), and a reservation value of zero. Obviously, if there is an equilibrium in this example it should give both seller types zero.[16] To show that there exists such an equilibrium, we have to demonstrate that no matter what mechanism the seller deviates to, there exists a belief of the buyer and an equilibrium play given this belief, that results in a vector of interim profits that it is not preferred by any type of the seller compared to (0,0).

Consider a deviation to the following mechanism where the buyer has only one message ($M_T = \{t_1\}$):

$\mathcal{M}^{direct}(s,t) =$

	t_1
$\{s_1\}$	p_1, x_1
$\{s_2\}$	p_2, x_2

The buyer either accepts or rejects such a deviation. It is easy to see there is a range of beliefs that supports the equilibrium outcome $(0, 0)$: For example, if $x_1, x_2 > 0$, then the buyer rejects the direct mechanism regardless of his beliefs; if $x_1, x_2 < 0$, then the buyer accepts whatever his belief, but the profit of the seller is negative whatever his type; if $x_1 > 0$ but $x_2 < 0$ ($x_1 < 0$ but $x_2 > 0$, respectively), then the buyer rejects if he believes that it is sufficiently likely that the seller's type is s_1 (s_2, respectively).

Now consider the situation in which the seller deviates to the following mechanism, where

[16] The arguments below used to show that $(0, 0)$ is indeed an equilibrium can be extended to any situation in which full-surplus extraction by the seller is feasible.

the set of messages for the buyer is $M_T = \{\text{Left}, \text{Right}\}$:[17]

$$\mathcal{M}^{LR} = \begin{array}{|c|c|c|} \hline & \text{Left} & \text{Right} \\ \hline \{s_1\} & p_{1L}, x_{1L} & p_{1R}, x_{1R} \\ \hline \{s_2\} & p_{2L}, x_{2L} & p_{2R}, x_{2R} \\ \hline \end{array}$$

Let π be the belief of the buyer on type s_1. If, for example, $x_{1L} = x_{2R} = -x_{1R} = -x_{2L} = x > 0$, then the buyer never rejects whatever his belief: he chooses Right if $\pi > 1/2$ and Left if $\pi < 1/2$. The deviation is therefore strictly profitable for seller type s_2 if $\pi > 1/2$, and for seller type s_1 if $\pi < 1/2$. To make the deviation not profitable for any seller type, the buyer's belief should be exactly $\pi = 1/2$, in which case he is indifferent between Left and Right and he should randomize equally between the two messages, so that the seller's interim profit is $\frac{1}{2}(x, -x) + \frac{1}{2}(-x, x) = (0, 0)$.

More generally, to find a belief and reporting strategy that deter deviation to any mechanism $\mathcal{M}^{LR}$, consider the auxiliary zero-sum game

$$\begin{pmatrix} x_{1L} & x_{1R} \\ x_{2L} & x_{2R} \end{pmatrix}$$

and let ϕ be the value of this game. Using basic properties of zero-sum games, it is immediate to conclude that, if $\phi \geq 0$, then there exists π such that the buyer rejects; likewise, if $\phi < 0$, then there exists π and an optimal reporting strategy for the buyer such that the seller's interim profit is lower than ϕ for every s.

In general, we cannot define an appropriate zero sum game, since some equilibrium profit vectors can be only achieved by mechanisms that restrict trade for some type realizations. This is where the concept of SPO comes in.

Figure 2 illustrates a few lines of interim profits achievable by various mechanisms as we vary the buyer's belief π. Because the buyer's valuation is zero and he can always reject, we cannot achieve strictly positive vectors of profits. The upper contour set of all achievable

[17] The arguments extend to any number of messages.

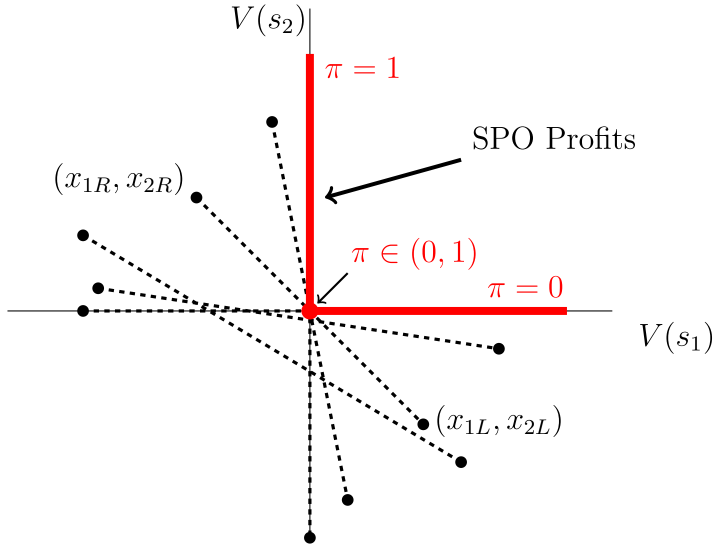


Figure 2: Seller's Interim Profit Vectors

interim profits is represented in bold red as a function of the belief π. This is precisely the set of SPO profits.

To see this, note that in this example (V_1, V_2) is a PO interim profit vector if and only if it maximizes $wV_1 + (1-w)V_2$ under the constraint $\pi V_1 + (1-\pi)V_2 = 0$ for some w, $\pi \in [0,1]$. This yields $\mathcal{V}^{PO} = ([-\mathcal{X}, 0] \times [0, \mathcal{X}]) \cup ([0, \mathcal{X}] \times [-\mathcal{X}, 0])$. The PO interim profits $(0,0)$ are obtained with $\pi = w = 1/2$, while the PO profits on the segments $[(0,0) - (0, \mathcal{X})]$ and $[(0,0) - (\mathcal{X}, 0)]$ are obtained only with extremal beliefs $\pi = 1$ and $\pi = 0$ respectively. Hence, $\mathcal{V}^{SPO} = \{((0,0), [(0,0) - (0,\mathcal{X})], [(0,0) - (\mathcal{X}, 0)]\}$, and the only SPO profits with interior beliefs are $(0,0)$–the full information outcome.

SPO profit vectors may be infeasible because by definition we ignore the seller's constraints. However, when they are feasible, as is the case under own-type certifiability, then they are equilibrium allocations. This is established next.

5.2 SPO allocations are Equilibrium allocations

Our main theorem establishes that under own-type certifiability, an SPO profit vector for the prior corresponds to an equilibrium outcome of the mechanism-proposal game.

Theorem 1 *Under own-type certifiability, every SPO allocation for the prior is an expecta-*

tional equilibrium of the mechanism-proposal game.

Proof. Let $(\hat{p}, \hat{x})$ be an SPO allocation for the (strictly positive) prior belief $\pi^0 \in \Delta(S)$. Let $\hat{V} = (\hat{V}(s))_{s \in S} \in \mathbb{R}_+^{|S|}$ denote the corresponding vector of interim profits.

We start by describing players' strategies along the equilibrium path. Assume that all seller types propose the same direct revelation mechanism[18]

$$\hat{\mathcal{M}} : \mathcal{E} \times T \to [0,1] \times \mathbb{R},$$

such that

$$\hat{\mathcal{M}} = \begin{cases} (\hat{p}, \hat{x})(s,t) & \text{if } E = \{s\}, \\ (0,0) & \text{otherwise.} \end{cases}$$

If the seller fully certifies his type and the buyer is truthful, then this mechanism implements the allocation $(\hat{p}, \hat{x})$. Since this proposal does not reveal any information about the seller's information, the buyer's belief continues to be the prior after the proposal. Also, since, by assumption, $(\hat{p}, \hat{x})$ is feasible (because it is SPO for the prior), the buyer accepts $(\hat{p}, \hat{x})$ and reports his type truthfully, and the seller has no incentive to deviate from full certification (because this would result in no trade and zero profit).

To show that proposing $\hat{\mathcal{M}}$ constitutes an expectational equilibrium we have to show that for any deviation to any generalized mechanism $\mathcal{M}$, there exist off-path beliefs for the buyer $\pi^* \in \Delta(S)$ such that there is an equilibrium outcome of $\mathcal{M}$ given these beliefs that yields an interim profit $V^*(s)$ for the seller type s that is not better than $\hat{V}(s)$ for every s. Since the continuation game induced by $\hat{\mathcal{M}}$ is finite, $\hat{\mathcal{M}}$ has at least one continuation equilibrium for any off-path belief $\pi \in \Delta(S)$. Let $\mathcal{V}(\pi)$ be the convex hull of the set of equilibrium profits of the principal when off-path beliefs are π.[19] Let $\mathcal{V} \subseteq \mathbb{R}_+^{|S|}$ be the convex hull of $\bigcup_{\pi \in \Delta(S)} \mathcal{V}(\pi)$.

[18]The corresponding set of cheap talk messages for the buyer is $\hat{M}_T = T$ and the set of cheap talk messages for the seller is a singleton. Notice that $\{s\} \in \mathcal{E}$ for every $s \in S$ because we assume own-type certifiability.

[19]In case of multiple equilibria, the random public signal observed by the seller and buyer before they play the mechanism $\mathcal{M}$ selects one.

For every profit vector $V = (V(s))_{s\in S} \in \mathcal{V}$ and belief $\pi \in \Delta(S)$ define the correspondence

$$(\pi, V) \to \arg\max_{\tilde{\pi}} \sum_{s\in S} \tilde{\pi}(s)(V(s) - \hat{V}(s)) \times \mathcal{V}(\tilde{\pi}). \tag{3}$$

The cross product of the belief and the profit sets, $\Delta(S) \times \mathcal{V}$, is convex and compact, and the correspondence is upper hemicontinuous and convex valued, so from Kakutani's fixed point theorem it has a fixed point $(\pi^*, V^*) \in \Delta(S) \times \mathcal{V}$. That is, there exists (π^*, V^*) such that $\pi^* \in \arg\max_\pi \sum_{s\in S} \pi(s)(V^*(s) - \hat{V}(s))$ and $V^* \in \mathcal{V}(\pi^*)$.

After any deviation to a mechanism $\mathcal{M}$, consider such an off-path belief π^* for the buyer, and the corresponding continuation equilibrium profit vector V^*. Let $I = \{s : V^*(s) > \hat{V}(s)\}$ and $J = \{s : V^*(s) \leq \hat{V}(s)\}$. First observe that $I \neq S$ because $\hat{V}$ is SPO. Assume by way of contradiction that I is nonempty; then $\pi^*_s = 0$ for all $s \in J$ because π^* maximizes $\sum \pi(s)(V^*(s) - \hat{V}(s))$. Since V^* is a continuation equilibrium profit given π^*, it is feasible given π^*. Hence the vector of interim profits $\tilde{V}$ with $\tilde{V}(s) = V^*(s)$ for $s \in I$ and $\tilde{V}(s) = \hat{V}(s) + \varepsilon > V(s)$ for $s \in J$ is also feasible given π^*. This implies that $\tilde{V}$ strictly Pareto dominates $\hat{V}$, and therefore $\hat{V}$ is not SPO, a contradiction. Thus, $I = \emptyset$, which means that V^* is not profitable compared to $\hat{V}$ for any seller type s. ■

In the following proposition we relate the ex-ante profit of an SPO allocation for beliefs π to the ex-ante optimal profits for the same belief and show that SPO profits are (weakly) higher. This result does not appear in Maskin and Tirole (1990), and the proof uses the fact that we assume that utilities are quasi-linear (an assumption that is not made in Maskin and Tirole, 1990).

Proposition 3 *Under own-type certifiability, every SPO allocation for the prior is ex-ante optimal.*

Proof. Let $\mathbf{V}(t) \equiv E_S V(s,t)$ and $\mathbf{V} \equiv E_{T,S} V(s,t)$. To prove the proposition, we show that if a vector of buyer π-feasible profits $(\hat{V}(s))_{s\in S}$ yields lower ex-ante expected profit than ex-ante optimal ones for π then it is not SPO. Let (p^*, x^*) be an ex-ante optimal allocation, with corresponding $(V^*(s))_{s\in S}$ and $\mathbf{V}^*$. If $(\hat{V}(s))_{s\in S}$ yields lower ex-ante profit we have $\mathbf{V}^* - \hat{\mathbf{V}} > 0$.

Let S_1 denote all seller types for which $\hat{V}(s) \geq V^*(s)$, and let S_2 be the complement of S_1. The set S_2 is non-empty because $\hat{V}$ is not ex-ante optimal.

Define an allocation $(\tilde{p}, \tilde{x})$ as follows:

$$\tilde{p}(s,t) = p^*(s,t) \text{ for all } s, t,$$

$$\tilde{x}(s,t) = \hat{V}(s) + p^*(s,t)v(s,t) \text{ for } s \in S_1, t \in T,$$

$$\tilde{x}(s,t) = \hat{V}(s) + p^*(s,t)v(s,t) + \frac{1}{\sum_{s' \in S_2} \pi^0(s')} \left[\mathbf{V}^*(t) - \hat{\mathbf{V}}\right] \text{ for } s \in S_2, t \in T.$$

Note that:

$$\tilde{V}(s,t) = \hat{V}(s) \text{ for } s \in S_1, t \in T,$$

$$\tilde{V}(s,t) = \hat{V}(s) + \frac{1}{\sum_{s' \in S_2} \pi^0(s')} \left[\mathbf{V}^*(t) - \hat{\mathbf{V}}\right] \text{ for } s \in S_2, t \in T.$$

The above two equations imply that $\tilde{V}(s) = \hat{V}(s)$ for $s \in S_1$ and $\tilde{V}(s) > \hat{V}(s)$ for $s \in S_2$ because $\mathbf{V}^* - \hat{\mathbf{V}} > 0$. The interim payment of the buyer at the allocation $(\tilde{p}, \tilde{x})$ is

$$\begin{aligned} E_S[\tilde{x}(s,t)] &= E_S[\hat{V}(s) + p^*(s,t)v(s,t)] + \left[\mathbf{V}^*(t) - \hat{\mathbf{V}}\right] \\ &= E_S[p^*(s,t)v(s,t)] + \mathbf{V}^*(t) = E_S[x^*(s,t)], \end{aligned}$$

so the resulting allocation $(\tilde{p}, \tilde{x})$ is feasible because (p^*, x^*) is. And it is better for all seller types, and strictly better for those in S_2, compared to $(\hat{V}(s))_s$. Hence, $(\hat{V}(s))_s$ is not SPO. Contradiction. ■

Proposition 3 states that under own-type certifiability, an SPO allocation for the prior is ex-ante profit-maximizing for the seller. However, as the introductory example illustrates only *one* ex-ante profit maximizing vector for the prior is SPO, namely the profit vector $(3, 2)$. The vectors $(4, 1)$ and $(2.5, 2.5)$ are ex-ante optimal but not SPO. We show in Section 6.2 that these vectors cannot be equilibria. In this sense, the Pareto optimality that characterizes SPO vectors functions like a "selection" to identify which ex-ante optimal vector is an equilibrium one.

Corollary 1 *Under own-type certifiability, there exists an ex-ante profit maximizing expecta-*

tional equilibrium.

Proof. Directly from Propositions 2, 3 and Theorem 1. ■

6 Extensions

6.1 Partial certifiability

Without own-type certifiability, an SPO allocation may not be be feasible: In the introductory example, the feasible allocations with soft information ($\mathcal{E} = \{S\}$) give the same interim profit to both seller types which is at most 2.5 for the prior. The intersection of this set with the SPO set is empty. However, if the certifiability structure is given by $\mathcal{E} = \{\{s_1, s_2\}, \{s_1\}\}$, then s_2 cannot mimic s_1 and the SPO vector of interim profits $(3, 2)$ is feasible. The next proposition shows that, if an SPO allocation for the prior is feasible under partial certifiability, then it remains an equilibrium of the mechanism-proposal game under partial certifiability.

Proposition 4 *If an SPO allocation for the prior is feasible under the certifiability structure $\mathcal{E}$, then it is an equilibrium of the mechanism-proposal game under $\mathcal{E}$.*

Proof. The proof is direct from the following simple observation. From Theorem 1, under full certifiability, i.e., if the certifiability structure is given by $\mathcal{E}^F = 2^S$, an SPO allocation for the prior is an equilibrium of the mechanism-proposal game. Since $\mathcal{E} \subseteq \mathcal{E}^F$, the set of possible deviations (mechanisms) of the seller in the mechanism-proposal game under $\mathcal{E}$ is strictly included in the set of possible deviations in the mechanism-proposal game under $\mathcal{E}^F$. Since the SPO allocation is feasible under $\mathcal{E}$, it remains an equilibrium under $\mathcal{E}^F$. ■

Together with Proposition 3 this proposition also implies that if an SPO allocation is feasible given some certifiability structure then it is an ex-ante optimal allocation.

6.2 Are all equilibria optimal?

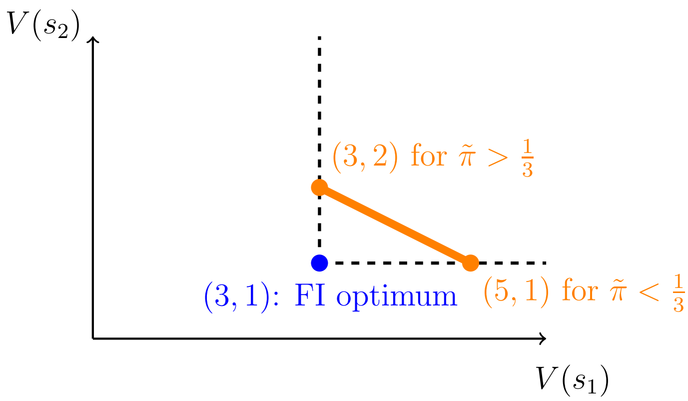
We have shown that being SPO is a sufficient condition for a vector of interim profits to constitute an equilibrium outcome of the mechanism-proposal game when the seller can provide enough evidence about his type. We have also shown that it is ex-ante optimal. If the seller were able to choose a mechanism before learning his type, he would clearly choose an ex-ante optimal mechanism. Hence, an SPO allocation has the strong property that it can be optimally chosen at the ex-ante stage, and it is immune to deviations by the seller at the interim stage. But are there other equilibria of the mechanism-proposal game that are not ex-ante optimal?

For the introductory example we now show that the SPO allocation is the unique equilibrium. This is in sharp contrast with equilibrium properties under "soft" information (Maskin and Tirole, 1992, Koessler and Skreta, 2016). Here, certifiability leads to a unique equilibrium outcome, which does not belong in the continuum of equilibrium outcomes under soft information. In this sense, certification increases the "power" of deviations.

To show that $(3, 2)$ is the unique vector of interim equilibrium profits for the uniform prior we show that every other feasible vector is dominated by the following deviation: the seller chooses the mechanism $\tilde{\mathcal{M}} : \mathcal{E} \times M_S \times M_T \to [0, 1] \times \mathbb{R}$ where M_S is a singleton, $M_T = \{\text{Left}, \text{Right}\}$ and

$\tilde{\mathcal{M}} =$

	Left	Right
$\{s_1\}$	$1, 5$	$1, 3$
$\{s_2\}$	$1, 1$	$1, 2$
$\{s_1, s_2\}$	$1, -10$	$1, -10$



In every continuation Nash equilibrium of $\tilde{\mathcal{M}}$, and for every buyer's belief $\tilde{\pi}$ (that $s = s_1$), the buyer never rejects the mechanism and the seller type s always certifies $\{s\}$. The buyer reports "Left" if $\tilde{\pi} < \frac{1}{3}$, he reports "Right" if $\tilde{\pi} > \frac{1}{3}$, and he is indifferent between the two reports when $\tilde{\pi} = \frac{1}{3}$. Hence, continuation interim equilibrium profits induced by $\tilde{\mathcal{M}}$ necessarily belong to the convex hull of $(3,2)$ and $(5,1)$, whatever the buyer's belief. Since only $(3,2)$ is feasible in this set, $(3,2)$ is the unique vector of interim equilibrium profits.

By varying the buyer's beliefs, the mechanism $\tilde{\mathcal{M}}$ above generates all SPO interim profits with interior beliefs. Hence, whatever the prior belief of the buyer, an equilibrium must be SPO, otherwise it is dominated by $\tilde{\mathcal{M}}$.

The interesting property of $\tilde{\mathcal{M}}$ is that for every belief, and no matter the equilibrium selected given that belief, the equilibrium outcome results in a profit vector on the frontier (SPO). Ideally we would like to design a game with such properties for any trading scenario covered in our model. That game would be a "canonical deviation" since a mechanism immune to that deviation would be immune to any deviation (since any alternative deviation can result at best at profit a vector on the frontier). We do not know if such a "canonical" (and finite) mechanism, generating the upper contour set of all feasible interim profits as beliefs vary, can be constructed in general. This is left for future research.[20]

[20]Maskin and Tirole (1990) get a uniqueness result but they have to assume a sorting assumption and to allow infinite mechanisms in an artificial extended game with a third player reporting the agent's beliefs to the principal.

6.3 Information certification by the buyer

In this paper we assumed that only the seller's type is certifiable, whereas the buyer's is not. This is a reasonable assumption for many applications when the buyer is privately informed about his taste for the different product types. What happens otherwise? If the seller and the buyer's information is perfectly certifiable in our environment, equilibrium outcomes are trivial: The seller can extract all surplus in each state by proposing the allocation $(p, x)(s, t) = (0, 0)$ if $v(s, t) \geq u(s, t)$, and $(p, x)(s, t) = (1, u(s, t))$ if $v(s, t) < u(s, t)$. It immediately follows that the resulting vector of interim profits is the unique equilibrium vector of interim profits.

A more general bargaining environment with full certifiability from the principal and the agent has been studied by De Clippel and Minelli (2004), without assuming transferable utilities. They show (their Proposition 1) that an allocation is an equilibrium allocation of the mechanism-proposal game if and only if, whatever the state, the interim expected payoffs of the principal and the agent are higher than their interim expected payoffs at the "best safe" allocation (i.e., the best allocation for the seller that satisfies ex-post participation constraints). With transferable utilities, the "best safe" allocation extracts all surplus in each state and cannot be dominated for the principal by any other feasible allocation (it is a strong solution in the sense of Myerson, 1983), so it is the unique equilibrium allocation. On the contrary, when utility is not transferable, the best safe allocation may be dominated (see Example 2 in De Clippel and Minelli, 2004). It would be interesting to study intermediate models in which information can only be certified by the principal but utilities are not transferable.

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A Online Appendix: Proof of Proposition 2

To show that for every $\pi \in \Delta(S)$ there exists a profile of strong Pareto optimal interim profits with belief π we follow Maskin and Tirole (1990) and Mylovanov and Tröger (2012). We define a fictitious competitive economy in which the seller's types are trading slacks on the ex-post incentive compatibility and participation constraints of the buyer. Existence of a Walrasian equilibrium relative to π in such an economy follows from standard arguments in general equilibrium theory (Lemma 2). Then, we show that a Walrasian equilibrium relative to π is SPO with prior π (Lemma 3). Therefore, we conclude that there exists at least one SPO with belief π for every $\pi \in \Delta(S)$.

Consider a fictitious economy in which the set of traders is the set S of seller's types. A bundle of goods for type s consists of a vector of slacks $(c(s,t), c(s,t,t'))_{t,t'}$, where $c(s,t) \in \mathbb{R}$ is a slack on the ex-post participation constraint of buyer type t, and $c(s,t,t') \in \mathbb{R}$, $t' \neq t$, are the slacks on the ex-post incentive constraints of buyer type t. Let $V_I(s \mid c)$ denote the indirect interim profit of seller type s when there are slacks $(c(s,t), c(s,t,t'))_{t,t'}$ in the ex-post incentive compatibility and participation constraints of the buyer:

$$V_I(s \mid c) = \max_{x(s,\cdot),p(s,\cdot)} \sum_{t \in T} \tau(t)(x(s,t) - p(s,t)v(s,t)),$$

under the constraints

$$p(s,t)u(s,t) - x(s,t) \geq p(s,t')u(s,t) - x(s,t') - c(s,t,t'), \text{ for every } t,t' \in T; \tag{4}$$

$$p(s,t)u(s,t) - x(s,t) \geq -c(s,t), \text{ for every } t \in T. \tag{5}$$

Let $C(s)$ be the (nonempty, closed and convex) set of slack vectors for which the feasible set of allocations $(x(s,\cdot), p(s,\cdot))$ of the maximization problem is nonempty.[21] The objective of the maximization problem is continuous and the feasible region is compact, and therefore from Weierstrass' theorem the solution value $V_I(s \mid c)$ exists for every $s \in S$ and for $(c(s,t), c(s,t,t'))_{t,t'} \in C(s)$.

Because (4) and (5) are linear, the feasible region of the maximization problem is continuous in the slacks. In addition, the objective is linear. Hence, from the Maximum Theorem, $V_I(s \mid c)$ is continuous and concave in the slacks.[22]

Initial endowment of each slack is 0. Given some exogenous prices $\gamma(t)$ and $\gamma(t,t')$ of the slacks $c(s,t)$ and $c(s,t,t')$, seller s's demand correspondence is given by

$$D(s \mid \gamma) \equiv \arg\max_{C(s)} V_I(s \mid c),$$

[21]Notice that, contrary to Maskin and Tirole (1990) and Mylovanov and Tröger (2012), this feasible region depends on the seller's type s because we do not assume private values (precisely, the buyer's valuation, $u(s,t)$, depends on s). However, this does not create any additional complication.

[22]In Mylovanov and Tröger (2012) the constraints are continuous but not necessarily linear in the allocations. Hence, they provide a more technical existence proof, which only uses the fact that $V_I(s \mid c)$ is upper semicontinuous.

subject to the budget constraint

$$\sum_{t\in T}\gamma(t)c(s,t)+\sum_{t,t'\in T}\gamma(t,t')c(s,t,t')\leq 0.$$

Lemma 1 *If* $(c(s,t),c(s,t,t'))_{t,t'}\in D(s\mid\gamma)$, *then* $\sum_{t\in T}\gamma(t)c(s,t)+\sum_{t,t'\in T}\gamma(t,t')c(s,t,t')=0$.

Proof. The lemma is the standard Walras's law, which holds for the same reason as in Maskin and Tirole (1990), and Mylovanov and Tröger (2012) (for non-satiated types).[23] If at the optimum for type s we have $\sum_{t\in T}\gamma(t)c(s,t)+\sum_{t,t'\in T}\gamma(t,t')c(s,t,t')<0$, then type s can increase the slacks of the participation constraints $c(s,t)$, $t\in T$, by a small constant, independently of t. Therefore increase the transfers $x(s,t)$, $t\in T$, by this same constant, independently of t, which would increase his indirect interim profit while still satisfying his budget constraint. ■

Definition 5 A *Walrasian equilibrium* relative to $\pi\in\Delta(S)$ is a vector of non negative prices $(\gamma(t),\gamma(t,t'))_{t,t'\in T}$ and slacks $(c(s,t),c(s,t,t'))_{s\in S,t,t'\in T}$ such that:

$$(c(s,t),c(s,t,t'))_{t,t'\in T}\in D(s\mid\gamma),\ \text{ for every } s\in S,$$

$$\sum_{s\in S}\pi(s)c(s,t)\leq 0,\ \text{ for every } t\in T, \tag{6}$$

$$\sum_{s\in S}\pi(s)c(s,t,t')\leq 0,\ \text{ for every } t,t'\in T. \tag{7}$$

The last two equations are the "market clearing" conditions, which ensure that a Walrasian equilibrium allocation satisfies the interim incentive and participation constraints of the buyer when the prior is π.

Lemma 2 *For any* $\pi\in\Delta(S)$ *there exists at least one Walrasian equilibrium relative to* π.

Proof. As in Maskin and Tirole (1990), the indirect interim profit $V_I(s\mid c)$ of each seller type s in the fictitious economy is continuous and concave, so existence follows by applying the techniques employed in Debreu (1959). We follow below the logic of the proof in Mylovanov and Tröger (2012).

Since transfers are bounded, we can replace $C(s)$ by a compact subset of slacks $C^*(s)\subset C(s)$. Hence, by the Maximum Theorem, $D(s\mid\gamma)$ is non-empty, compact-valued, and upper hemicontinuous in γ. It is also convex-valued because $V_I(s\mid c)$ is concave.

Let Δ be the simplex of price vectors, i.e., prices such that $\gamma(t),\gamma(t,t')\geq 0$ and $\sum_{t\in T}\gamma(t)+\sum_{t,t'}\gamma(t,t')=0$. Consider the correspondance $h:\prod_{s\in S}C^*(s)\to\Delta$, where

$$h(c)=\arg\max_{\gamma\in\Delta}\sum_{s\in S}\pi(s)\left(\sum_{t\in T}\gamma(t)c(s,t)+\sum_{t,t'\in T}\gamma(t,t')c(s,t,t')\right).$$

[23] In Mylovanov and Tröger (2012), this property only applies to traders who are not "satiated"; but since we have monetary transfers, at least one participation constraint of the buyer is always binding in our model.

The correspondence h is convex-valued, and by the Maximum Theorem it is upper hemicontinuous. Consider the correspondence

$$(\prod_{s\in S} C^*(s)) \times \Delta \to (\prod_{s\in S} C^*(s)) \times \Delta$$

$$(c, \gamma) \mapsto (\prod_{s\in S} D(s \mid \gamma)) \times h(c).$$

By Kakutani's theorem, this correspondence has a fixed point (c^*, γ^*). By construction we have $(c^*(s,t), c^*(s,t,t'))_{t,t'\in T} \in D(s \mid \gamma^*)$ for every $s \in S$. So, to show that (c^*, γ^*) is a Walrasian equilibrium it remains to show (6) and (7). Assume by way of contradiction that (6) fails (the same logic applies for (7)), i.e.,

$$\sum_{s\in S} \pi(s) c^*(s, \tilde{t}) > 0, \text{ for some } \tilde{t} \in T.$$

Consider the price vector γ such that $\gamma(\tilde{t}) = 1$ (and 0 for every other slack). This yields

$$\sum_{s\in S} \pi(s) \left(\sum_{t\in T} \gamma(t) c^*(s,t) + \sum_{t,t'\in T} \gamma(t,t') c^*(s,t,t') \right) = \sum_{s\in S} \pi(s) c^*(s, \tilde{t}) > 0.$$

But the budget constraints imply

$$\sum_{s\in S} \pi(s) \left(\sum_{t\in T} \gamma^*(t) c^*(s,t) + \sum_{t,t'\in T} \gamma^*(t,t') c^*(s,t,t') \right) \leq 0,$$

which yields a contradiction with $\gamma^* \in h(c^*)$. ■

Lemma 3 *Any Walrasian equilibrium outcome relative to π is SPO with belief π.*

Proof. Let $(\gamma(t), \gamma(t,t'))_{t,t'\in T}$ and $(c(s,t), c(s,t,t'))_{s\in S, t,t'\in T}$ be a Walrasian equilibrium relative to π, with interim profits $V_I(s \mid c)$, $s \in S$. Assume by way contradiction that it is not SPO; then, there exists a PO mechanism $(\hat{p}, \hat{x})$ with belief $\hat{\pi}$ such that

$$\hat{V}(s) \geq V_I(s \mid c), \tag{8}$$

for every $s \in S$, with a strict inequality for some s, and a strict inequality for every s if $\hat{\pi} \notin \operatorname{int} \Delta(S)$. Let $(\hat{c}(s,t), \hat{c}(s,t,t'))_{s\in S,t,t'\in T}$ be slack variables associated with $(\hat{p}, \hat{x})$, i.e., slacks such that

$$\sum_{s\in S} \hat{\pi}(s) \hat{c}(s,t) \leq 0 \text{ and } \sum_{s\in S} \hat{\pi}(s) \hat{c}(s,t,t') \leq 0, \text{ for every } t, t' \in T. \tag{9}$$

Since $(\gamma(t), \gamma(t,t'))_{t,t'\in T}$ and $(c(s,t), c(s,t,t'))_{s\in S,t,t'\in T}$ is a Walrasian equilibrium, (8) and Lemma 1 imply that:

$$\sum_{t\in T}\gamma(t)\hat{c}(s,t)+\sum_{t,t'\in T}\gamma(t,t')\hat{c}(s,t,t')\geq\sum_{t\in T}\gamma(t)c(s,t)+\sum_{t,t'\in T}\gamma(t,t')c(s,t,t')=0$$

for every s, with a strict inequality whenever $\hat{V}(s)>V_I(s\mid c)$.

Hence,

$$\sum_{s\in S}\hat{\pi}(s)\left[\sum_{t\in T}\gamma(t)\hat{c}(s,t)+\sum_{t,t'\in T}\gamma(t,t')\hat{c}(s,t,t')\right]>0.$$

Thus, $\sum_{s\in S}\hat{\pi}(s)\left[\gamma(t)\hat{c}(s,t)+\gamma(t,t')\hat{c}(s,t,t')\right]>0$ for some (t,t'). This implies that $\sum_{s\in S}\hat{\pi}(s)\hat{c}(s,t)>0$ or $\sum_{s\in S}\hat{\pi}(s)\hat{c}(s,t,t')>0$ for some (t,t'), a contradiction with (9). ■