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HAMILTONIAN METHODS: BRST, BFV

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RESUMEN El rango de abstracción de la aplicabilidad de los métodos de Hamilton para evaluar las teorías son muy diversas y abarcan las áreas de investigación de la fenomenología y de la física matemática. Repasamos algunas de las áreas desarrolladas en México en las últimas décadas, comprenden el estudio de los métodos simplécticos, BRST-BFV y enfoques BV, el programa proyector de Klauder, y la no “perturbaciones” técnicas empleadas en el estudio de los estados relacionados con las teorías relativistas.

ABSTRACT The range of applicability of Hamiltonian methods to gauge theories is very diverse and cover areas of research from phenomenology to mathematical physics. We review some of the areas developed in México in the last decades. They cover the study of symplectic methods, BRST-BFV and BV approaches, Klauder projector program, and non perturbative technics used in the study of bound states in relativistic field theories.

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1 Quantization of Gauge Systems, Dirac Method, BRST BFV and BV

The theories that describe the fundamental interactions -electromagnetic, electroweak, strong and gravitational- are gauge theories. Recently another gauge theory, string theory, has attracted a lot of attention because it could be used to construct a consistent theory of quantum gravity¹. It is important to mention here that classical general relativity as a field theory can be quantized using the methods that we will describe here. This approach has

¹String theory will be reviewed in another entry of this volume.

been pursued by Ashtekar and collaborators in the last two decades. It is known as Loop Quantum Gravity. A basic property of such theories is that they have constraints among the fields and its conjugated momenta in phase space or among fields and its "velocities" in configuration space. This implies, in particular, that the physical degrees of freedom are not the same as the ones used to construct the theory from first principles. This type of theories are based on variational principles and symmetries that are cornerstones in the theoretical construction of any physical acceptable interacting theory. They have peculiar symmetries called gauge symmetries that are deeply connected with the fact that these theories have constraints. The interactions are constructed following the principle of gauge invariance. The systematic research of these type of theories was initiated by Dirac² whose aim was to construct a general procedure to quantize the general theory of relativity. From this seminal work a wide trend of research was opened: the study of the classical constrained dynamics, its consistency conditions and quantization.

Nowadays we have at our disposal some methods to analyze the dynamical consistency, the symmetries, physical content and quantization of a given constrained field theory. They are the Dirac method and its extensions (see below), loop quantization, geometric quantization, Klauder projector program and symplectic covariant quantization.

Gauge theories can not be quantized using the standard canonical quantization in the operator or path integral approaches. One of the reasons is that not all the degrees of freedom of the theory are physical. Some of them must be eliminated because they are not gauge invariant observables, i.e., they transform under the gauge transformations. At the end when theory is compared with experimental results these degrees of freedom must be eliminated using a covariant or noncovariant method. The freedom of covariant methods is that the gauge symmetry (and some other global symmetries like Poincaré symmetry) can be used to prove the renormalizability and unitarity of the given theory.

With the aim to try to understand in better grounds the structure and physical content of a gauge theory we can analyze the intrinsic properties (independently from the constraint algebra) of the gauge theory and the gauge fixing method. This perspective helps also in the understanding of anomalies, renormalizability, consistent interactions, structure of the path integral measure, among others. A systematic Hamiltonian approach based on the BRST symmetry³ developed by E. S. Fradkin and his collaborators⁴,

²"Generalized Hamiltonian Dynamics" *Can. J. Math* 2,1950, 129-148; "Lectures on Quantum Mechanics", 1964.

³C. Becchi, A. Rouet and R. Stora "Renormalization Of Gauge Theories", 1976; C. Becchi, A. Rouet and R. Stora "The Abelian Higgs-Kibble Model. Unitarity Of The S Operator", 1974; I. A. Batalin and G. A. Vilkovisky "Relativistic S Matrix Of Dynamical Systems With Boson And Fermion Constraints", 1977; I. A. Batalin and I. V. Tyutin "Existence theorem for the effective gauge algebra in the generalized canonical formalism with Abelian conversion of second class constraints", 1991.

⁴E. S. Fradkin and T. E. Fradkina "Quantization Of Relativistic Systems With Boson And Fermion First And Second Class Constraints", 1978; I. A. Batalin and E. S. Fradkin "A Generalized Canonical Formalism And Quantization Of Reducible Gauge Theories", 1983; I.

the BFV (Batalin, Fradkin, Vilkoviskii) method take full advantage of this perspective. Some of the properties of this powerful approach based on the Dirac method are

- It can be applied to open algebras (algebras that closes when the equations of motion are taken into account)
- The BRST transformation is based on intrinsic properties of the constraint surface. The Noether generator of the transformation is a classical object that can be quantized using the standard canonical approach. It is nilpotent to implement the constraint surface and the Poisson structure in the dynamics.
- It is fully based on the Hamiltonian formalism and allows the use of Hamiltonian technics like Liouville measure, canonical transformations, definition of the kinematic Hilbert space, among others.

As the Dirac method, the BFV approach can be very difficult to implement in systems with general covariance or in systems where the first and second class constraints can not be separated in a covariant way. This last point is crucial in the Green-Schwarz approach to the superstring.

An alternative Lagrangian approach know as BV (Batalin, Vilkoviskii)⁵, is covariant and implement the dynamics on cohomology through the Kozul-Tate resolution. This deep property allow this method to be used for the study of anomalies, consistent deformations of a given theory, and renormalizability using covariant technics.

In this context a wide range of applications and intrinsic studies of these methods was developed in the last decades. On one hand, the study of the classical and quantum properties of constrained systems, and intrinsic properties of the Dirac method⁶. On the other, applications of Dirac method to Ashtekar formulation of general relativity⁷ and relational dynamics⁸. The

A. Batalin and E. S. Fradkin "Operator Quantization Of Relativistic Dynamical Systems Subject To First Class Constraints", 1983 [Sov. J. Nucl, 1984; YAFIA, 1984; YAFIA, 1985].

⁵I. B. Batalin and G.A. Vilkovisky, 1983.

⁶M. Henneaux, C. Teitelboim and J. D. Vergara "Gauge invariance for generally covariant systems", 1992 [arXiv:hep-th/9205092].

M. Montesinos and J. D. Vergara "Linear constraints from generally covariant systems with quadratic constraints", 2002 [arXiv:gr-qc/0111006].

J. A. Garcia and J. M. Pons "Lagrangian Noether symmetries as canonical transformations" 2001 [arXiv:hep-th/0012094].

J. M. Pons and J. A. Garcia "Rigid and gauge Noether symmetries for constrained systems", (2000) [arXiv:hep-th/9908151].

J. A. Garcia and J. M. Pons "Equivalence of Faddeev-Jackiw and Dirac approaches for gauge theories", 1997 [arXiv:hep-th/9610067].

J. A. Garcia and J. M. Pons "Faddeev-Jackiw approach to gauge theories and ineffective constraints", 1998 [arXiv:hep-th/9803222].

⁷H. A. Morales-Tecotl, L. F. Urrutia and J. D. Vergara "Reality conditions for Ashtekar variables as Dirac constraints", 1996 [arXiv:gr-qc/9607044].

M. Montesinos and J. D. Vergara "Gauge invariance of complex general relativity", 2001 [arXiv:gr-qc/0010113].

M. Montesinos, H. A. Morales-Tecotl, L. F. Urrutia and J. D. Vergara "Real sector of the non-minimally coupled scalar field to self-dual gravity", 1999 [arXiv:gr-qc/9903043].

BFV method has been extended to systems with time dependent constraints⁹, and its formulation in terms of the Schwinger quantum principle was studied in¹⁰. An application of the BV approach to the study of the general form of the strict gauge invariant observables of exotic gauge theories (theories with field tensors with mixed type symmetry) is¹¹, This type of theories are relevant in recent studies about duality. In particular, they are dual to the Fierz-Pauli Lagrangian. From this perspective, the structure and coupling of this theories and the corresponding Dirac analysis of them in 4D was studied in¹².

2 Klauder Projector Method

As an alternative approach to Hamiltonian Quantization, the physical Projector Operator Approach initiated and implemented by Klauder using coherent state techniques¹³, has been applied to some simple gauge invariant quantum mechanical models. In this approach gauge fixing is not necessary and thus, it could avoid¹⁴ potential Gribov ambiguities¹⁵ which arise in the quantization of gauge invariant systems. As is well known, some gauges may suffer Gribov ambiguities. In fact it is only for an admissible gauge fixing that we can define the correct dynamical description of the system in reduced space. This admissible gauges must be well defined globally and this property is crucial in the description of non perturbative phenomena. The aim of the Projector Operator Approach is to construct a systematic method to project the dynamics in the space defined by the solutions to a given field theory. In this way the gauge fixing procedure is avoided.

These aspects of gauge invariant systems were explicitly analyzed within a solvable $U(1)$ gauge invariant quantum mechanical model¹⁶ related to the

A. Ashtekar, L. Bombelli and A. Corichi "Semiclassical states for constrained systems", 2005 [arXiv:gr-qc/0504052].

⁸M. Montesinos, C. Rovelli and T. Thiemann "An $SL(2, \mathbb{R})$ model of constrained systems with two Hamiltonian constraints", 1999 [arXiv:gr-qc/9901073].

⁹J. A. Garcia, J. D. Vergara and L. F. Urrutia "BRST-BFV method for nonstationary systems", 1995 [arXiv:hep-th/9608178].

¹⁰J. A. Garcia, J. D. Vergara and L. F. Urrutia "BRST-BFV quantization and the Schwinger action principle", 1996 [arXiv:hep-th/9511092].

¹¹J. A. Garcia and B. Knaepen "Couplings between generalized gauge fields", 1998 [arXiv:hep-th/9807016].

¹²H. Casini, R. Montemayor and L. F. Urrutia "Dual theories for mixed symmetry fields. Spin-two case: (1,1) versus (2,1) Young symmetry type fields", 2001 [arXiv:hep-th/0102104].

H. Casini, R. Montemayor and L. F. Urrutia "Duality for symmetric second rank tensors. I: The massive case", 2002 [arXiv:hep-th/0206129]. H. Casini, R. Niontemayor and L. F. Urrutia "Duality for symmetric second rank tensors. II; The linearized gravitational field", 2003 [arXiv:hep-th/0304228].

¹³"Coherent states quantization of constraint systems", By J.R. Klauder, 1997; "Universal procedure for enforcing quantum constraint's, by J. Klauder, 1999; "Quantization of constrained systems", By J.R. Klauder, 2001.

¹⁴"Projector operator approach to constrained systems", By J. Govaerts, 1997.

¹⁵"Quantization of non-abelian gauge theories", by V.N. Gribov, 1978.

¹⁶"A soluble gauge model with gribov type copies", by R. Friedberg, T.D. Lee, Y. Pang and H.C. Ren, 1996.

dimensional reduction of Yang-Mills theory. In this model, even at the classical level, one can parameterize the space of gauge orbits in terms of a classical parameter called the Teichmüller parameter¹⁷. It is through this parameter that all the gauge orbits are included in the quantization of the system, in agreement with the Friedberg *et al.*¹⁸ point of view that all such gauge copies should be included in a correct quantization of gauge systems. These points were discussed and analyzed in Refs.¹⁹.

During the process of quantization of physical systems through Hamiltonian procedures, it was necessary to investigate the general construction of self-adjoint configuration space representations of the Heisenberg algebra over arbitrary manifolds not necessarily cartesian or parameterized with cartesian coordinates. All such inequivalent representations are parameterized in terms of the topology classes of flat $U(1)$ bundles over the configuration space manifold. In the case of Riemannian manifolds, these representations are also manifestly diffeomorphic covariant. The general discussion, illustrated by some simple examples in non relativistic quantum mechanics, is of particular relevance to systems whose configuration space is parameterized by curvilinear coordinates or is not simply connected, which thus include for instance the modular spaces of theories of non abelian gauge fields and gravity. This was the main motivation of Ref.²⁰.

Finally, in order to study Hamiltonian gauge invariance, a Hamiltonian version of the Noether theorem for constrained systems is formulated in²¹. In particular, a novel method is presented to show that the gauge transformations are generated by the conserved quantities associated with the first class constraints. These results are applied to the relativistic point particle, to the Friedberg *et al.* model and, with special emphasis, to two time physics.

3 Symplectic Geometry in Gauge Theory

The symplectic geometry constitutes a modern Hamiltonian scheme in the study of symmetries and quantization of gauge theories from a geometrical point of view. The basic idea of this scheme is the construction of a Hamiltonian structure on the phase space of the theory which contains all physically relevant information, and does not require the choice of phase space coordinates p 's and q 's as in the traditional approach. Geometrically the Hamiltonian structure plays the role of a field strength, obtained by

¹⁷"Hamiltonian quantization and constrained dynamics", J. Govaerts, 1991.

¹⁸Ibid 16

¹⁹"Quantizing gauge theories without gauge fixing: The physical projector", V.M. Villanueva, J. Govaerts, J.L. Lucio 2000.

"Quantization without gauge fixing: avoiding Gribov ambiguities through the physical projector" by Victor M. Villanueva, Jan Govaerts, José-Luis Lucio-Martinez, 1999, 2000.

²⁰"Topology classes of flat $U(1)$ bundles and diffeomorphic covariant representations of the Heisenberg algebra", by Jan Govaerts, Victor M. Villanueva, 1999, 2000.

²¹"Hamiltonian Noether theorem for gauge systems and two time physics", by V.M. Villanueva, J.A. Nieto, L. Ruiz, J. Silvas, 2005.

(*exterior*) derivative from a symplectic potential, which can be considered as a gauge field on the phase space.

The symplectic scheme was originally introduced by Witten *et al.* with applications to Yang-Mills theory, General Relativity, and string field theory²². The applications to string/brane theory have been given recently revealing a rich underlying geometrical structure of the theory²³. Additionally we have undertaken the study of topological gauge theories, which are relevant in the context of formulating background-independent theories, and in the construction of topological invariants of four-manifolds. Specifically the symplectic scheme reveals that the topological action related with the Euler characteristic of the world-sheet in string theory mimics the geometrical structure of a two-dimensional gauge theory²⁴. Moreover, the corresponding lower bound state for that topological string action is given by a loop state, described by a Wilson loop along the spatial configuration of the string²⁵.

On the other hand, the symplectic scheme allows us to prove that some properties of instantons in Yang-Mills theory traditionally associated with their self-duality, actually come from their topological nature. Specifically the only solution for the Schrödinger equation for quantum Yang-Mills theory known as the Chern-Simons wave functional and associated with instantons, exists actually for the (four-dimensional) topological Yang-Mills theory, reducing the self-dual property to a spurious condition²⁶. This result can be generalized for topological actions and Chern-Simons functionals in spaces of even dimension.

Classically all topological action associated with curvatures coming from a gauge connection can be expressed as the (*exterior*) derivative of a Chern-Simons form; the quantum Hamiltonian of the topological action has as its lower state a functional of the corresponding Chern-Simons form²⁷, in such a way that the results previously described for topological Yang-Mills theory (and consequently for the absolute minimum of conventional Yang-Mills theory) corresponds only to a particular case. Furthermore, starting from the topological Yang-Mills theory, it can be proved that a moduli space of

²²E. Witten "Nucl", 1986; C. Crnkovic and E. Witten "Three Hundred Years of Gravitation", 1987; C. Crnkovic, "Nucl", 1987.

²³R. Cartas-Fuentevilla, "Identically closed two-form for covariant phase space quantization of Dirac-Nambu-Goto p-branes in a curved spacetime", 536,283, 2002a; "Identically closed two-form for covariant phase space quantization of Dirac-Nambu-Goto p-branes in a curved spacetime", 536,289, 2002b; "Identically closed two-form for covariant phase space quantization of Dirac-Nambu-Goto p-branes in a curved spacetime", 563,107, 2003b; "Class and Quantum Gravity", 19, 3571, 2002c; B. Cárter, "Symplectic structure in brane mechanics", 12,1317, 2003a.

²⁴R. Cartas-Fuentevilla, math-ph/0404011.

²⁵R. Cartas-Fuentevilla, hep-th/0411251, to be published "Wilson loops and topological phases in closed string theory", 2006

²⁶R. Cartas-Fuentevilla, and F. Tlapanco-Limón, "The Kodama state for topological quantum field theory beyond instantons", 623,165, 2005.

²⁷R. Cartas-Fuentevilla "Quantization of the Chern invariant polynomial and its topological quantum ground state", 2006.

finite dimension can be obtained without invoking self-duality, leading to the idea of *fluctons*²⁸.

4 Hamiltonian non perturbative Methods in Gauge Theories

A given Hamiltonian H is split into a "free" (solvable) part H_0 and an "interacting" part H_1 , $H = H_0 + H_1$. Then a generalization of the Gell-Mann-Low Theorem²⁹ provides a similarity transform U_{BW} from any H_0 -invariant subspace Ω_0 of the Hilbert space to an exactly H -invariant subspace Ω . Consequently, the diagonalization of H in Ω is equivalent to the diagonalization of an "effective" Hamiltonian H_{BW} in Ω_0 . The map U_{BW} , and hence the Hamiltonian H_{BW} , are given in terms of a perturbative series. However, the results of the application of the generalized Gell-Mann-Low theorem will typically be nonperturbative.

This framework has so far been applied to the bound state problem in quantum field theory, by taking Ω_0 as the subspace of Fock space consisting of all H_0 (and momentum-) eigenstates of the would-be constituents as free particles. The effective Hamiltonian then consists of the relativistic kinetic energies of the constituents and an effective potential for their interaction generated as a perturbative series. The solution of the corresponding Schrödinger equation yields (to any finite order of the perturbative series an approximation to) the bound state energies of the full theory and the wave functions of the constituents (for N -particle bound states, the N -particle components of the full states in Fock space).

Applications to two-particle bound states in the Wick-Cutkosky model, Yukawa theory and Coulomb gauge QED, including a determination of the lowest-order fine and hyperfine structures in these theories, can be found in Ref.³⁰. The most important results, particularly in comparison with the Bethe-Salpeter approach, are

- UV divergencies (as far as they have appeared in the lowest-order calculations) can be absorbed in a renormalization of the parameters. The renormalization procedure can be set up entirely in the Hamiltonian framework.
- The nonrelativistic and one-body limits are particularly transparent and lead to the correct results.
- Even in Yukawa theory, the effective Schrödinger equation is a well-defined eigen-value equation.
- No abnormal solutions have been found. All solutions are consistent with physical expectations (symmetry properties).

²⁸R. Cartas-Fuentevilla, and J.M. Solano-Altamirano, "Fluctons"; in preparation (2006).

²⁹A. Weber "Particles and Fields-Seventh Mexican Workshop", 2000.

³⁰A. Weber and N.E. Ligterink (2002), "The Generalized Gell-Mann-Low Theorem for Relativistic Bound States", *Phys. Rev. D* 65

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