UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4001

SEMESTER: Autumn 2010/11

MODULE TITLE: Engineering Mathematics 1

DURATION OF EXAMINATION: $2\frac{1}{2}$ hours

LECTURER: Dr. N. Kopteva

PERCENTAGE OF TOTAL MARKS: 70%

INSTRUCTIONS TO CANDIDATES:
Answer question 1 and any other *two* questions from questions 2, 3, 4 and 5.

To obtain maximum marks you must show all your work clearly and in detail.
Standard mathematical tables are provided by the invigilators. Under no circumstances should you use your own tables or be in possession of any written material other than that provided by the invigilators.
Non-programmable, non-graphical calculators that have been approved by the lecturer are permitted. **WriteView calculators or similar advanced calculators are not permitted.** There will be a spot check of calculators during the exam.
You must obey the examination rules of the University. Any breaches of these rules (and in particular any attempt at cheating) will result in disciplinary proceedings. For a first offence this can result in a year's suspension from the University.

1 (a) A particle has displacement $x(t) = (2t^2 - 8t + 9)\, e^t$ for times $0 \le t \le 4$. Find its velocity $v(t)$ and determine for what times the particle is moving in the *negative* x direction. 4%

(b) Find $\sin^{-1}(\sin(\frac{27\pi}{7}))$. 4%

(c) Find a root of $e^{-x} = x^2$ to 7 decimal places using *Newton's method* with the initial guess $x_0 = 0$. 4%

(d) Evaluate $\lim\limits_{x\to 0} (1+x)^{\frac{1}{\sin(2x)}}$.
(Hint: represent this limit as $\lim\limits_{x\to 0} e^{f(x)}$ for some function $f(x)$ and then use l'Hôpital's rule to evaluate $\lim\limits_{x\to 0} f(x)$.) 4%

(e) Find the first derivative of $x^{\ln(x+1)}$. 4%

(f) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^4 \cdot 3^n}$. 4%

(g) If $\mathbf{u} = 3\mathbf{i} - \mathbf{j} - 6\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 7\mathbf{j} + 2\mathbf{k}$, and $\mathbf{w} = 2\mathbf{i} + 3\mathbf{j}$, find the triple vector product $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$. 4%

(h) Prove that if $f'(x) < 0$ on (a, b), then $f(x)$ is decreasing on (a, b). (Hint: use the Mean-Value Theorem.) 4%

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2 Consider the function $y = f(x) = x\, e^{-x^2/8}$.

(a) What is the domain of $f(x)$? Find the x- and y-intercepts of $f(x)$, if any. 2%

(b) Discuss the behaviour of $f(x)$ as $x \to \infty$ and as $x \to -\infty$. Find all *vertical* and *horizontal asymptotes* of $f(x)$, if any. 3%

(c) Find the critical points and singular points of $f(x)$, if any. Find the x-values for which $f(x)$ is increasing/decreasing. Find all local maxima and minima of $f(x)$, if any. (Decimal approximations will suffice.) 5%

(d) Find the x-values for which $f(x)$ is concave up/down. Find the points of inflection of $f(x)$, if any. (Decimal approximations will suffice.) 5%

(e) Sketch the graph of $y = f(x)$. 4%

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3 (a) Suppose that the intensity of a point light source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source. Two point light sources with strengths of S and $8S$ are separated by a distance of 90 cm. Where on the line segment between the two sources is the intensity a minimum? 7%

(b) Test the series below for convergence, stating which test you are using:

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt[3]{n^4+1}}$, (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2\, 5^n}$, (iii) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt[3]{n^4+1}}$. 4%+4%+4%

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4 (a) By direct differentiation and using Taylor's formula, find the ***first four non-trivial terms*** of the Maclaurin series for the function

$$e^{\sin x}$$

9%

(b) Hence find the ***first three non-trivial terms*** of the Maclaurin series of

$e^{\sin(4x)}$, $e^{\sin(2x^3)}$, and of $e^{\sin(2x^2-4x)}$. 3%+3%+4%

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5 (a) Evaluate the limit: $\displaystyle\lim_{x\to\infty} \frac{1}{x-\sqrt{x^2-5x-1}}$. 4%

(b) Consider the planes $x + 5z = 8$ and $2x - 3y + 4z = -12$. For each of them, find a normal vector. Then find the angle between these two planes. 5%

(c) Find an equation of the plane passing through the point $(2, 0, 1)$ that is perpendicular to the line containing the points $(2, 0, 7)$ and $(3, 5, -1)$. 3%

(d) Find the area of the triangle with vertices: $(2, 2, -3)$, $(4, 1, 5)$ and $(4, 6, -1)$. 4%

(e) Find the equation of the plane through $(2, 2, -3)$, $(4, 1, 5)$ and $(4, 6, -1)$. 3%

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