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Gradient descent method(extra)

Introduction

The class before this week, we discussed about the Golden section search, the Newton's method and the Secant method. They are ways to find the minimizer but they are limited in two dimensions. The Gradient decent solved this problem. It used 'Gradient' to derivative a function into several dimensions.

$\boldsymbol{X}_{k+1} = \boldsymbol{X}_k$ - $\alpha_k * \nabla f(\boldsymbol{X}_k)$, which αk is step size and $\nabla f(\boldsymbol{X}_k)$ is search direction.

Question 1: f(x,y)=3*(x^2+y^2)+4*x*y+5*x+6*y+7

As a result, Figure 1 is the figure and difference for every step by the steepest descent algorithm. My step size = 0.2, x* = -0.7542 , y* = -1.2458 and f(x*,y*) = 5.8750.

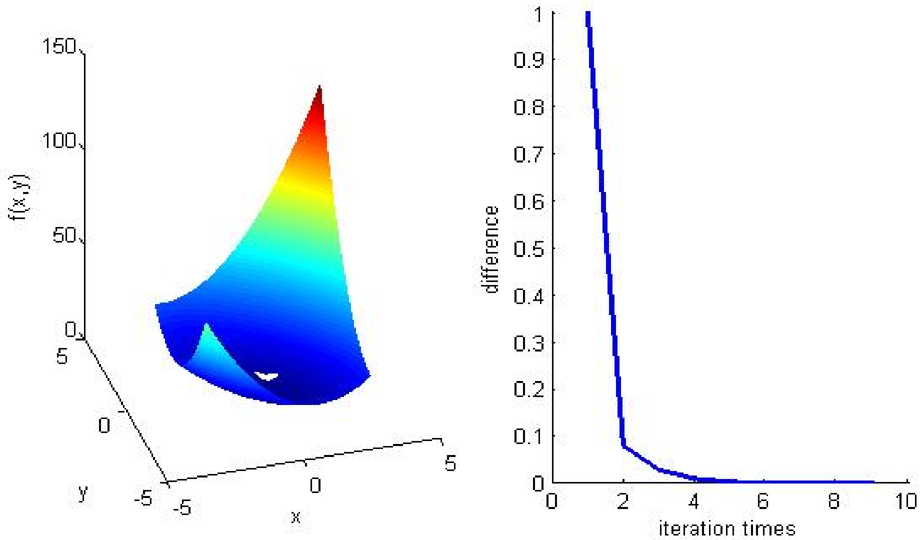

Figure 1. output of question 1

To find Hessian Matrix of Scalar Function, we can use the code

syms x; syms y;

f =(3.*(x.^2+y.^2)+4.*x.*y+5.*x+6.*y+7);

hessian(f,[x,y])

>> [6, 4]

>> [4, 6]

We can see that if Hessian Matrix is positive-definite, our function must have a local minimizer. On the other word, if it is negative-definite, our function must have a local maximizer.

● Question 2: f(x,y)=2*(x^2+y^2)+4*x*y+5*x+6*y+7

As a result, Figure 2 is the figure and difference for every step. My step size = 0.2 and initial point is (2, 1). We can see it cannot converge to the final point. There are infinite points we can find, and then the difference is almost the same.

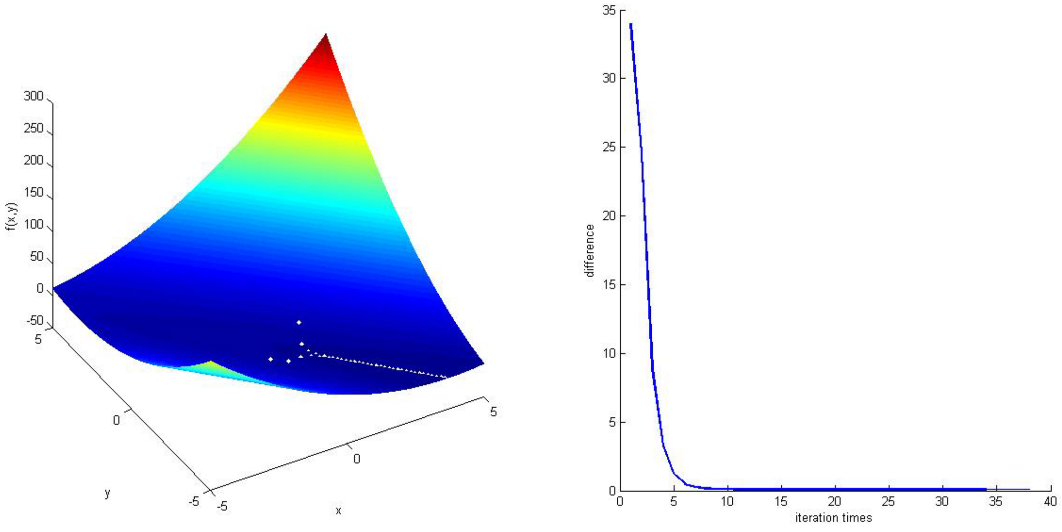

Figure 2. output of question 2

Figure 3 presents value of x and y in the left side and x+y in the right side. To observe the left figure, the unbounded situation will let our point x approached positive infinity and point y approached negative infinity. Furthermore, x+y will be a constant value as -1.3750, shown in the right hand side.

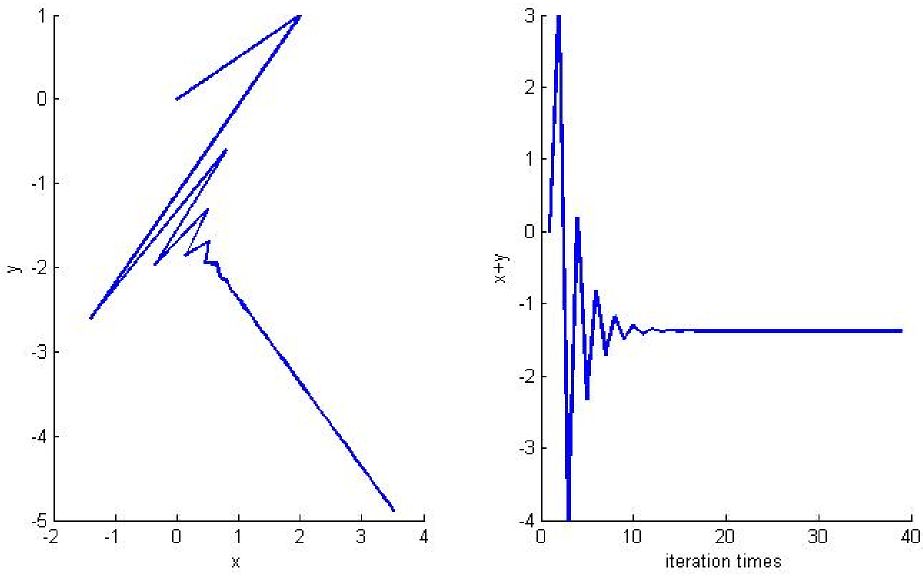

Figure 3. output of x and y