Diamond Cubic Truss

/ Gilman's Tetrahedral Truss /

Lorimerlite Framework

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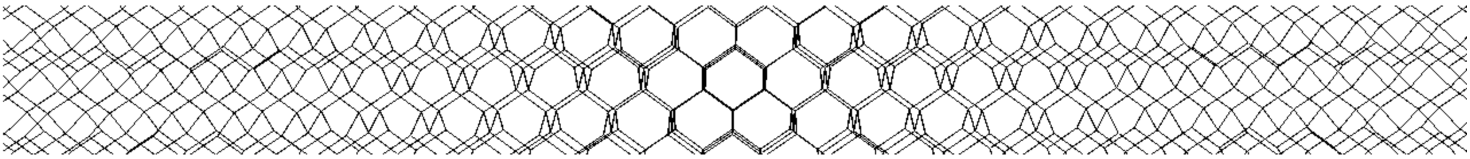


ABSTRACT

This short text describes a support framework that utilises an isotropic geometry optimised for resisting compressive forces with a minimum of structural material.

This framework has been considered in the past for structural engineering purposes by material scientist John J. Gilman in the 1980's[1] and Ralph W. Kraft in the 1960's[2]. However Kraft's intention was the development of a structure that would remain as rigid as possible while being lightweight, and Gilman explicitly refers to the inherent rigidity of the skeletal triangle as the source of the structures strength[3].

Here, by contrast, the intention in the independent development of this structure was primarily to resist compression with the least amount of structural material using a geometry of shortest paths. Empirical exploration through prototypes has revealed the structures effectiveness for this purpose, but also its inefficiency as a structure for the purpose of providing geometric rigidity.

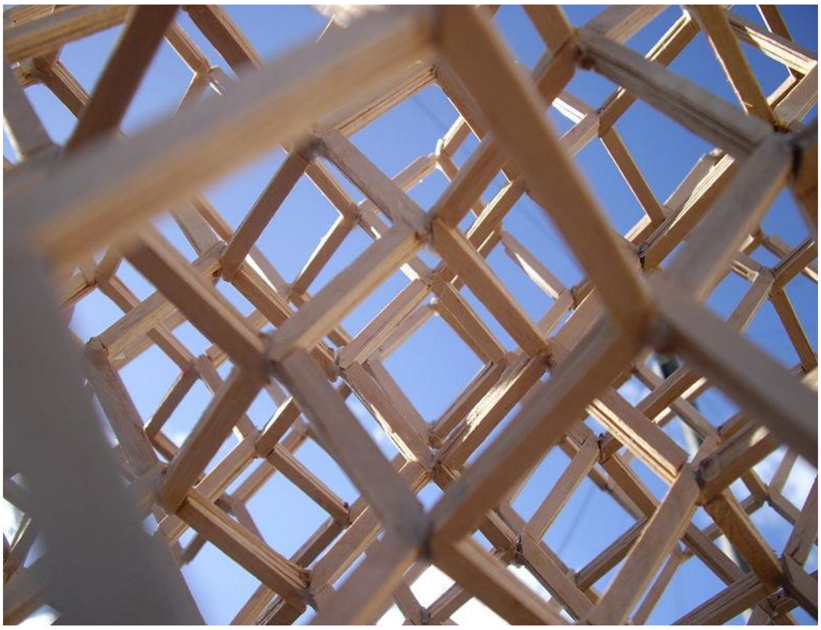
This text will go on to outline the development of this compression structure while drawing comparisons with the Weaire-Phelan Structure, as both have been informed by the efficient geometry of soap bubbles.

Keywords: Diamond Cubic Truss, Gilman's Tetrahedral truss, Lorimerlite Framework, Shortest Path, Compression Structure, John J. Gilman, Alexander O.D. Lorimer, Weaire-Phelan Structure

Context

"…a repeating pattern of straight beams of equal length that meet at tetrahedral joints, four at a time at 109.5 degree angles"[4]

The geometry described above is that of the molecular structure of a carbon diamond, and has been considered at various points in the past for structural engineering purposes. Ralph W. Kraft in the 1960's explored the possibility of such a framework sandwiched between flat panels in order to provide one rigid composite panel.[5] In John J. Gilman's 1980's patent application, he refers to "the inherent rigidity of the skeletal triangle"[4] and "the rigidity of the basic tetrahedron"[6] as the source of the structure's strength.



Lorimer, A. Diamond Cubic Truss: Made of Coffee-stirrers, 2012

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This structure does not consist of triangles on any plane and consequently may be subject to geometric distortion under tension or shear.

The structure's compressive strength is achieved through its shortest path geometry. For a predefined volume of space to fill and predefined number of joints in the structure, each beam finds the shortest un-braced path through space[7] as an isotropic arrangement (in the same way as a hexagonal grid in two dimensions[8]).

The smaller the length of individual beams the greater the resistance to axial compression[9], reducing the danger of lateral buckling and thus enhancing the compressive strength of the entire structure.

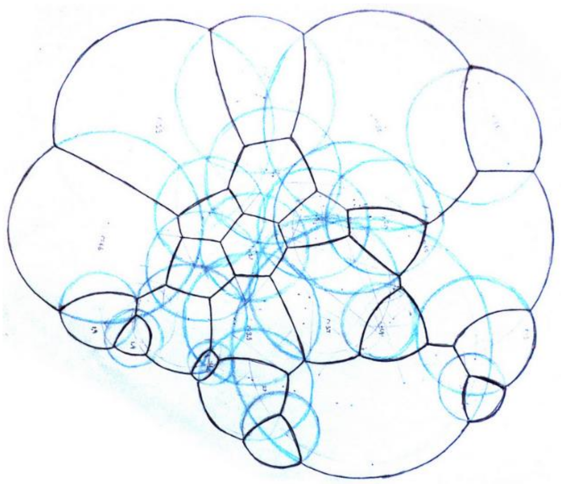
All Loads are distributed at each joint evenly with every beam meeting at 109.5 degree angles.

Lorimer, A. '[ch]air' (made from laser-cut cast acrylic), 2012

Development

While happening to follow an identical geometry to that of the molecular structure of a pure carbon diamond the development of the diamond cubic truss was informed by the efficient geometry of soap bubbles.

Bubbles of equal volume that are trapped between two panes of glass form into a cluster of perfect hexagons[10], widely recognised as the efficient geometry of the bee's honeycomb and other natural phenomena. While a hexagonal grid would not provide the most rigid structure as it may be subject to geometric distortion, unlike triangles[11], it does however present a geometry of shortest-paths[12] and therefore optimally resists purely compressive forces across a two-dimensional plane relative to other repeating geometries.



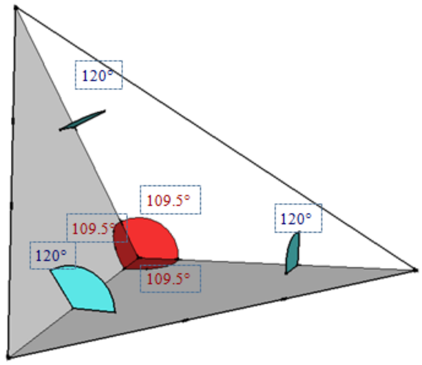
Lorimer, A. 'Original Analysis Drawing', 2011

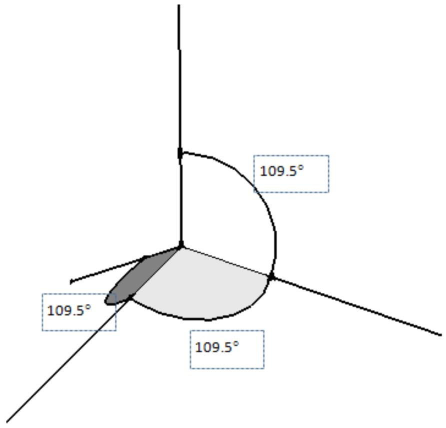
This soap bubble form finding technique is very useful for displaying an underlying principle governing natural systems; however it may also be used to solve various complex problems.

For example, if bubbles of varying volumes are introduced, the cluster will instantly adapt to ensure that all corners in the grid continue to connect only three lines at 120° angles.

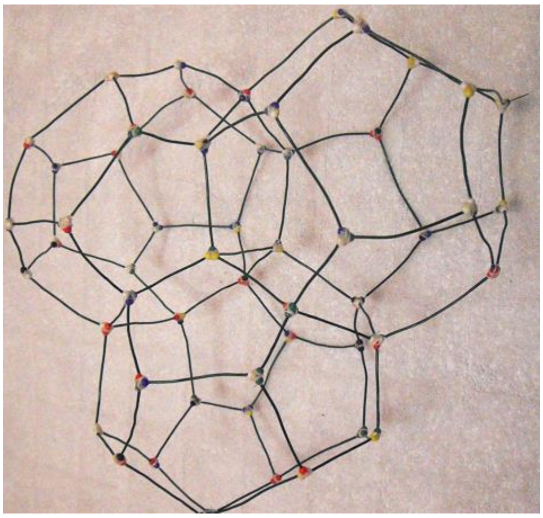
The cluster becomes a display or curving lines and perfect angles, mathematically determined for minimal material usage while containing the predefined volumes[13].

Considering three dimensions; in a cluster of bubbles the walls of each cell will meet consistently at 120° angles, bowing either inward or outward to maintain this optimum angle. The edges of each cell, however, will meet at a vertex, four at a time, at an optimum angle of 109.5°; a tetrahedral geometry.[14]





Lorimer, A. 'Analysis Diagram', 2011



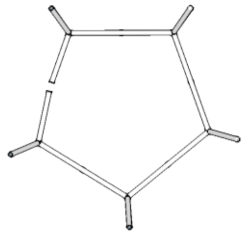
Lorimer, A. 'Wire Weaire-Phelan Model', 2011

The Weaire-Phelan structure, developed in 1993, was informed by bubble geometry[15] and could be considered a perfect soap bubble foam, equivalent to the grid of perfect hexagons in two dimensions. The structure is the best known solution to the Kelvin Conjecture originally posed in 1887, questioning how cells of equal volume could be partitioned in a cluster with minimal surface areas.[16]

Recently the geometry has become adopted as the structural support framework of the Beijing National Aquatics Centre, 2008.

It is interesting to consider whether the economy of this geometry translates into a similarly efficient compression structure, as with the two-dimensional hexagonal grids.

While the Weaire-Phelan structure is the optimum geometry for minimum surface area, it is of course instead the edge lengths of each cell which must be minimised as it is these that are translated into the structural components of the skeletal support framework. Also each edge in the Weaire-Phelan structure is required to bow slightly so as to maintain the ideal tetrahedral vertices, however a support structure with bent structural elements under axial compression is obviously less than ideal.

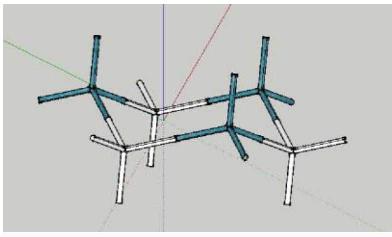


Lorimer, A. 'Open Pentagon with 109.5° Angles', 2011

In fact the support structure used in the Beijing National Aquatics Centre is actually a slight alteration of the true Weaire-Phelan structure.[17] All beams have been straightened, resulting in distorted vertices which no longer display the perfect tetrahedral angles.

"The problem with the 109.5° angle is that it belongs to no regular straight edged shape. Each corner in a regular pentagon has 108° but this is the closest angle that can be achieved"[18] without bending the edges.

There is however another option; 109.5° angles can be used to create a hexagon whose corners do not lie on the same plane. These non-coplanar hexagons can then be used to construct a framework composed of prefect tetrahedral vertices with all straight edges.

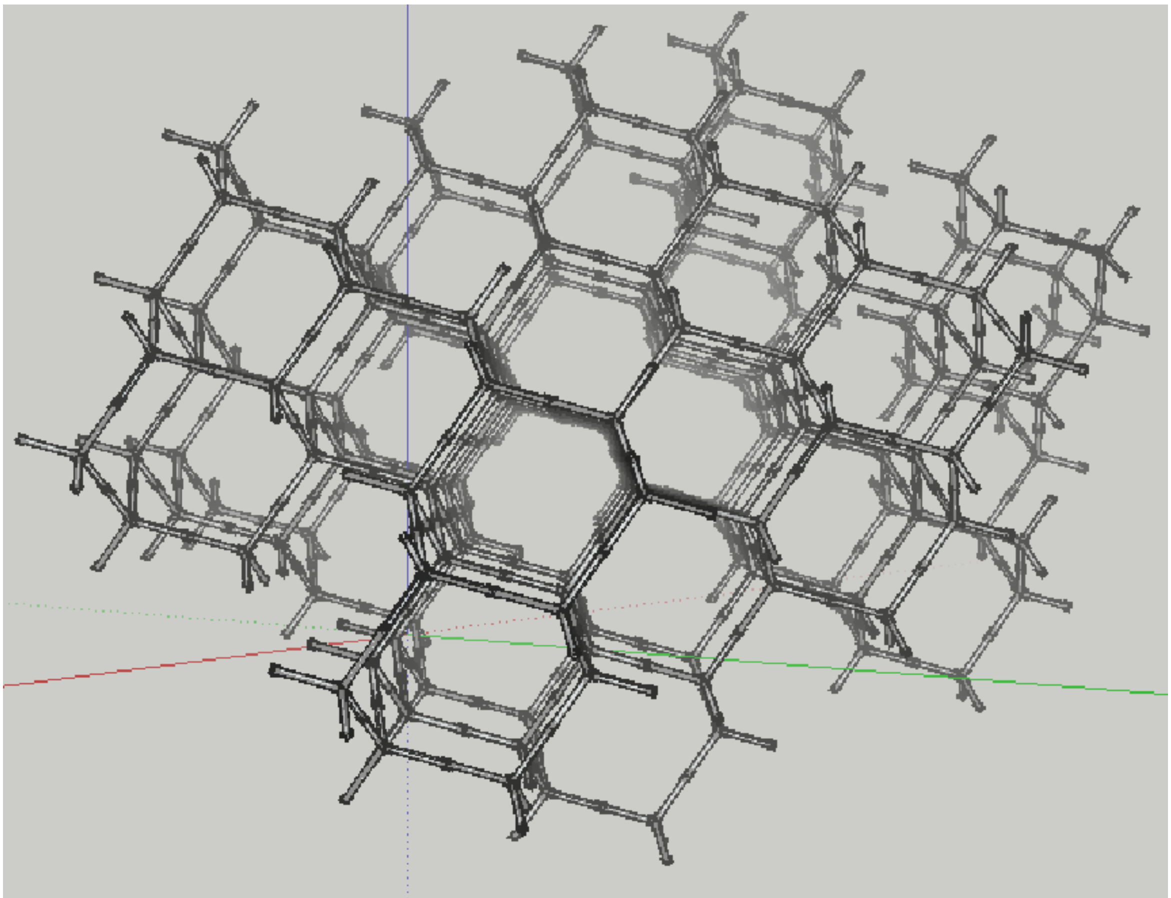


Lorimer, A. 'Non-Coplanar Hexagon', 2011

Summary

The resulting structure fills a predefined volume of space with the short possible length of a predefined number of struts, as an isotropic arrangement. All struts are able to remain straight while maintaining 109.5 degree joint angles which distribute loads evenly, forming an optimum isotropic structure for resisting compressive forces.

This structure does not consist of triangles on any plane and so, perhaps contrary to Kraft's and Gilman’s expectations, it may be subject to geometric distortion rending it structurally inefficient in circumstances where a rigid framework is required.



Lorimer, A. ‘Diamond Cubic Truss, CAD’, 2011

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