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Newton’s method

Introduction

Newton’s method is the way we can use it to find our roots. Meantime, we know the minimizer or maximizer appear in roots of the derivative (solutions to f'(x)=0). According to Figure 1,

$f'(x) = \frac{\Delta y}{\Delta x} = \frac{f(x1)}{x1-x2}$, $f'(x) * (x2-x1) = f(x1)$

$x2 = x1 - \frac{f(x1)}{f'(x1)}$, so $x_{k+1} = x_k - \frac{f(xk)}{f'(xk)}$.

The function which is twice differentiable is f(x). The minimum value appears in f' (x) = 0, so we can write $x_{k+1} = x_k - \frac{f'(xk)}{f''(xk)}$.

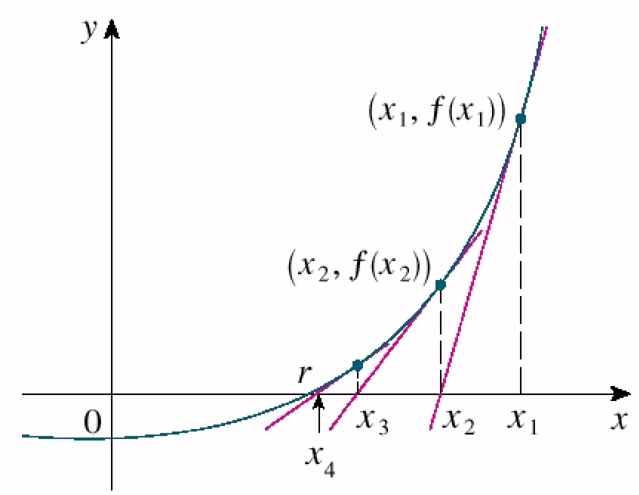

Figure 1Newton’s method

Question1

Implement a Newton’s method with the following specs:
Input: iteration number, initial point
Output: the minimizer, graph of the objective function with marks that illustrate the algorithm process

(a) f(x) = x^2/2-sin(x)

```
t = 0:0.01:3;                          %plot problem1
plot(t,(t.^2)/2-sin(t));
times = 5;                             %iteration number
xk = 1.5;                              %initial point
hold on;
plot(xk,(xk.^2)/2-sin(xk),'rx')        %plot initial point

syms x;
g = (x.^2)/2-sin(x);                   %problem1

g1 = diff(g);                          %find first differential equation
g2 = diff(g1);                         %find second differential equation

for i=1:times;                         %for times of loop
    x=xk;                              %substitute variable
    value1=subs(g1);                   %get first differential value
    value2=subs(g2);                   %get second differential value
    xk = xk - value1/value2;           %newton's method
    plot(xk,(xk.^2)/2-sin(xk),'rx')    %plot point
end
```

By using iteration number = 5 and initial point = 1.5, I got Figure 2 as my output. The minimizer is at x = 0.739085133215161.

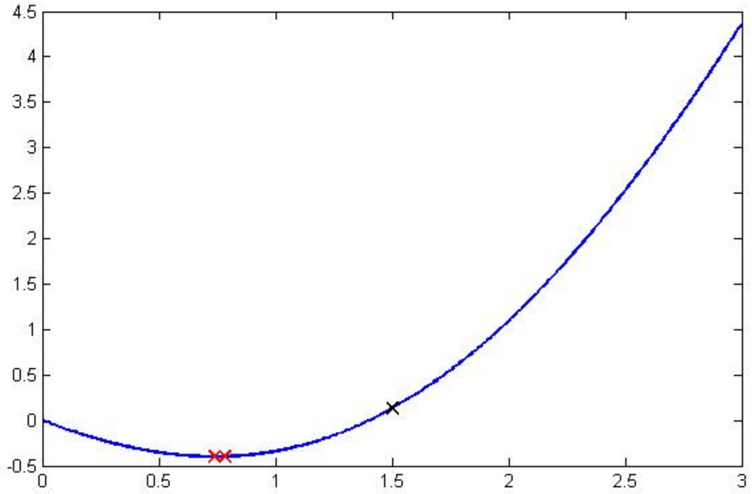

Figure 2output of problem 1

(b) f(x) = x^4-14x^3+60x^2-70x

```
t = 0:0.01:2;                                   %plot problem2
plot(t,t.^4 - 14*t.^3 + 60*t.^2 -70*t);
times = 5;                                      %iteration number
xk = 1.5;                                       %initial point
hold on;
plot(xk,xk.^4 - 14*xk.^3 + 60*xk.^2 -70*xk,'rx') %plot initial point

syms x;
g = x.^4 - 14*x.^3 + 60*x.^2 -70*x;             %problem2

g1 = diff(g);                                   %find first differential equation
g2 = diff(g1);                                  %find second differential equation

for i=1:times;                                  %for times of loop
    x=xk;                                       %substitute variable
    value1=subs(g1);                            %get first differential value
    value2=subs(g2);                            %get second differential value
    xk = xk - value1/value2;                    %newton’s method
    plot(xk,xk.^4 - 14*xk.^3 + 60*xk.^2 -70*xk,'rx')      %plot point
end
```

By using iteration number = 5 and initial point = 1.5, I got Figure 3 as my output. The minimizer is at x = 0.780884051178671.

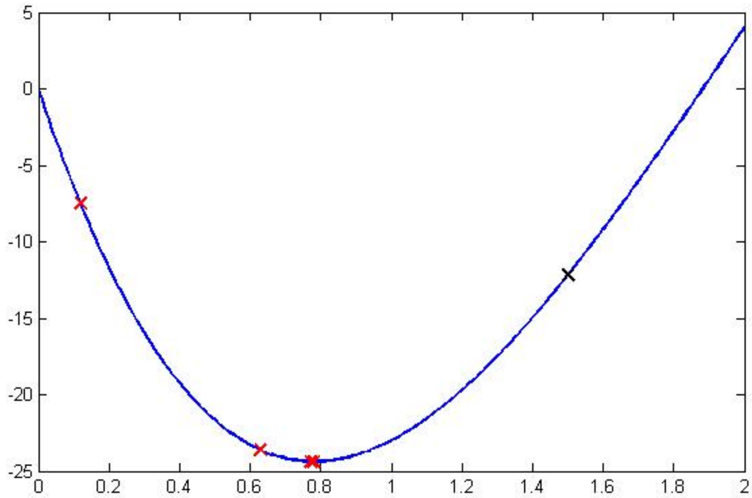

Figure 3 output of problem 2

(c) f(x) = 8e^(1-x)+7ln(x)

```
t = 1:0.01:2;                                   %plot problem3
plot(t,8*exp(1-t) + 7*log(t));
times = 5;                                      %iteration number
xk = 1.5;                                       %initial point
hold on;
plot(xk,8*exp(1-xk) + 7*log(xk),'rx')           %plot initial point

syms x;
g = 8*exp(1-x) + 7*log(x);                      %problem3

g1 = diff(g);                                   %find first differential equation
g2 = diff(g1);                                  %find second differential equation

for i=1:times;                                  %for times of loop
    x=xk;                                       %substitute variable
    value1=subs(g1);                            %get first differential value
    value2=subs(g2);                            %get second differential value
    xk = xk - value1/value2;                    %newton's method
    plot(xk,8*exp(1-xk) + 7*log(xk),'rx')       %plot point
end
```

By using iteration number = 5 and initial point = 1.5, I got Figure 4as my output. The minimizer is at x = 1.609381067723078.

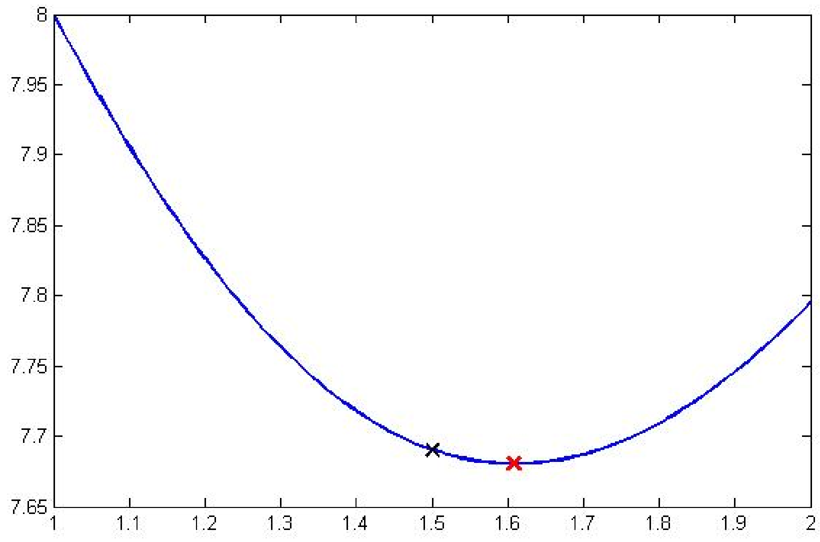

Figure 4 output of problem 3

● Question2

Describe a few situations in which your Newton's method can fail.

There are three situations as following: If our function is f(x), then

(a) There are no roots in f'(x), for example, $f'(x) = (x-2)^2+1$.

Using iteration number = 3 and initial point = 1.5 presents in Figure 5.

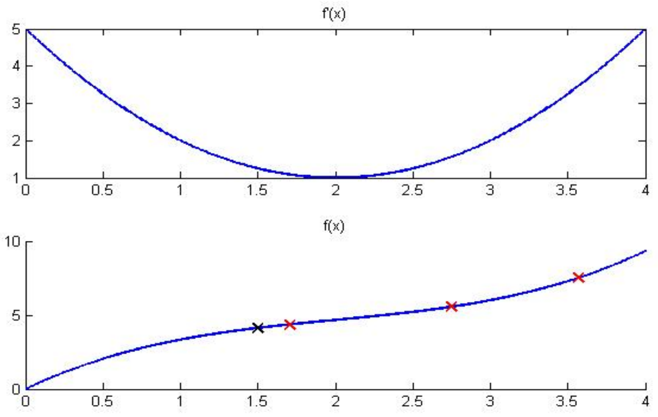

Figure 5 f'(x) = (x-2)2+1

(b) There are two roots in f'(x), for example, $f'(x) = (x-2)^2-1$.

Using iteration number = 3 and initial point = 1.5 presents in Figure 6. There are two roots in 1 and 3. If we choose initial point closed to 1, the minimizer is 1.

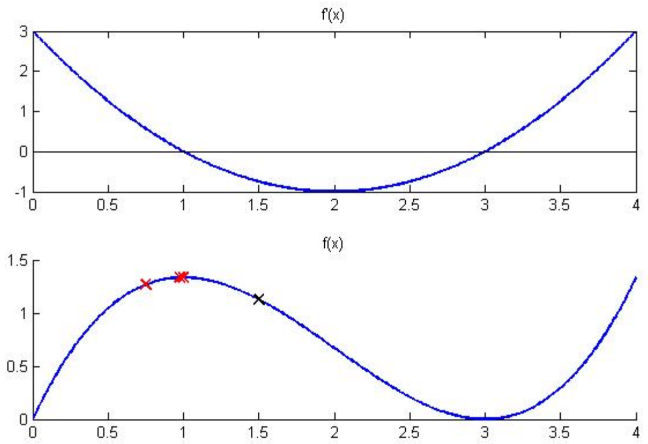

Figure 6 f'(x) = (x-2)2-1

(c) There are more roots in f'(x), for example, f'(x) = sin(x).

It's the same with (b). Our maximizer or minimizer will depends on which point we selected. We can find the final point which is the most close to our root.

(d) If we select the value on the point, f''(x) = 0.

For the function aboded mentioned, $f(x) = (x-2)^2-1$. Using x = 2, then it won't work.

(e) If the function cannot be differentiable, like f(x) = $x^{1/5}$ at x =0.

Using iteration number = 2 and initial point = 0.5 presents in Figure 7. It cannot converge successfully.

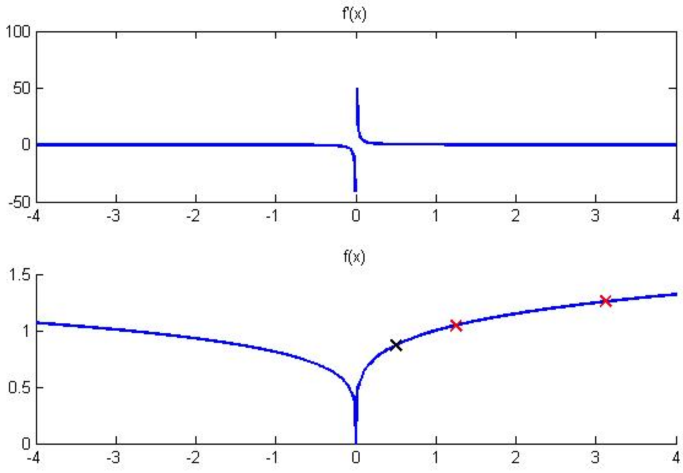

Figure 7 f(x) = $x^{1/5}$

- ## Question3

Describe the type of functions that your Newton's method can converge to the minimizer using only one iteration.
If f '(x) = linear function and f''(x) = constant, then it only needs once iteration.
For example,
f(x) = $3x^2$+6x
f'(x) = 6x + 6
f''(x) = 6
We can see Figure 8 as a result by iteration number = 5 and initial point = 0.5.

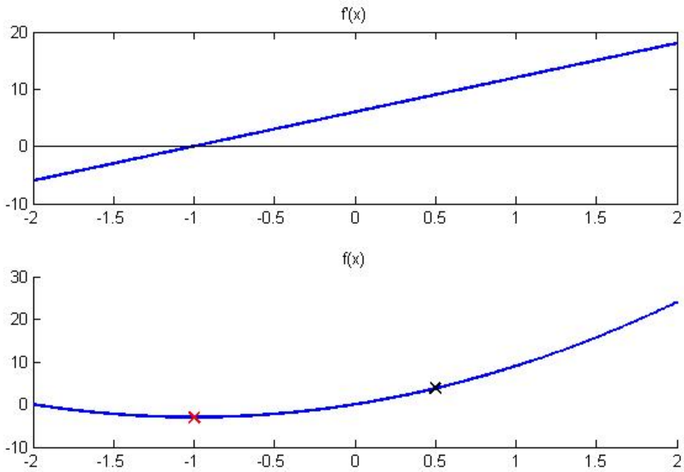

Figure 8 f(x) = $3x^2$+6x