

Golden Section Search

● Introduction

For Golden section search, it needs to exist in only one local maximizer or minimizer. Figure 1 presents the function interval is from a to b , if we have to find the minimum point. According to properties, if $f(x_1) > f(x_2)$, then local minimizer is between a and x_2 . In other words, if $f(x_1) < f(x_2)$, then local minimizer is between x_1 and b . In Figure 2, we can select our section points by Golden ratio. We have

$$\frac{c}{a} = \frac{a}{b}, \quad \frac{b}{a} = \varphi \quad \text{and} \quad \varphi = \frac{1 + \sqrt{5}}{2}.$$

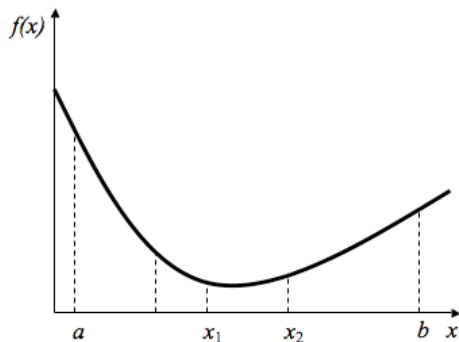


Figure 1 Diagram of a golden section search

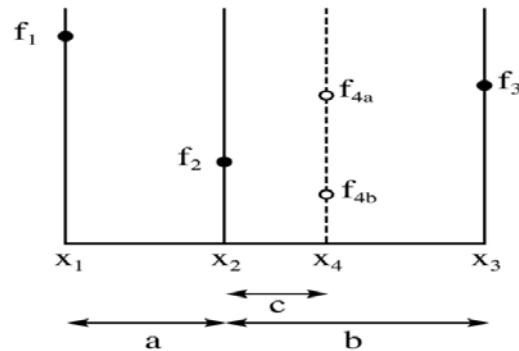


Figure 2 Golden section

● Question

Use the Golden section search to find a local minimizer for below function,

(a) $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ over $[0, 2]$

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B = 2;   A = 0;   times = 0;   range = 0.1;   %set the interval is from A to B
x = 0:0.01:2;   %range set 0.1
y = x.^4 - 14*x.^3 + 60*x.^2 - 70*x;
plot(x,y)   %use x and y to plot the function
while B - A > range   %set the condition to stop the loop
    a = (B-A)*2/(5+5^(1/2));   %value a
    b = (B-A) - a;   %value b
    x1 = A + a;   %value for section point 1
    x2 = A + a + a^2/b;   %value for section point 2
    x = x1;
    y1 = x.^4 - 14*x.^3 + 60*x.^2 - 70*x;   %To get y1
    x = x2;

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x2 = A + a + a^2/b;           %value for section point 2
x = x1;
y1 = 8*exp(1-x) + 7*log(x);   %To get y1
x = x2;
y2 = 8*exp(1-x) + 7*log(x);   %To get y2
if y1<y2                       %Based on the value magnitude to
    B = x2;                     % adjust interval
elseif y1>y2
    A = x1;
end
times = times + 1             %calculate how many times of loop
hold on; plot(A,A^4 - 14*A^3 + 60*A^2 -70*A,'rx');
hold on; plot(B,B^4 - 14*B^3 + 60*B^2 -70*B,'rx');
end
A; B;                         %print value A and B
times                          %print times
t = 7.65:0.001:8;
hold on; plot(A,t)
hold on; plot(B,t)

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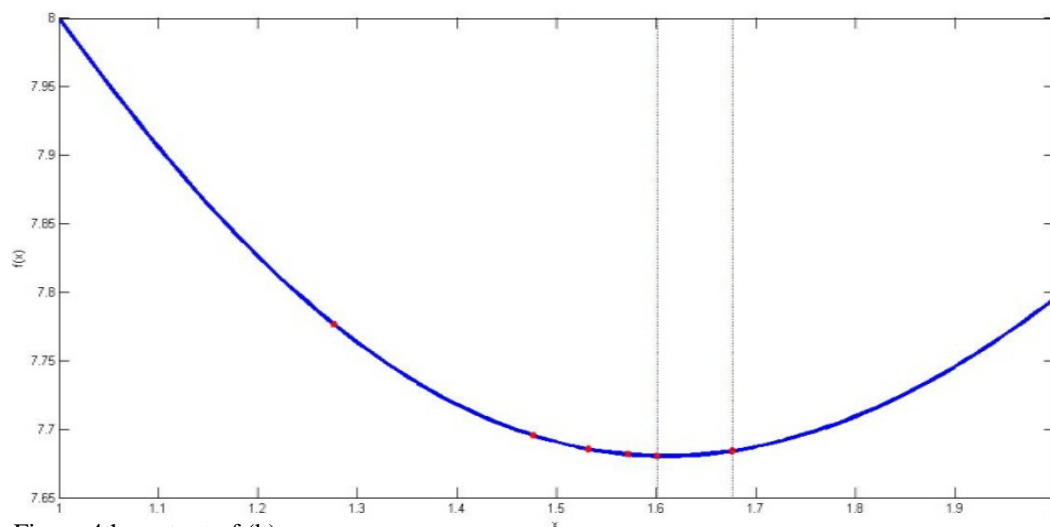


Figure 4the output of (b)

We can see the output of A and B in Figure 4.

A = 1.6006

B = 1.6764

Because I set the condition for stop is the interval less than 0.1, times = 6.

Remark:

If I set the interval is 0.000001,

A = 1.609380697125004 and B = 1.609381659249061. It's very close to minimizer.

times = 29.