

# Multi-objective optimization

## ● Define Pareto efficiency

To solve multi-object optimization problem, we use Pareto efficiency to identify which point is much better. In the beginning, we have to understand these meaning.

Let  $f: \Omega \rightarrow \mathbb{R}^l$  and  $x \in \Omega$  be given. For the optimization problem

$$\min f(x), \text{ s.t. } x \in \Omega$$

a point  $x^* \in \Omega$  is called a Pareto minimizer if there exists no  $x \in \Omega$  such that for  $i = 1, 2, \dots, l$ ,  $f_i(x) \leq f_i(x^*)$ , and for at least one  $i$ ,  $f_i(x) < f_i(x^*)$ .

**Pareto domination:**  $x$  dominate  $x^*$ , for all  $f(x) \leq f(x^*)$

**Pareto optimal point:** The point which is solution can dominate at least one other point.

**Pareto optimal set:** Its definition is the same with Pareto optimal point.

**Pareto front:**  $f(\text{Pareto optimal set})$  and it's the vector.

## ● Question 2

**Construct a multi-objective optimization problem (MOP) by defining objective functions and the constraint set. Find the associated Pareto front.**

$$\begin{cases} f_1 = x, f_2 = -x + y, f = (f_1, f_2), \text{ find the minimizer.} \\ \text{Subject to for all } x \text{ and } y \text{ is integer, } 2 \leq x \leq 5, 0 \leq y \leq 5. \end{cases}$$

Table I show us all the function value.

Table I function value

(x,y)	f(x,y)	(x,y)	f(x,y)	(x,y)	f(x,y)	(x,y)	f(x,y)
(2,0)	(2,-2)	(3,0)	(3,-3)	(4,0)	(4,-4)	(5,0)	(5,-5)
(2,1)	(2,-1)	(3,1)	(3,-2)	(4,1)	(4,-3)	(5,1)	(5,-4)
(2,2)	(2,0)	(3,2)	(3,-1)	(4,2)	(4,-2)	(5,2)	(5,-3)
(2,3)	(2,1)	(3,3)	(3,0)	(4,3)	(4,-1)	(5,3)	(5,-2)
(2,4)	(2,2)	(3,4)	(3,1)	(4,4)	(4,0)	(5,4)	(5,-1)
(2,5)	(2,3)	(3,5)	(3,2)	(4,5)	(4,1)	(5,5)	(5,0)

By definition, the Pareto set is shown in Table II,

Table II Pareto Set

(x,y)	f(x,y)
(2,0)	(2,-2)
(3,0)	(3,-3)
(4,0)	(4,-4)
(5,0)	(5,-5)

Figure 1 presents function values by blue nodes, Pereto front is pink line which is Pereto sets are connected with each other.

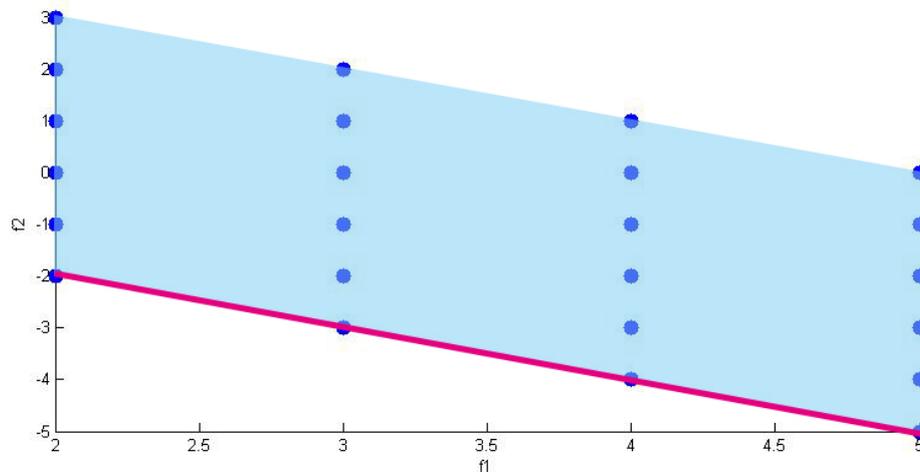


Figure 1 output of question2

### ● Question3

Use HMODE (codes can be downloaded from my website) to solve two algebraic MOPs.

I used this equation to be my example by HMODE.

$$\begin{cases} f_1 = x, f_2 = -(x^2 + y^2), f = (f_1, f_2), \text{ find the minimizer.} \\ \text{Subject to for all } x \text{ and } y \text{ is integer, } 2 \leq x \leq 5, 0 \leq y \leq 5. \end{cases}$$

→main.m

```
Np=50;           % size of population
tmax=200;       % # of iteration
eta_c=0.2;      % crossover
max_range=[5 5];
min_range=[2 0];
cstInd=1; % indicator for constraints; cstInd=1 => inequality constraint
is involved
```

```
[EP_alpha, EP_f, num_obj_eval]=
HMODE(Np,tmax,eta_c,min_range,max_range,cstInd);
```

```
plot(EP_f(:,1),EP_f(:,2),'or')
xlabel('f_1')
ylabel('f_2')
```

→obj\_eval.m

```
x=alpha_c(1);
```

```

y=alpha_c(2);
myobj(1)= x;
myobj(2)= -(x^2+y^2);
g1= -1;
g2= -1;
myobj(3)=max([g1 0])+max([g2 0]);

```

→ **plot the integer value**

```

for x = 2:1:5
    for y = 0:1:5
        f1 = x;
        f2 = -(x^2+y^2);
        plot(f1,f2,'o');
    end
end
end

```

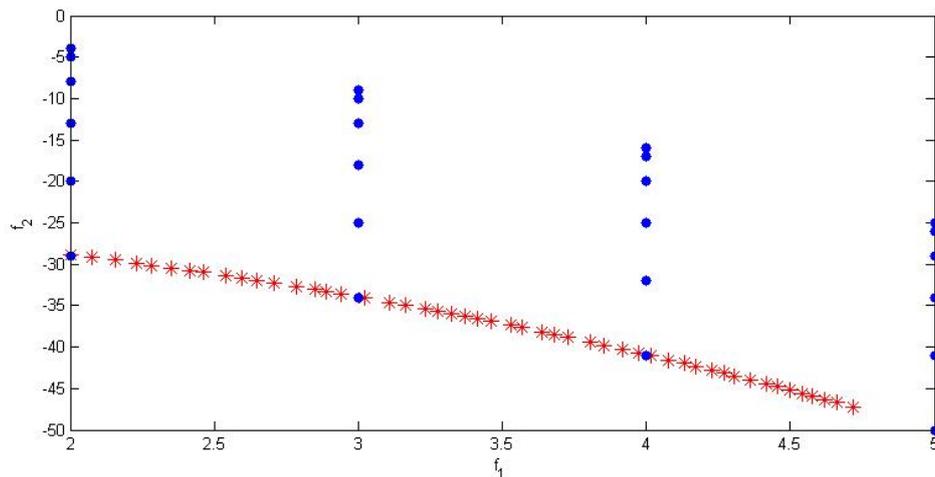


Figure 2 output of question3

We can see Pareto front as red line in Figure 2.