Uniqueness of the U.S. Dollar *

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[Very preliminary – Comments are welcome]

Abstract

It is widely documented that the U.S. dollar appreciates against other currencies when the U.S. economy experiences large bad shocks ("in crisis"), while the currencies of other countries depreciate against the U.S. dollar when similar shocks hit these countries. In addition, bad economic news in "normal" times makes the U.S. dollar depreciate but after similarly bad or worse news in crisis times, the U.S. dollar appreciates. We propose a simple safe-haven based equilibrium model that can help explain these puzzles. Our empirical analysis supports new predictions of our model.

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1 Introduction

Extensive empirical literature has found that U.S. dollar tends to appreciate against other currencies when U.S. economy experiences large bad shocks ("in crisis"), while the currencies of other countries depreciate against U.S. dollar when similar shocks hit these countries (e.g., Bock and Filho (2015)). In addition, bad economic news in normal economic condition times makes U.S. dollar depreciate but when similarly bad or worse news hit in crisis times, U.S. dollar appreciates against currencies of countries which are in normal or good economic conditions (e.g., Fratzscher (2009)). We propose a simple safe-heaven based equilibrium model that can help explain these puzzles. Our empirical analysis supports new predictions of our model.

Specifically, we consider a simple two-period, pure exchange general equilibrium model with two countries (U.S. and Foreign), multiple states of the economy, and two consumption goods one of which is tradable across borders and the other one is not. We analyze two extreme cases of financial markets: 1. only insurance is traded and thus market is incomplete, 2. sufficient number of securities are traded such that the market is complete. In the worst state ("Disaster state"), the foreign country has very little endowment of the tradable good, while the U.S. has relatively more. The marginal utility goes to infinity as the tradable good consumption goes to zero for both representative agents in the two countries and the transition probability into the Disaster state can vary with the current state.

Consistent with empirical evidence, we show that when economic conditions are within normal fluctuations ("Normal state"), bad economic news (i.e., a drop in the initial endowment) in any country makes the currency of the country depreciate against the other currency. This is because to smooth consumption the demand for the other country's tradable goods and thus also currency increases ("wealth effect"). However, when the economic condition is so bad that that the U.S. gets into a "Crisis state", the impact of bad news on the exchange rate can be reversed for the U.S. Intuitively, the foreign country always has a demand for U.S. dollars because owning U.S. dollar can hedge against the low endowment in the Disaster state ("hedging effect"). A bad news in Crisis state can significantly increase the probability of getting into the Disaster state and thus because of the increased concern over the worst scenario, and the foreign country may increase the demand for the U.S. dollar if the hedging effect dominates the wealth effect. In contrast, if the foreign country gets into the Crisis state, then its currency depreciates even more against dollar because both the endowment effect and the hedging effect work in the same direction for the foreign country. The unique pattern of U.S. dollar exchange rate is consistent with the finding of Habib and Stracca (2012): the larger the size of the economy, or the stock market capitalization, relative to world GDP, the higher the currency excess returns in times of financial stress.

We show that these qualitative results hold across both the incomplete market and the complete market cases. With a complete market risk sharing is more efficient. Still, the incentive to hedge against the Disaster state remains the same and if the probability of getting into the Disaster state is increased, then the demand for dollar and thus the dollar exchange rate can increase as in the incomplete market case. We also consider extensions to dynamic equilibrium models and show that our main results also hold in these dynamic models.

Our empirical analysis confirms the empirical evidence on the change rate patterns found in the existing literature. In addition, we show that even the usually safe-haven currencies like Japanese yen and Swiss Franc depreciate against U.S. dollar when these countries experience bad economic shocks.

In the literature, there exist several explanations for the movement of US exchange rates in the business cycles. Maggiori (2013) attributes safe-haven characteristics of UD Dollar to the most-developed financial sector of the US economy, while Maggiori and Gabaix (2015) allude to different liquidity of different currencies involved. Our paper interprets safe-haven feature of a currency as the claim on a safe-haven economy, which is more direct and fundamental than above mentioned factors. Rare disasters have been also attributed to the exchange rate dynamics in e.g., Burnside et al. (2009), Emmanuel and Gabaix (2008), following an insight of a Peso problem raised originally by Rietz (1988) and recently re-examined by Barro (2006). These papers rely on complete financial markets to compute exchange rate (as ratio of stochastic discount factors of two countries involved). Our paper instead allows for both complete and incomplete market settings.

The paper is organized as follows. Section 2 presents previously-known and new empirical regularities concerning US dollar exchange rate movement in different periods, as well as around US macro-economic news announcements. Section 3 presents a baseline model of endowment economy to study US exchange rates. Section 4 considers US exchange rates in incomplete market setting. Section 5 considers US exchange rates in incomplete market setting. Section 6 summarizes main findings of the paper. Appendices contain technical derivations for all results of the main text.

2 Stylized Movements of Exchange Rates through Crises

2.1 Data

We perform two sets of analysis to highlight and support the main theoretical results of the paper. We begin by showing how US exchange rates react in crises stemming both within and outside the US. The graphical depiction in Figures 1 and 2 shows the US dollar has appreciated against a basket of currencies in several crises and recessions, including those located solely in the United States. In other words, in a variety of crises we see that the US Dollar (USD) tends to appreciate. This highlights the fact that the USD plays a unique role in the global market.

The next set of analysis shows the impact of US news in good times and bad. We consider ten important macroeconomic indicators concerning the US economy and the associated forecasts.[1] We find negative macroeconomic news in the US has a differential impact depending on whether it comes during a crisis or during normal times. This empirical fact again lends support for the assertion that the USD is unique in its position as a safe-haven currency.

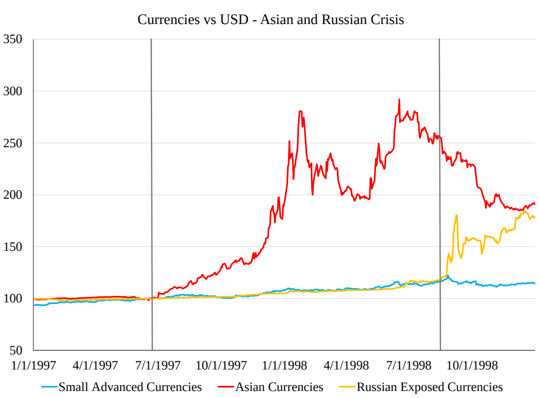
We begin by collecting exchange rate data from the Federal Reserve Economic Data (FRED) and Bloomberg over the period 1973-2014. All exchange rates are defined as the amount of foreign currency per USD. Therefore, an increase in this rate implies it takes more foreign currency to purchase one USD, and hence, the dollar is appreciating. Following Kohler (2010), we create baskets of currencies normalized to 100 at the beginning of various crises to understand how the dollar performs in crises. Replicating Kohler (2010), in Figure 1 we look at the Asian, Russian, and Financial crises. In each graph we define the basket of small currencies as those comprised of Australia, Canada, New Zealand, Norway, and Sweden. Asian currencies include South Korea, Malaysia, Thailand, and the Philippines. Finally the currencies exposed to Russia include Brazil, Chile, Russia, and South Africa. When we deviate from these groupings, we will highlight the alternative basket and the motivation for inclusion.

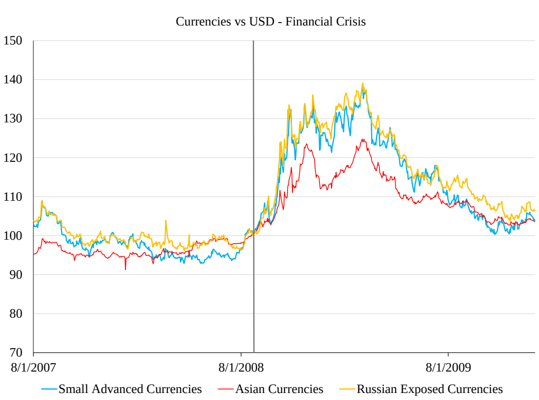
In Panel (a) shows the impact of the Asian Financial Crisis and the Russian Default Crises of the late 1990s. In Panel (b) shows the impact of the recent Financial crisis. In each case the USD appreciates. In particular, in the Financial crisis, we see that the USD appreciates against all three baskets.

[1] The ten macroeconomic indicators are; Industrial Production, Gross Domestic Product (GDP), Non-Farm Payroll, Unemployment, Institute for Supply Management's Manufacturing Index (NAPM/ISM), Consumer Confidence, Housing Starts, Consumer price index (CPI), Producer Price Index (PPI), US Trade Balance.

Figure 1: **Exchange Rates Vis-a-vis USD in Crises**

The figures below display a time-series plot of the daily average of scaled foreign exchange rate movements around various crises. The vertical gray line denotes the beginning date of each crisis as denoted in the panel titles. Each country's exchange rate, as expressed in US Dollars, is scaled to 100 on the day of the crisis. An increase in the average denotes an appreciation of the US Dollar. Each group represents the arithmetic average of the scaled exchange rate. Small advanced currencies include Australia, Canada, New Zealand, Norway, and Sweden. Asian currencies include Indonesia, Korea, Malaysia, and Thailand. Finally, the Russian-exposed currencies include Russia, South Africa, Brazil, and Chile. Panel (a) captures the Asian financial crisis of 1997 with a start date of July 2, 1997 and the Russian Default Crisis of 1998 with a start date of August 17, 1998. Finally, Panel (b) captures the Financial Crisis of 2008 with a start date of August 21, 2008. These specifications follow Kohler (2010).





The second analysis requires US macro-economic news. Following Fratzscher (2009), we define news as the difference between the median survey and the reported value for each of ten different macroeconomic indicators. These data can be taken from Bloomberg and the Appendix includes details regarding the specifics about the indices used.

2.2 US Exchange Rate in Different Crises: Cross-sectional Perspective

We extend the analysis of Kohler (2010) by examining the performance of the USD in other earlier crises: the Oil crisis of 1973, the US Stagflation Crisis of 1982, and the Tech crisis of 2001. As previously stated, we compute the arithmetic average of a basket of scaled currencies. Our three baskets of currencies in Figure 1 include a basket of small advanced currencies, Asian currencies, and currencies with exposure to Russia. Figure 2 shows the appreciation of the USD in times of distress associated with these earlier crises.

Altogether, Figures 1 and 2 demonstrate that the USD generally appreciates in times of distress, regardless of where the distress originates. The appreciation in the USD is unique and can be interpreted as the USD acting as a safe-haven currency in times of uncertainty. Furthermore, the state of the US economy allows other countries to update their likelihood of arriving in a crisis or disaster state. As the likelihood of entering a disaster state increases, the safe-haven aspect of the USD becomes more appealing and tend to cause the dollar to appreciate. Next, we build off Fratzscher (2009) to show that the foreign exchange rates are inextricably linked macro fundamentals and their implied relationship with the US economy.

2.3 US Exchange Rate in Great Recession: Time-series Perspective

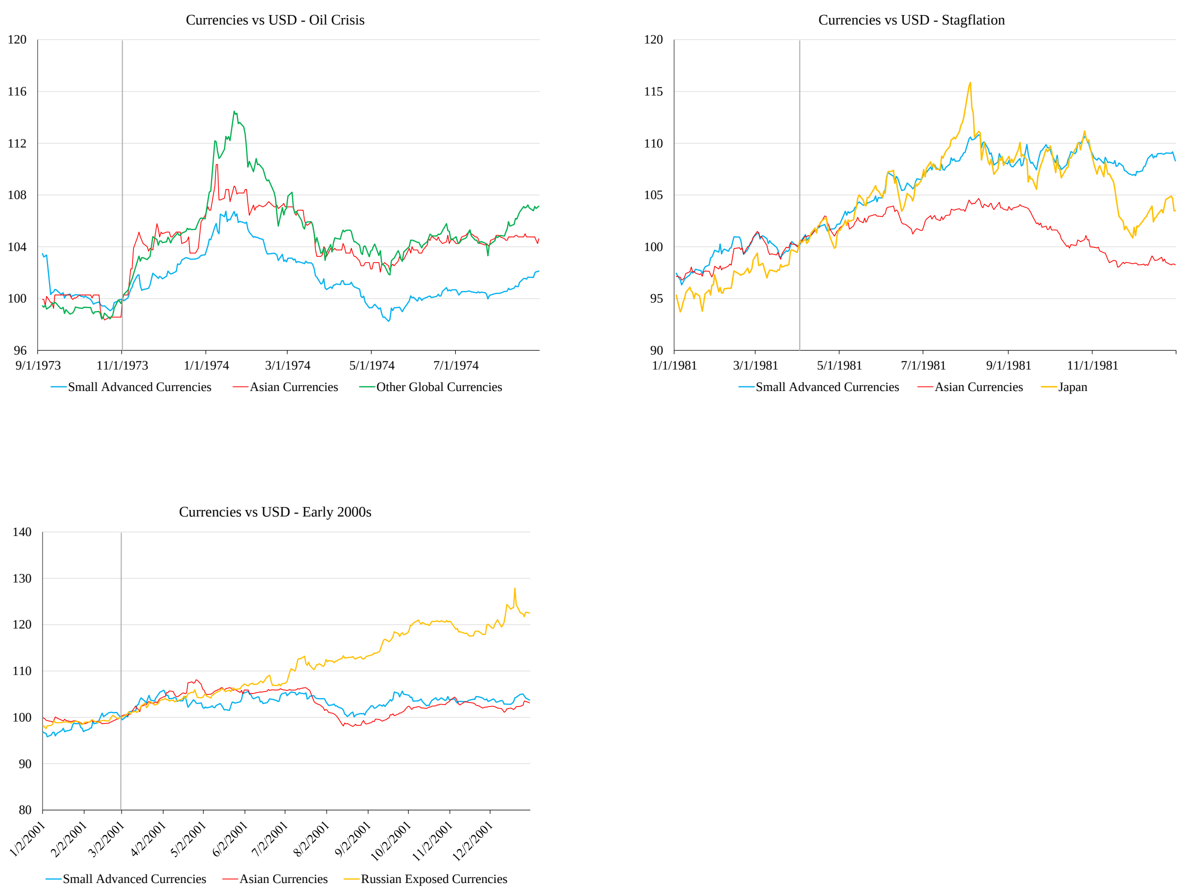
Bad news about the state of the US economy has different implications for the other economies and USD exchange rates when news arrive in different states of the US economy. This is because the market plausibly updates its expectations conditionally on the current state of the US economy. This point is alluded to in Fratzscher (2009) who shows that negative news during the crisis is associated with the appreciation of the dollar across a broad array of currencies. We replicate and extend regression results from Fratzscher (2009) below.

We separate the effects of the period prior to the crisis (1/1/1996-6/30/2008) and the crisis period (7/1/2008-1/31/2010). For each of the major US macro-economic news releases we create a dummy ("Negative News") that takes the value of one if the reported value was below the median survey estimate for that period. Using the panel data from 54 countries, we regress countries' currency return on news dummies,

$$Return_{t,c} = \alpha + \sum_{news=1}^{10} \beta_{news} \text{Negative News}_t + \gamma_c + \varepsilon_{t,c}.$$

Figure 2: **Exchange Rates Vis-a-vis USD in Crises**

The figures below display a time-series plot of the daily average of scaled foreign exchange rate movements around various crises. The vertical gray line denotes the beginning date of each crisis as denoted in the panel titles. Each country's exchange rate is scaled to 100 on the day of the crisis. Each country's exchange rate, as expressed in US Dollars, is scaled to 100 on the day of the crisis. An increase in the average denotes an appreciation of the US Dollar. Each group represents the arithmetic average of the scaled exchange rate. Small advanced currencies include Australia, Canada, New Zealand, Norway, and Sweden. Asian currencies include Indonesia, Korea, Malaysia, and Thailand. Finally, the Russian-exposed currencies include Russia, South Africa, Brazil, and Chile. These specifications follow Kohler (2010). Panel (a) captures the Oil Crisis of 1973 with a start date of November 1, 1973. Panel (b) captures the US Stagflation Crisis of 1982 with a start date of July 1, 1982. Finally, Panel (c) captures the Tech Crisis of 2001 with a start date of March 1, 2001.



Regression results are reported in Table 1. We are interested on the sign of the slope coefficients associated with news dummies, and its possible change across pre-crisis and crisis periods.

The intuition is that if a country is currently in a crisis state, bad news originated from a safe-haven economy may signal even worse news for the country's economy. Therefore, bad US macro-

Table 1: U.S. Macroeconomic News In and Out of Crises

The table displays the effect of negative U.S. macroeconomic news on the average daily return of 54 different currencies. We separate the effects of the period prior to the crisis (1/1/1996-6/30/2008) and the crisis period (7/1/2008-1/31/2010). For each of the major US macro-economic news releases we create a dummy that takes the value of one if the reported value was below the median survey estimate for that period, taken from Bloomberg. The regression includes fixed effects and clusters standard errors at the currency-level. ***, **, * indicates significance at the 1%, 5%, and 10% levels, respectively. The regression, below, has an observation for each country (c) and each day (t) during the sample.

$$Return_{t,c} = \alpha + \sum_{news=1}^{10} \beta_{news} \text{Negative News}_t + \gamma_c + \varepsilon_{t,c}$$

	Pre-Crisis	**Crisis**	**Difference**	**P-Value**
Industrial Production	0.0469***	0.0371	-0.01	0.659
	(0.01)	*(0.02)*		
GDP	0.0283*	0.0794**	0.05	0.243
	(0.02)	*(0.03)*		
Non-Farm Payroll	0.0782***	-0.0060	-0.08	0.063
	(0.01)	*(0.04)*		
Unemployment	0.0118	0.0006	-0.01	0.435
	(0.01)	*(0.04)*		
NAPM/ISM	0.0844***	-0.1297***	-0.21	0.000
	(0.01)	*(0.04)*		
Consumer Confidence	0.0482***	-0.2516***	-0.30	0.000
	(0.01)	*(0.04)*		
Housing Starts	0.0497***	0.1371***	0.09	0.001
	(0.01)	*(0.03)*		
CPI	0.0215**	-0.2315***	-0.25	0.000
	(0.01)	*(0.06)*		
PPI	0.0345***	-0.0519*	-0.09	0.021
	(0.01)	*(0.03)*		
US Trade Balance	0.0364***	-0.0360	-0.07	0.341
	(0.01)	*(0.04)*		
Intercept	-0.0153***	-0.0237***		
	(0.00)	*(0.00)*		
Observations	177,683	21,353		
R2	0.003	0.004		

economic news arriving in pre-crisis period would weaken the USD ($\beta_{news} > 0$), while similarly bad US macro-economic news arriving in crisis period might strengthen the USD ($\beta_{news} < 0$). Table (1) empirically confirms this pattern for almost all US macro-economic indicators under consideration.

3 A Stylized Model of International Asset Pricing

We consider a setting of international endowment economy in discrete time with two countries $i \in \{h, f\}$. We employ the standard filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_t, \mathbf{P}\}$ to model uncertainties in international markets, wherein $\{\mathcal{F}_t\}_t$ is the natural filtration associated with the time-evolution of countries' endowments. We consider a stylized two-period setting, $t \in \{0, 1\}$. The choice of a two-country two-period setting is for illustration and conveniences, and can be generalized to a multiple-country and multiple-period setting at the cost tractability.

Endowments

Each country $i \in \{h, f\}$ is endowed with both tradable (T) endowments $\{e_{iT0}, e_{iT1}(s)\}$ and non-tradable (N) endowments $\{e_{iN0}, e_{iN1}(s)\}$ in periods $t \in 0, 1$ respectively, with $s \in \Omega \equiv \{1, \ldots, N\}$ denoting the state at $t = 1$. Tradable endowment is a consumption good that is consumed by all countries. Whereas, nontradable endowment is a country-specific consumption good that can only be consumed by the host country. All goods are perishable and must be consumed in the same period in which their endowments arrive. Therefore, the following resource constraints hold at all times and states,

$$c_{hNt}(s) = e_{hNt}(s), \qquad c_{fNt}(s) = e_{fNt}(s), \quad \forall t \in \{0, 1\}, s \in \Omega, \tag{1}$$

for nontradable endowments, and

$$c_{hTt}(s) + c_{fTt}(s) = e_{hTt}(s) + e_{fTt}(s) \equiv e_{Tt}(s), \quad \forall t \in \{0, 1\}, s \in \Omega, \tag{2}$$

for tradable endowments. $e_{Tt}(s)$ denotes aggregate tradable endowment at time t and state s. Time-one endowments $\{e_{iN1}(s), e_{iT1}(s)\}$, $i \in \{h, f\}$, are the sole source of uncertainty in the economy.

Preferences

At macro level, each country is represented by a (representative) agent $i \in \{h, f\}$. The agents (interchangeably, countries) differ in their risk aversions and time preferences. There is no information asymmetry. Countries maximize expected utilities of consuming tradable and nontradable consumption goods,

$$U_i = \sum_{t=0}^{1} \sum_{s \in \Omega} p(s) \beta_i^t u(c_{it}(s)), \qquad i \in \{h, f\}, \tag{3}$$

where $\{p(s)\}$ denotes the distribution of future states, $\beta_i \leq 1$ country i's time discount factor. Strictly increasing, convex and continuously differentiable function $u(c_{it})$ denotes country i's period utility (or felicity) over consumption aggregator c_{it}, the latter being a function of tradable and nontradable consumptions.

$$\begin{cases} c_{it}(s) \equiv c_{iTt}^{\epsilon_i}(s)\, c_{iNt}^{1-\epsilon_i}(s), & \epsilon_i \in [0,1], \\ \frac{-c_{it}(s) u''(c_{it}(s))}{u'(c_{it}(s))} = \gamma_i\left(c_{it}(s)\right), & \gamma_i\left(c_{it}(s)\right) > 0, \end{cases} \qquad \forall i \in \{h, f\}, t \in \{0, 1\}, s \in \Omega. \tag{4}$$

Coefficient $\gamma_i\left(c_{it}(s)\right)$ (or simply, γ_{it}) denotes country i's (possibly state-dependent and time-varying) relative risk aversion. We assume that γ_{it} is strictly positive over the domain of positive consumption agregator $c_{it} \in \mathbf{R}^+$. Whereas ϵ_i characterizes i's taste for tradable consumption good. Note that the consumption aggregator c_{it} is a homogeneous function (of degree one) of tradable and nontradable consumptions.[2]

[2]For every country i, the elasticity of substitution between tradable and nontradable consumption goods is unit (independent of taste ϵ_i) in each time and state,

$$\phi_i \equiv \frac{\partial \log \frac{c_{iTt}(s)}{c_{iNt}(s)}}{\partial \log \frac{p_{iNt}(s)}{p_{iTt}(s)}} = \frac{\partial \log \frac{c_{iTt}(s)}{c_{iNt}(s)}}{\partial \log \frac{u'_{iNt}(s)}{u'_{iTt}(s)}} = 1, \quad \forall i \in \{h, f\}.$$

where the last equality follows from (5). The elasticity of substitution ϕ_i characterizes how the relative expenditure $\frac{p_{iTt}(s)c_{iTt}(s)}{p_{iNt}(s)c_{iNt}(s)}$ on consumption goods changes as relative price $\frac{p_{iTt}(s)}{p_{iNt}(s)}$ changes,

$$\frac{\partial\left(p_{iTt}(s)c_{iTt}(s)/p_{iNt}(s)c_{iNt}(s)\right)}{\partial\left(p_{iTt}(s)/p_{iNt}(s)\right)} = \frac{c_{iTt}(s)}{c_{iNt}(s)}\left(1 - \phi_i\right) = 0.$$

Consumption Good Prices

Because tradable consumption good is common to both countries, it is convenient to employ it as the numeraire and express value of other goods and assets in unit of the tradable consumption good.[3] In equilibrium, price of country i's nontradable good equals ratio of country i's marginal utilities of consuming nontradable and tradable goods,

$$p_{iNt}(s) = \frac{u'_{iNt}(s)}{u'_{iTt}(s)} = \frac{1-\epsilon_i}{\epsilon_i}\,\frac{c_{iTt}(s)}{c_{iNt}(s)}, \qquad \forall t \in \{0.1\}, s \in \Omega, \tag{5}$$

where u' denotes first-order partial derivative of utility function with respect to consumption (see Appendix A.1) evaluated at equilibrium.

Price Indices and Real Exchange Rate

Given the nontradable consumption good price $p_{iNt}(s)$ (in unit of tradable consumption good), country i's price index $Q_{it}(s)$ at time t and state s is the minimum-cost consumption basket $\{c^*_{iT}, c^*_{iN}\}$ that delivers some constant notional amount of utility,[4]

$$Q_{it}(s) \equiv \min_{c_{iT}, c_{iN}} c_{iT} + p_{iNt}(s)c_{iN}, \qquad \text{subject to} \qquad c_i = c_{iT}^{\epsilon_i}\, c_{iN}^{1-\epsilon_i} = 1,$$

where we have used the specification (4) for the aggregator c_i. The resulting basket compositions are

$$c^*_{iT} = \left(\frac{\epsilon_i}{1-\epsilon_i}\right)^{1-\epsilon_i} p_{iNt}^{1-\epsilon_i}(s), \qquad c^*_{iN} = \left(\frac{\epsilon_i}{1-\epsilon_i}\right)^{-\epsilon_i} p_{iNt}^{-\epsilon_i}(s),$$

and the price index (in unit of tradable consumption goods) is, $\forall i \in h, f, t \in 0, 1, s \in \Omega$,

$$Q_{it}(s) = (1-\epsilon_i)^{\epsilon_i - 1}\, \epsilon_i^{-\epsilon_i}\, p_{iNt}^{1-\epsilon_i}(s) = (1-\epsilon_i)^{\epsilon_i - 1}\, \epsilon_i^{-\epsilon_i} \left(\frac{u'_{iNt}(s)}{u'_{iTt}(s)}\right)^{1-\epsilon_i}. \tag{6}$$

In consumption setting, the real exchange rate $S_t(s)$ is the ratio of countries' price indices. We adopt the per-currency-h convention for the exchange rate; a unit of country h's consumption

[3] In this convention, price of tradable consumption good at all time and state is identically one, $p_{Tt}(s) \equiv 1$, $\forall s, t$.

[4] See e.g., Obstfeld and Rogoff (1996). Because $u(c_i)$ is a strictly monotone function of c_i, and the later is homogenous function of degree one of the basket (c_{iT}, c_{iN}), optimizing the basket subject to a constant notional amount of utility is equivalent to optimizing the basket subject to a constant notional amount of the consumption aggregator.

basket values as many as $S_t(s)$ units of f's basket at time t and state s,

$$S_t(s) = \frac{Q_{ht}(s)}{Q_{ft}(s)} = \frac{(1-\epsilon_h)^{\epsilon_h - 1}}{(1-\epsilon_f)^{\epsilon_f - 1}} \, \frac{\epsilon_h^{-\epsilon_h}}{\epsilon_f^{-\epsilon_f}} \, \frac{p_{hNt}^{1-\epsilon_h}(s)}{p_{fNt}^{1-\epsilon_f}(s)} = \frac{\epsilon_h^{-1}}{\epsilon_f^{-1}} \left(\frac{c_{hTt}(s)}{e_{hNt}(s)}\right)^{1-\epsilon_h} \left(\frac{c_{fTt}(s)}{e_{fNt}(s)}\right)^{\epsilon_f - 1}, \quad (7)$$

where in the last equality we have used nontradable consumption good prices (5) and resource constraints (1). Note that this expression for the real exchange rate holds for either complete or incomplete financial market, because it is derived as the ratio of countries' price indices.[5]

We note that the exchange rate (or relative value of home currency) increases with home tradable consumption and decreases with home nontradable endowment. The intuition is as follows. First, when home equilibrium tradable consumption increases, home marginal utility of tradable consumption decreases and home nontradable consumption good price p_{hNt} (in unit of tradable consumption good) increases. As a result, home consumption basket (or price index) Q_{ht} is more expensive (in unit of tradable consumption good), and exchange rate increases. Second, when home nontradable endowment increases, home nontradable consumption good price p_{hNt} drops, and so does the home price index Q_{ht}. As a reasult exchange rate decreases. By the same intuition, exchange rate decreases with foreign tradable consumption and increases with foreign nontradable endowment.

The Ex-ante Value of the US Dollar

The focus of this paper is the current exchange rate S_0, or the *ex-ante value of the US Dollar*, at current time $t = 0$ in equilibrium. We perform a *comparative analysis* on equilibrium exchange rate S_0 across various economic premises, which differ from one another in current endowments and equilibrium asset holdings, as well as the distribution (expectations and supports) of future endowments. This comparative analysis is useful when we wish to compare *snapshots* of the world economy at different points in time. For tranquil period, e.g., the economic conditions countries h and f face in 2004 and their expectations about future economy going into 2005, differ from economic conditions in 2008, and expectation going into 2009, for the turmoil period. Consequently, compared to itself, each country settles on different consumption and saving choices in competitive equilibrium, resulting in different equilibrium exchange rates across different periods.

[5] In international asset pricing, exchange rate is usually derived as the ratio of countries' stochastic discount factors, which is valid only when financial market is complete.

Specifically, the variation of ex-ante (at $t = 0$) equilibrium exchange rate is,[6]

$$\frac{dS_0}{S_0} = -\left\{(1-\epsilon_h)\frac{de_{hN0}}{e_{hN0}} - (1-\epsilon_f)\frac{de_{fN0}}{e_{fN0}}\right\} + \left\{(1-\epsilon_h)\frac{dc_{hT0}}{c_{hT0}} - (1-\epsilon_f)\frac{dc_{fT0}}{c_{fT0}}\right\}. \quad (8)$$

The first two terms capture the direct effect of the variation of nontradable endowments on exchange rate. Whereas the last two terms capture the indirect effects: variations of all exogenous quantities[7] induce variations of equilibrium tradable consumptions, which in turn foster variations of the exchange rate. The intuition is identical to that discuss below (7), an increase in home equilibrium tradable (resp., nontradable) consumption increases (resp., decreases) home nontradable consumption good price, and thus the home price index and exchange rate.

How equilibrium tradable consumptions vary with exogenous quantities in the economy depend on the assets that countries can trade in international financial markets. Below we study the exchange rate S_0 in different settings of complete and incomplete financial markets.

4 Incomplete Financial Markets

4.1 Asset Markets: The Insurance Contract

In this section, to focus on the safe-haven aspects of country h's currency, we consider the an international financial market setting in which the only traded asset at $t = 0$ is the insurance contract of zero-net supply. Each insurance contract pays one unit of tradable consumption good in any state s at time $t = 1$. Therefore, the insurance is a bond which is risk-free in tradable consumption good denomination. Trading this insurance allows countries to lock in sure payoff in tradable consumption good. The payoff thus does not contain any country's non-tradable consumption good that delivers no utility to the other country. In this sense, the contract captures the insurance value in delivering surely useful consumption.

Insurance price is I_0 at $t = 0$ in unit of tradable consumption good. Country i ($i \in \{h, f\}$) buys $\alpha_i \in \mathbf{R}$ units of insurance at $t = 0$. In this convention, when $\alpha_i < 0$, country i sells $-\alpha_i$ units of

[6] Similarly, the variation of ex-post (at $t = 1$) equilibrium exchange rate is,

$$\frac{dS_1(s)}{S_1(s)} = -\left\{(1-\epsilon_h)\frac{de_{hN1(s)}}{e_{hN1(s)}} - (1-\epsilon_f)\frac{de_{fN1(s)}}{e_{fN1(s)}}\right\} + \left\{(1-\epsilon_h)\frac{dc_{hT1(s)}}{c_{hT1(s)}} - (1-\epsilon_f)\frac{dc_{fT1(s)}}{c_{fT1(s)}}\right\}, \quad \forall s \in \Omega.$$

[7] The exogenous quantities in the economy are countries' curent (tradable and nontradable) endowments, as well as the distribution (expectations and supports) of future endowments.

insurance at $t = 0$. Insurance markets clear at $t = 0$,

$$\alpha_f = -\alpha_h. \tag{9}$$

Countries choose consumption plan $\{c_{iT0}, c_{iT1}(s)\}$ and insurance holding α_i to maximize their expected utilities (3) subject to their budget constraints,

$$c_{iT0} + \alpha_i I_0 = e_{iT0}, \quad c_{iT1}(s) = e_{iT1}(s) + \alpha_i, \quad \forall i \in \{h, f\}, s \in \Omega. \tag{10}$$

The resulting first order conditions (FOC) determine the insurance price,

$$\beta_h \sum_s p(s) \frac{u'_{hT1}(s)}{u'_{hT0}} = I_0 = \beta_f \sum_s p(s) \frac{u'_{fT1}(s)}{u'_{fT0}}. \tag{11}$$

Insurance Demand

Following the above expressions, several observations about the equilibrium insurance demand in incomplete international financial market are in order. First, if foreign country's tradable endowment $e_{f1}(s)$ is sufficiently low in some future state s, then f must buy insurance from home country h. Indeed, in the budget constraint (10) at $t = 1$ for state s, consumption $c_{fT1}(s)$ must be strictly positive,[8] which requires strictly positive $\alpha_f > 0$ (or country f buys insurance) when $e_{f1}(s)$ is small enough.[9]

Second, in this situation, foreign country f needs to contractually buy tradable consumption good from h in all future state; $c_{fT1}(s) - e_{fT1}(s) = \alpha_f > 0$, $\forall s \in \Omega$. This property illustrates the inefficiency and countries' imperfect risk sharing posed by incomplete financial market. Once f faces a possibility of a disaster state in future, f must contractually accept consumption transfer from h in all future states, including those in which f's endowments are abundant. Thus the incomplete financial market limits the ability of countries to smooth their consumptions across states. At $t = 1$, every country broadly sticks with their endowment in equilibrium because endowments and equilibrium consumptions have exactly the same ordering,

$$e_{iT1}(s) > e_{iT1}(s') \Longleftrightarrow c_{hT1}(s) > c_{hT1}(s'), \quad \forall i \in \{h, f\}, s, s' \in \Omega. \tag{12}$$

[8] The specification (4) indicates that f's marginal utility blows up as $c_{fT1}(s) \to 0$.

[9] By identical argument, technically, we also need to assume that home endowments $e_{h1}(s)$ are sufficiently positive for all states s.

We make an assumption that foreign country buys insurance initially. In light of the above discussion, this assumption holds naturally when foreign country faces a possible future state with very low output. Furthermore, the common practice of countries holding the US Treasuries is also in line with this assumption.[10]

Assumption 1 (Insurance demand) *Country f has net positive holdings of insurance in period $t = 0$,*

$$\alpha_f > 0.$$

We present below a simple sufficient condition for Assumption 1.

Proposition 1 *If countries' exogenous endowments $\{e_{hTt}(s), e_{fTt}(s)\}$ satisfy the following inequality,*

$$\beta_h \sum_s p(s) \frac{u'_{hT1}(s)\big|_{e_{hT1}(s)}}{u'_{hT0}\big|_{e_{hT0}}} < \beta_f \sum_s p(s) \frac{u'_{fT1}(s)\Big|_{e_{fT1}(s)}}{u'_{fT0}\Big|_{e_{fT0}}}, \tag{13}$$

then Assumption 1 holds.

The intuition underlying the above result is simple. The inequality (13) implies that the no-trade consumption configuration $c_{iTt}(s) = e_{iTt}(s)$, $\forall i \in \{h, f\}$, $t \in \{0, 1\}$, $s \in \Omega$, can not sustain the equilibrium condition (11). As a result, in equilibrium it must be that $c_{fT0} < e_{fT0}$, and $c_{fT1}(s) > e_{fT1}(s)$, $\forall s$, or Assumption 1 is fulfilled.

The incomplete market, on one hand, makes the insurance contract highly valuable as the only available asset to hedge against possible future disasters. On the other, the insurance contract exhibits the above-mentioned costly limitations, and thus would also be less valuable, in the incomplete market. Which of these two opposing factors prevails depends on countries' endowment distributions. An unambiguous relationship between value of insurance contract and the endowment distributions is economically related to, and therefore can be established under, an assumption about the price elasticity of insurance demand. We elucidate the role of this price elasticity next, before revisiting the impacts of endowment distributions on insurance demands and the exchange rate.

[10]In the our current simplified setting, the traded insurance largely substitutes for bonds.

Price Elasticity of Insurance Demand

We notice a close relationship between the price elasticity of insurance demand (PED),

$$\theta_f \equiv \frac{\partial \alpha_f / \alpha_f}{\partial I_0 / I_0}, \tag{14}$$

and the sensitivity of country f's insurance expenditure to insurance price,

$$\frac{\partial \left(\alpha_f I_0\right)}{\partial I_0} = \alpha_f \left(1 + \theta_f\right). \tag{15}$$

Thus, when country f buys insurance ($\alpha_f > 0$), f's elastic demand for insurance suffices to ensure that its insurance expenditure decreases with insurance price, $\theta_f < -1 \rightarrow \frac{\partial(\alpha_f I_0)}{\partial I_0} < 0$. Accordingly, we make the following assumption.

Assumption 2 (Elastic insurance demand) *Country f has elastic demand for insurance* (14),

$$\theta_f < -1.$$

The assumption of elastic insurance demand can be expressed more explicitly in term of equilibrium consumptions,

$$\alpha_f \left(1 + \theta_f\right) = \frac{\frac{I_0}{u''_{fT0}} \beta_f K_f}{I_0^2 + \beta_f \sum_{s \in \Omega} p(s) \frac{u''_{fT1}(s)}{u''_{fT0}}},$$

where

$$K_f \equiv \sum_s p(s) \left[u'_{fT1}(s) + \alpha_f u''_{fT1}(s)\right]. \tag{16}$$

Clearly, under the premise that foreign country buys insurance initially (Assumption 1), the positivity of the elastic insurance demand in Assumption 2 is equivalent to the following condition,

$$K_f > 0. \tag{17}$$

Finally, we note that positive and elastic insurance demand by foreign country (Assumptions 1 and 2) imply unambiguously a negative sign for the following quantity $\mathcal{T}$,

$$\left.\begin{aligned} \theta_f &< -1, \\ \alpha_f &> 0, \end{aligned}\right\} \Longrightarrow \mathcal{T} < 0. \tag{18}$$

with

$$\mathcal{T} \equiv I_0^2 \left(\frac{u'_{hT0}}{u''_{hT0}} + \frac{u'_{fT0}}{u''_{fT0}} \right) + \sum_s p(s) \left[\beta_h \frac{u''_{hT1}(s)}{u''_{hT0}} \left(\frac{u'_{fT0}}{u''_{fT0}} + \alpha_h I_0 \right) + \beta_f \frac{u''_{fT1}(s)}{u''_{fT0}} \left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0 \right) \right]. \tag{19}$$

This quantity $\mathcal{T}$ plays key technical role in the variations of exchange rate under changes in exogenous endowments and expectations. Economically, $\mathcal{T}$ characterizes the tatonnement stability of the economy (Appendix A.2).

4.2 Variations of Price and Demand of Insurance

In the current setting, we are interested in variations of insurance price I_0 and demand α_h because they induce variations of the ex-ante value of the US Dollar. Expression (31) below shows how variations of insurance expenditure $\alpha_h I_0$ contributes to variations of exchange rate. We explore the variations of insurance market due to exogenous changes in endowments (endowment effect) and expectations (expectation effect) in turn.

Endowment Effect on Insurance Market

The following lemma shows how insurance price and demand change with exogenous variations of foreign country's endowments.

Lemma 1 (Variations of insurance price and demand) *Assume home country has sufficiently low current risk aversion $\gamma_{h0} < 1$. Also assume either (i) elastic insurance demand $\theta_f < -1$ (Assumption 2), or (ii) foreign country has sufficiently low current risk aversion, $\gamma_{f0} < 1$, or (iii) foreign country currently (at $t = 0$) buys insurance, $\alpha_f > 0$ (Assumption 1). All else being equal,*

1. *when foreign country's current (either nontradable or tradable) endowments are higher, both current insurance price I_0 and current insurance demand α_f by foreign country are higher,*

$$\{\textit{either} \quad de_{fT0} > 0, \quad \textit{or} \quad de_{fN0} > 0\} \Longrightarrow \{dI_0 > 0, \quad \textit{and} \quad d\alpha_f > 0\}.$$

2. *when foreign country's current (either nontradable or tradable) endowments are lower in any future state s, both current insurance price I_0 and current insurance demand α_f by foreign*

country are lower,

$$\{either \quad de_{fT1}(s) < 0, \quad or \quad de_{fN1}(s) < 0\} \Longrightarrow \{dI_0 > 0, \quad and \quad d\alpha_f > 0\}, \quad \forall s \in \Omega.$$

The proof of these results is given in Appendix A.4. When the above assumptions hold, Lemma 1 simply implies that country's f expenditure $\alpha_f I_0$ on insurance unambiguously increases with its current and decreases with its future endowments. It is these variations of the insurance expenditure, but not the variations of insurance price or demand separately, that contribute to the variations of exchange rate (8) in equilibrium. In comparative statics sense, the intuition underlying Lemma 1 is as follows. First, all else being equal, when f has higher current tradable or nontradable endowments, it is wealthier and seeks to transfer more of current consumptions to the next period by buying more insurance, pushing the insurance price up in the equilibrium. Second, when f faces the potential drop in future tradable or nontradable endowments, it also demands more insurance to hedge against the worsening future prospects, again pushing up the insurance price.

Given elastic insurance demand $\theta_f < -1$ (Assumption 2) and $\alpha_f > 0$ (Assumption 1), we note that insurance price decreases, and foreign country's demand increases, with home country's future tradable endowment,

$$\frac{\partial I_0}{\partial e_{hT1}(s)} < 0, \qquad \frac{\partial \alpha_f}{\partial e_{hT1}(s)} > 0, \qquad \forall s \in \Omega. \tag{20}$$

Furthermore, under the same assumptions, foreign country's insurance expenditure increases with home country's endowments,

$$\frac{\partial(\alpha_f I_0)}{\partial e_{hT1}(s)} = \mathcal{T}^{-1} \frac{\beta_h \beta_f}{u''_{hT0} u''_{fT0}} \left(\sum_{x \in \Omega} p(x) \left[u'_{fT1}(x) + \alpha_f u''_{fT1}(x) \right] \right) p(s) u''_{hT1}(s) \tag{21}$$

for all states $s \in \Omega$, then foreign country's insurance expenditure increases with home country's future tradable endowment.

Expectation Effect on Insurance Market

Insurance price and demand depend on countries' perceived belief about prospects of future economy. Because the latter varies along the business cycle, so do the former. For simplicity, we model the changes in expectation (from state distribution $\{p(s_1)\}$ to state distribution $\{p(s_1)+dp(s_1)\}$) by varying the expectation only in two transitions, keeping expectation concerning all other transitions are unchanged,

$$dp(\overline{s}_1) = -dp(\underline{s}_1) \equiv dp > 0, \qquad dp(s_1) = 0, \quad \forall s_1 \in \Omega \backslash \{\overline{s}_1, \underline{s}_1\}. \tag{22}$$

where state designation is composite, $\underline{s}_1 = (\underline{s}_{h1}, \underline{s}_{f1})$, $\overline{s}_1 = (\overline{s}_{h1}, \overline{s}_{f1})$. The specification $dp > 0$ does not incur the loss of generality, because $\underline{s}_1$ $\overline{s}_1$ can be any states in Ω. Evidently, these variations preserve the probability normalization $\sum_{s_1 \in \Omega}[p(s_1) + dp(s_1)] = 1$. We employ the following shorthand notation for changes in marginal utilities across these two target states,

$$\Delta u'_h \equiv u'_{hT1}(\overline{s}_{h1}) - u'_{hT1}(\underline{s}_{h1}), \qquad \Delta u'_f \equiv u'_{fT1}(\overline{s}_{f1}) - u'_{fT1}(\underline{s}_{f1}). \tag{23}$$

The changes (22) in expectation about future economy induce the following variation of current insurance price,[11]

$$dI_0 = \mathcal{T}^{-1}\left[\frac{\beta_h}{u''_{hT0}}\left(I_0^2 + \beta_f \sum_{s\in\Omega} p(s)\frac{u''_{fT1}(s)}{u''_{fT0}}\right)\Delta u'_h + \frac{\beta_f}{u''_{fT0}}\left(I_0^2 + \beta_h \sum_{s\in\Omega} p(s)\frac{u''_{hT1}(s)}{u''_{hT0}}\right)\Delta u'_f\right] dp, \tag{24}$$

where $\{\Delta u'_i\}$ are defined in (23). The variation of the insurance price depends on changes in expectation and can be in either directions. E.g., under Assumptions 2 and 1, the concavity of preferences, the insurance is unambiguously more valuable ($dI_0 > 0$) when perceive chance of recession is higher ($dp > 0$). It is also intuitive that the increase in insurance price is proportional to the difference in countries' marginal utilities $\Delta u'_i$ in recession and normal states.

The changes (22) in expectation about future economy also induce variation of current insurance demand by foreign country,[12]

$$d\alpha_f = \mathcal{T}^{-1}\left[\frac{\beta_h}{u''_{hT0}}\left(\frac{u'_{fT0}}{u''_{fT0}} - \alpha_f I_0\right)\Delta u'_h - \frac{\beta_f}{u''_{fT0}}\left(\frac{u'_{hT0}}{u''_{hT0}} + \alpha_f I_0\right)\Delta u'_f\right] dp. \tag{25}$$

For illustration, assume elastic insurance demand $\theta_f < -1$ (Assumption 2) and $\alpha_f > 0$ (Assumption 1). Under increasing perception of disaster, the differential marginal utility $\Delta u'_i > 0$ characterizes country i's propensity to insurance demand. In this circumstance, it could be expensive for f to buy *additional* insurance from h, specially when h is sufficiently risk averse. Indeed,

[11] To obtain this, we substitute the specification (22) into the general variation (62) of the insurance price.
[12] To obtain this, we substitute the specification (22) into the general variation (63) of insurance demand.

(25) implies that when $\frac{u'_{hT0}}{u''_{hT0}} + \alpha_f I_0 > 0$, α_f incerases with p. Equivalently, using (50), we have this relationship reads,

$$\gamma_{h0} > 1 + \frac{e_{hT0}}{\epsilon_h \left(c_{hT0} - e_{hT0}\right)} \quad \Longrightarrow \quad \frac{\partial \alpha_f}{\partial p} < 0,$$

That is, foreign insurance demand decreases with the perceived chance of (global) recession when home country h has sufficiently high risk aversion and is not willing to sell more insurance. Only in the case that foreign country's propensity to insurance demand dominates that of home country,[13] foreign insurance demand increases with the perceived chance of recession $\frac{\partial \alpha_f}{\partial p} > 0$.

The combination of (24) and (25) gives us the overall variation of foreign country's insurance expenditure,

$$\begin{aligned} d(\alpha_f I_0) = \quad & \mathcal{T}^{-1} \frac{\beta_h \beta_f}{u''_{hT0} u''_{fT0}} \left\{ \left[\left(\alpha_f \sum_{s \in \Omega} p(s) u''_{fT1}(s) + \frac{I_0}{\beta_f} u'_{fT0} \right) \Delta u'_h \right. \right. \\ & \left. \left. + \left(\alpha_f \sum_{s \in \Omega} p(s) u''_{hT1}(s) - \frac{I_0}{\beta_h} u'_{hT0} \right) \Delta u'_f \right] \right\} dp \\ = \quad & \mathcal{T}^{-1} \frac{\beta_h \beta_f}{u''_{hT0} u''_{fT0}} \left\{ \left(\sum_{s \in \Omega} p(s) \left[u'_{fT1}(s) + \alpha_f u''_{fT1}(s) \right] \right) \Delta u'_h \right. \\ & \left. - \left(\sum_{s \in \Omega} p(s) \left[u'_{hT1}(s) + \alpha_h u''_{hT1}(s) \right] \right) \Delta u'_f \right\} dp \end{aligned} \tag{26}$$

Assume elastic insurance demand $\theta_f < -1$ (Assumption 2) and $\alpha_f > 0$ (Assumption 1). Under increasing perception of global disaster (both $\Delta u'_h > 0$, $\Delta u'_f > 0$), foreign country's expenditure on insurance increases with chance of global disaster, $\frac{\partial(\alpha_f I_0)}{\partial p} > 0$ when either foreign country is sufficiently risk averse (on average),

$$-\frac{\sum_{s \in \Omega} p(s) u''_{fT1}(s)}{u'_{fT0}} > \frac{I_0}{\alpha_f \beta_f}, \tag{27}$$

or foreign country's propensity to insurance demand dominates that of the home country,

$$\frac{\Delta u'_f}{\Delta u'_h} > \frac{\beta_h}{\beta_f} \frac{u'_{fT0}}{u'_{hT0}}. \tag{28}$$

[13] This is quantified by the inequality,

$$\frac{\Delta u'_f}{\Delta u'_h} > \frac{\frac{\beta_h}{u''_{hT0}} \left(\frac{u'_{fT0}}{u''_{fT0}} - \alpha_f I_0 \right)}{\frac{\beta_f}{u''_{fT0}} \left(\frac{u'_{hT0}}{u''_{hT0}} + \alpha_f I_0 \right)}.$$

4.3 Variations of the Exchange Rate

Using budget constraints (10), we can express the variation of the exchange rate (8) as follows,

$$\frac{dS_0}{S_0} = (1-\epsilon_h)\left(\frac{de_{hT0}}{c_{hT0}} - \frac{de_{hN0}}{e_{hN0}}\right) - (1-\epsilon_f)\left(\frac{de_{fT0}}{c_{fT0}} - \frac{de_{fN0}}{e_{fN0}}\right) + \left(\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}\right) d(\alpha_f I_0). \quad (29)$$

The first four terms exhibit direct effects of exogenous and contemporaneous (at $t = 0$) endowment variations on changes in the current exchange rate. The last terms show that all exogenous variations, including the above, also induce changes in exchange rate indirectly through changes in insurance expenditure. The exchange rate variations (31) are intuitive. First, higher home current nontradable endowment depresses current home nontradable consumption good price, which lowers home basket value and decreases current exchange rate. Second, higher home current tradable endowment tends to directly increase home equilibrium current tradable consumption and thus also increases price of home nontradable consumption good (in unit of tradable consumption good). Third, an increase in foreign country's expenditure on insurance also contributes to a surge in home country's current tradable consumption,[14] which again leads to higher home country price index and exchange rate.

To explain the observed exchange rate movements across business cycles, as well as after macro news announcements, we adopt the following modeling framework.

1. We model bad US news as a drop in h's future endowment (in some state s); $de_{hT1}(s) < 0$. Symmetrically, bad foreign news is characterized by a drop in f's future endowment; $de_{fT1}(s) < 0$.

2. We model US recession state as an elevation in the perceived chance of future disaster state; $dp > 0$.

3. Thus, bad US news arriving in US recession state is characterized by simultaneous variations in future endowment and expectation; $de_{fT1}(s) < 0$ and $dp > 0$.

[14] See budget counstraint (10).

Bad US News Arrive in US Normal State

Follow from (32)

$$\frac{dS_0}{S_0} = \mathcal{T}^{-1}\left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}\right]\frac{\beta_h\beta_f}{u''_{hT0}u''_{fT0}}K_f p(s)u''_{hT1}(s)de_{hT1}(s).$$

Clearly, when $K_f > 0$ (18), the exchange rate S_0 decreases following bad US news ($de_{hT1}(s) < 0$). The intuition is that, when $K_f > 0$, foreign country has an elastic demand of insurance demand $\theta_f < -1$. Bad US news ($de_{hT1}(s) < 0$) then leads to a drop in f's insurance expenditure ($d(\alpha_f I_0) < 0$) as indicated by (21), which in turns leads to a drop in exchange rate as indicated by (31).

We also recall from (20) that as long as foreign country by insurance initially $\alpha_f > 0$ (Assumption 1), insurance price I_0 always decreases with, while f's insurance holding α_f always increases with, home future endowment $e_{hT1}(s)$. Thus elastic demand of insurance demand assumption $\theta_f < -1$ (or equivalently, $K_f > 0$) assures that α_f increases with $e_{hT1}(s)$ faster than I_0 decreases with $e_{hT1}(s)$ when US is in normal state (future disaster chance p is low enough). This is plausible because we can write K_f as,

$$K_f \equiv \sum_{s\in\Omega} p(s)u'_{fT1}(s)\left\{1 - \alpha_f\frac{(1-\epsilon_f) + \epsilon_f\gamma_{f1}}{c_{fT1}(s)}\right\} > 0. \tag{30}$$

Note that when f's consumption c_{fT1} is low, and the associated marginal utility $u'_{fT1}(s)$ is high and positive. But when US is in normal state, the probability $p(s)$ of disaster state s, in which c_{fT1} is low, is virtually zero. As a result, our condition (30) holds precisely because the US currently is in normal state.

Bad US News Arrive in US Recession State

The variation of exchange rate following simultaneously $de_{hT1}(s) < 0$ (bad US news) and $dp > 0$ (US in recession) reads (following relationships (32) and (35)),

$$\frac{dS_0}{S_0} = \mathcal{T}^{-1}\left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}\right]\frac{\beta_h\beta_f}{u''_{hT0}u''_{fT0}} \times \left\{K_f\, p(s)u''_{hT1}(s)de_{hT1}(s) + \left(K_f\Delta u'_h - K_h\Delta u'_f\right)dp\right\}, \tag{31}$$

where similar to (16), K_h is defined as follows,

$$K_h \equiv \sum_s p(s) \left[u'_{hT1}(s) + \alpha_h u''_{hT1}(s) \right]$$

The assumptions of elastic insurance demand $\theta_f < -1$ and f's initial positive insurance holding ($\alpha_f > 0$), immediately imply $K_h > 0$.[15] Consequently, exchange rate S_0 (31) increases following bad US news ($de_{hT1}(s) < 0$) when US is in recession ($dp > 0$). This is because when US is in recession, change in f's marginal utility $\Delta u'_f \gg 0$ (23) easily dominates all other terms.

While the effect of the insurance expenditure on exchange rate is quite mechanical, the deeper issue is how exogenous variations of the expectation, and current and future endowments foster variations of insurance expenditure. In light of equation (31), our analysis of the insurance price and demands in Section 4.2 yields the following additional results on exchange rate variations.

Further Results: Endowment Effect on the Exchange Rate

The variation of current exchange rate induced by the variation of home country's future tradable endowment,[16]

$$\frac{dS_0}{S_0} = \mathcal{T}^{-1} \left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}} \right] \frac{\beta_h \beta_f}{u''_{hT0} u''_{fT0}} \left[\sum_{x \in \Omega} p(x) \left\{ u'_{fT1}(x) + \alpha_f u''_{fT1}(x) \right\} \right] p(s) u''_{hT1}(s) de_{hT1}(s). \tag{32}$$

Our key observation is that while equilibrium (consumptions and prices) is the same under different configurations of exogenous economic conditions (endowments' supports and expectations), the sensitive of exchange rate to these exogenous state variables vary substantially. Because different endowment distributions (supports and expectation) represent different modes of the global economy, opposite tendencies in exchange rate variations arise in normal and crisis states.

Proposition 2 (Home tradable endowment effect on the exchange rate) *Assume elastic insurance demand $\theta_f < -1$ (Assumption 2) and $\alpha_f > 0$ (Assumption 1). In equilibrium,*

1. **home recession state**: *when home country's current endowment (or foreign country's future*

[15] We recall that $\alpha_h = -\alpha_f$, thus $\alpha_f > 0$ is equivalent to $\alpha_h < 0$, which then implies $K_h > 0$.

[16] In the current incomplete-market setting, the sensitivity (32) of exchange rate to home future tradable endowment variations has same sign for all future states $s \in \Omega$.

endowment) is sufficiently low,

$$u'_{fT1}(s) + \alpha_f u''_{fT1}(s) < 0, \quad \textit{and} \quad \frac{\partial S_0}{\partial e_{hT1}(s)} < 0, \quad \forall s \in \Omega,$$

so home currency decreases with home tradable endowment in all future states.

2. **home normal state***: when home country's current endowment (or foreign country's future endowment) is sufficiently high,*

$$u'_{fT1}(s) + \alpha_f u''_{fT1}(s) > 0, \quad \textit{and} \quad \frac{\partial S_0}{\partial e_{hT1}(s)} > 0, \quad \forall s \in \Omega,$$

or home currency increases with home tradable endowment in all future states.

To understand these results, we consider the following changes in the endowments, which mimic the changes in global economic conditions. From ex-post view, given the equilibrium insurance price I_0, we shift δI_0 units of tradable consumption good from home's to foreign's endowment at $t = 0$, and shift δ units of tradable consumption good from foreign's to home's endowment in every state $s \in \Omega$ at $t = 1$,

$$\left\{ \begin{array}{ll} e_{hT0}, & e_{fT0}, \\ e_{hT1}(s), & e_{fT1}(s), \end{array} \right. \longrightarrow \left\{ \begin{array}{ll} \widehat{e}_{hT0} \equiv e_{hT0} - \delta I_0, & \widehat{e}_{fT0} \equiv e_{fT0} + \delta I_0, \\ \widehat{e}_{hT1}(s) \equiv e_{hT1}(s) + \delta, & \widehat{e}_{fT1}(s) \equiv e_{fT1}(s) - \delta, \end{array} \right.$$

From ex-ante view, these shifts preserve both country-specific budget constraints and resource constraints. As a result, the equilibrium is invariant under these changes in endowments, while the sensitivity of exchange rate to home future tradable endowments varies substantially (and can change sign) because insurance positions change,

$$\{\alpha_h \ , \ \alpha_f\} \longrightarrow \{\widehat{\alpha}_h \equiv \alpha_h - \delta I_0 \ , \ \widehat{\alpha}_f \equiv \alpha_f + \delta I_0\}.$$

In particular, let $\overline{s}$ (resp. $\underline{s}$) be the state of highest (resp. lowest) foreign tradable consumption at $t = 1$ in equilibrium.[17] By virtue of (51), when endowment reconfiguration δ is such that foreign insurance holding $\widehat{\alpha}_f$ is above a lower bound, or equivalently home current tradable endowment

[17] So that $c_{fT1}(\overline{s}) \geq c_{fT1}(s) \geq c_{fT1}(\underline{s})$, $\forall s \in \Omega$. Note that $\mathcal{T}$ is not invariant under the endowment reconfiguration because it also depends on insurance holdings. However, given the assumption that (in all endowment configuration) foreign country buys insurance initially ($\alpha_h < 0$), first condition set of (58) implies that we need only that home risk aversion $\gamma_{h0} < 1$ (and home initial tradable endowment $e_{hT0} > 0$) (which we assume) to ensure $\mathcal{T} < 0$.

is below an upper bound (home recession), all terms x in (32) are negative and exchange rate decreases with home future tradable endowments,

$$\widehat{e}_{hT0} < c_{hT0} - \frac{I_0 \times c_{fT1}(\overline{s})}{1 - \epsilon_f + \gamma_{f1}\epsilon_f} \implies \widehat{\alpha}_f > \frac{c_{fT1}(\overline{s})}{1 - \epsilon_f + \gamma_{f1}\epsilon_f} > 0$$

$$\implies u'_{fT1}(x) + \alpha_f u''_{fT1}(x) < 0, \ \ \forall x \in \Omega \implies \frac{\partial S_0}{\partial e_{hT1}(s)} < 0,$$

and vice versa,[18]

$$\widehat{e}_{hT0} > c_{hT0} - \frac{I_0 \times c_{fT1}(\underline{s})}{1 - \epsilon_f + \gamma_{f1}\epsilon_f} \implies 0 < \widehat{\alpha}_f < \frac{c_{fT1}(\underline{s})}{1 - \epsilon_f + \gamma_{f1}\epsilon_f}$$

$$\implies u'_{fT1}(x) + \alpha_f u''_{fT1}(x) > 0, \ \ \forall x \in \Omega \implies \frac{\partial S_0}{\partial e_{hT1}(s)} > 0.$$

Similarly, the variation of current exchange rate induced by the variation of foreign country's future tradable endowment,

$$\frac{dS_0}{S_0} = -\mathcal{T}^{-1}\left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}\right]\frac{\beta_h\beta_f}{u''_{hT0}u''_{fT0}}\left[\sum_{x\in\Omega} p(x)\left\{u'_{hT1}(x) + \alpha_h u''_{hT1}(x)\right\}\right]p(s)u''_{fT1}(s)de_{fT1}(s).$$

which implies the following results.

Proposition 3 (Foreign tradable endowment effect on the exchange rate) *Assume elastic insurance demand $\theta_f < -1$ (Assumption 2) and $\alpha_f > 0$ (Assumption 1). In equilibrium, home currency always decreases with foreign tradable endowment in all future states,*

$$\alpha_f > 0 \implies u'_{hT1}(s) + \alpha_h u''_{hT1}(s) > 0, \ \ \forall s \in \Omega \implies \frac{\partial S_0}{\partial e_{fT1}(s)} < 0, \ \ \forall s \in \Omega.$$

The first implication above follows from fact that $\alpha_h = -\alpha_f > 0$ and preference is increasing and concave in tradable consumption (46). We remark an important asymmetry between Propositions 2 and 3. Given that foreign country currently (at $t = 0$) buys insurance from home country, while lower home country's future tradable endowments can induce either higher (in recession state) or lower (in normal state) exchange rate, lower foreign country's future tradable endowments invariably induce higher exchange rate.

[18] The endowment reconfigurations are chosen to respect the assumption that foreign country buys insurance initially, $\alpha_f, \widehat{\alpha}_f > 0$, in all endowment configurations.

The variation of current exchange rate induced by the variation of foreign country's future nontradable endowment,

$$\frac{dS_0}{S_0} = \mathcal{T}^{-1}\left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}\right]\frac{\beta_h\beta_f}{u''_{hT0}u''_{fT0}}\left[\sum_{x\in\Omega} p(x)\left\{u'_{fT1}(x) + \alpha_f u''_{fT1}(x)\right\}\right] p(s)u''_{hT1,hN1}(s)de_{hN1}(s). \tag{33}$$

This sensitivity differs from the sensitivity (32) of exchange rate to home tradable endowment only by the second-order (cross-) derivative $u''_{hT1,hN1}(s)$. From (33) follows the next results.

Proposition 4 (Home nontradable endowment effect on the exchange rate) *Assume (i) home country has sufficiently low current risk aversion $\gamma_{h0} < 1$, (ii) foreign country has sufficiently low risk aversion at all times and states ($\gamma_{ft} < 1$, $\forall t \in \{0,1\}, s \in \Omega$), and (iii) foreign country initially buys insurance ($\alpha_f > 0$).*

1. **Home recession state**: *If home country has sufficiently low risk aversion in a future state $s \in \Omega$, then the current value of home currency decreases with home nontradable endowment in that future state s,*

$$\gamma_{h1}(s) < 1 \implies u''_{hT1,hN1}(s) > 0 \implies \frac{\partial S_0}{\partial e_{hN1}(s)} < 0.$$

2. **Home normal state**: *If home country has sufficiently high risk aversion in a future state $s \in \Omega$, then the current value of home currency increases with home nontradable endowment in that future state s,*

$$\gamma_{h1}(s) > 1 \implies u''_{hT1,hN1}(s) < 0 \implies \frac{\partial S_0}{\partial e_{hN1}(s)} > 0.$$

An important feature of the above results (in difference from those of Propositions 2, 3) is that exchange rate varies differently with different future states of nontradable endowments. This is because, to home country, the substitutability of tradable and nontradable consumptions can vary from state to state. In states in which home marginal utility (of tradable consumption) increases with nontradable consumption (home recession state), home currency is more valuable when home nontradable endowment is lower. Vice versa, in states in which home marginal utility (of tradable consumption) decreases with nontradable consumption (home normal state), home currency is less valuable when home nontradable endowment is lower.

Further Results: Expectation Effect on the Exchange Rate

The variation of current exchange rate induced by the variation of expectation about a particular future state $s \in \Omega$ is,

$$\frac{dS_0}{S_0} = \mathcal{T}^{-1} \left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}} \right] \frac{\beta_h \beta_f}{u''_{hT0} u''_{fT0}} \times \tag{34}$$

$$\times \left\{ \left[\sum_{x \in \Omega} p(x) \left(u'_{fT1}(x) + \alpha_f u''_{fT1}(x) \right) \right] u'_{hT1}(s) - \left[\sum_{x \in \Omega} p(x) \left(u'_{hT1}(x) + \alpha_h u''_{hT1}(x) \right) \right] u'_{fT1}(s) \right\} dp(s)$$

For simplicity, we consider the scenario of changes in the expectation specified in (22), which fixes initial state s_0 and concerns variations in transitions to only two states $\underline{s}$ and $\bar{s}$. In this premise, the variation of exchange rate reads,

$$\frac{dS_0}{S_0} = \mathcal{T}^{-1} \left[\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}} \right] \frac{\beta_h \beta_f}{u''_{hT0} u''_{fT0}} \times \tag{35}$$

$$\times \left\{ \left[\sum_{x \in \Omega} p(x) \left(u'_{fT1}(x) + \alpha_f u''_{fT1}(x) \right) \right] \Delta u'_h - \left[\sum_{x \in \Omega} p(x) \left(u'_{hT1}(x) + \alpha_h u''_{hT1}(x) \right) \right] \Delta u'_f \right\} dp$$

where $\Delta u'_h$, $\Delta u'_f$ are differentials of the marginal utilities (23) across two states $\underline{s}$, $\bar{s}$ under consideration. Note that variation of exchange rate is proportional to and thus have same sign as the variation of insurance expenditure (26), so that either conditions (27) or (28) implies an increase in current value of home currency. Furthermore, we have,

Proposition 5 (Expectation effect on the exchange rate) *Assume elastic insurance demand $\theta_f < -1$ (Assumption 2) and $\alpha_f > 0$ (Assumption 1),*

1. **Home recession state***: When foreign country initially (at $t = 0$) buys sufficiently large amount of insurance, home currency's current value decreases faster with expectation of home country recession when recession is more severe,*

$$\alpha_f > \frac{-I_0 \times u'_{fT0}}{\beta_f \sum_{x \in \Omega} p(x) u''_{fT1}(x)} > 0 \implies \frac{\partial S_0}{\partial p} > 0 \quad \textit{and increases in} \quad \Delta u'_h.$$

2. **Foreign recession state***: When foreign country initially (at $t = 0$) buys sufficiently large amount of insurance, home currency's current value increases faster with expectation of for-*

eign country recession when recession is more severe,

$$\alpha_f > \frac{I_0 \times u'_{hT0}}{\beta_h \sum_{x\in\Omega} p(x) u''_{hT1}(x)} > 0 \implies \frac{\partial S_0}{\partial p} > 0 \quad \textit{and decreases in} \quad \Delta u'_f.$$

These results arise directly from either the expression for insurance expenditure (26), or its the exchange rate variation (35) (which are proportional to one another).

5 Complete Financial Markets

We consider now the complete financial market setting in which at $t = 0$ countries trade a complete set of Arrow-Debreu securities to to hedge all shocks in tradable and nontradable endowments. In equilibrium, countries achieve maximal risk sharing by equalizing their marginal utilities of tradable consumptions at all times and states,

$$u'_{hT0} = \lambda u'_{fT0}, \qquad \beta_h u'_{hT1}(s) = \lambda \beta_f u'_{fT1}(s), \quad \forall s \in \Omega, \tag{36}$$

where λ is a constant (the Pareto weight) that is determined by the endowment distributions of the two countries (see below). Countries are subject to both resource constraints (2) and budget constraints,[19]

$$c_{iT0} + \sum_{s\in\Omega} c_{iT1}(s)q(s) = e_{iT0} + \sum_{s\in\Omega} e_{iT1}(s)q(s), \quad i \in \{h, f\}, \tag{37}$$

where $q(s)$ denotes the price at $t = 0$ of the s-th AD security, which pays a unit of tradable consumption good at $t = 1$ if and only if the state then is s. AD prices are shadow costs of resource constraints (2) at $t = 1$ of the corresponding states,

$$\beta_h p(s) \frac{u'_{hT1}(s)}{u'_{hT0}} = q(s) = \beta_f p(s) \frac{u'_{fT1}(s)}{u'_{fT0}}. \tag{38}$$

The complete-market equilibrium (consumptions, AD prices, Pareto weight, and exchange rate) are solutions to the system of FOC conditions (36), the resource constraints (2), the budget constraints (37).[20] In our variational analysis of the equilibrium, we first take the Pareto weight λ as given, and

[19]Budget constraints bind in equilibrium as a result of countries' strictly monotonic preferences. Nontradable endowments are consumed entirely in the host country, $c_{iNt}(s) = e_{iNt}(s)$, $\forall i, t, s$, so they are canceled out in the budget constraints.

[20]Let $S \in \mathbf{N}$ be the numer of states at $t = 1$: $S = dim(\Omega)$. The equilibrium system has $2S + 3$ unknowns and $2S + 3$ equations The unknowns are $2S + 2$ tradable consumptions $\{c_{iT0}, c_{iT1}(s)\}$, and the Pareto weight λ. The

express the variations of equilibrium consumptions in term of the variation of the Pareto weight. The budget constraint then determines the latter endogenously.

Following this procedure, we obtain the variations of equilibrium tradable consumption in term of the variations of exogenous endowments and the Pareto weight (see Appendix A.3), $\forall t \in \{0, 1\}$, $\forall s \in \Omega$,

$$dc_{hTt}(s) = \left(\frac{u''_{hTt}(s)}{u'_{hTt}(s)} + \frac{u''_{fTt}(s)}{u'_{fTt}(s)}\right)^{-1} \left[\frac{d\lambda}{\lambda} - \frac{u''_{hTt,hNt}(s)}{u'_{hTt}(s)} de_{hNt}(s) + \frac{u''_{fTt,fNt}(s)}{u'_{fTt}(s)} de_{fNt}(s) + \frac{u''_{fTt}(s)}{u'_{fTt}(s)} de_{Tt}(s)\right],$$
$$dc_{fTt}(s) = \left(\frac{u''_{hTt}(s)}{u'_{hTt}(s)} + \frac{u''_{fTt}(s)}{u'_{fTt}(s)}\right)^{-1} \left[-\frac{d\lambda}{\lambda} + \frac{u''_{hTt,hNt}(s)}{u'_{hTt}(s)} de_{hNt}(s) - \frac{u''_{fTt,fNt}(s)}{u'_{fTt}(s)} de_{fNt}(s) + \frac{u''_{hTt}(s)}{u'_{hTt}(s)} de_{Tt}(s)\right]. \quad (39)$$

The variations of equilibrium tradable consumptions are intuitive.[21] First, a (former) country's tradable consumption increases directly with aggregate tradable endowment,[22] with the proportional coefficient being the concavity of the other (latter) country's utility. This is because when the latter country's utility is more concave, its utility increases less with consumption.[23] Thus, in equilibrium, the latter country is willing to let the former country to consume larger share of the surge in aggregate tradable endowment. Second, tradable consumption decreases directly with nontradable endowment of the same country, because a drop in nontradable endowment makes tradable consumption relatively cheaper, and vice versa. Finally, equilibrium consumptions also indirectly move with exogenous endowments via the movement in the Pareto weight. Home (resp., foreign) country's tradable consumption decreases (resp., increases) with the variation of λ. This is because the Pareto weight equals the ratio of home to foreign marginal utilities of tradable consumption. When λ surges, it must be that foreign marginal utility decreases (relative to home marginal utility), or foreign tradable consumption increases and home tradable consumption decreases relatively. The following results, which immediately arise from the variation of the Pareto weight (39), formalize the last intuition.

Lemma 2 *In complete market, keeping aggregate endowments unchanged, equilibrium consump-*

equations are $S+1$ FOCs (36) at $t=0$ and $t=1$, $S+1$ resource constraints (2) for tradable consumption good at $t=0$ and $t=1$, and a country's budget constraint (37) with $q(s)$ substituted by (38) (the other country's budget constraint is redundant by Walras's law). All other quantities, including AD prices $\{q(s)\}$ and exchange rate S_0, can be determined from equilibrium consumptions.

[21]Note that all second-order derivatives are negative, see Appendix A.1.

[22]The tradable endowments appear in aggregate quantity, $de_T = de_{hT} + de_{fT}$ (but not country-specific), because tradable consumption good is identical everywhere, and trades in this good are frictionless.

[23]Incidently, the same concavity indicates that the latter country has stronger intertemporal consumption smoothing desire.

tions of a country unambiguously move in the same direction in all states and time,

$$d\lambda \geq 0 \Rightarrow \begin{cases} dc_{hT0},\ dc_{hT1}(s) \leq 0, \\ dc_{fT0},\ dc_{fT1}(s) \geq 0, \end{cases} \qquad d\lambda < 0 \Rightarrow \begin{cases} dc_{hT0},\ dc_{hT1}(s) > 0, \\ dc_{fT0},\ dc_{fT1}(s) < 0, \end{cases} \qquad \forall s \in \Omega$$

These results indicate that, in complete market, the Pareto weight *uniformly* governs the (indirect) variations of a (any) country's equilibrium consumptions across times and states. Therefore, movements in λ necessarily induces variations of equilibrium exchange rate. Combining expressions (39) and (8) yields the following current exchange rate's variation for the complete-market setting,

$$\frac{dS_0}{S_0} = \frac{\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \times \frac{d\lambda}{\lambda} + \frac{\frac{1-\epsilon_h}{c_{hT0}}\frac{u''_{fT0}}{u'_{fT0}} - \frac{1-\epsilon_f}{c_{fT0}}\frac{u''_{hT0}}{u'_{hT0}}}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \times de_{T0} \tag{40}$$

$$- \left[\frac{1-\epsilon_h}{e_{hN0}} + \frac{\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \frac{u''_{hT0,hN0}}{u'_{hT0}}\right] de_{hN0} + \left[\frac{1-\epsilon_f}{e_{fN0}} + \frac{\frac{1-\epsilon_h}{c_{hT0}} + \frac{1-\epsilon_f}{c_{fT0}}}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \frac{u''_{fT0,fN0}}{u'_{fT0}}\right] de_{fN0}.$$

We note that this expression for the exchange rate variation mirror the variation (31) of the complete market, therein the endogenous insurance expenditure $\alpha_f I_0$ corresponds to the endogenous Pareto weight of the incomplete market. Again, variations in endowments either influence exchange rate directly, or indirect through their impacts on the Pareto weight. Higher home (resp., foreign) nontradable endowment directly decreases home (resp., foreign) nontradable good price and thus depresses home (resp., foreign) currency value relatively. An increase in aggregate tradable endowment increases tradable cosumptions in both countries, though the increases may differ across countries. When foreign country's utility is more concave ($u''_{fT0} < u''_{hT0} < 0$), home country's tradable consumption increases more than foreign country's. As a result, home nontradable consumption good price increases (relatively to that of foreign nontradable consumption good), and home price index and exchange rate appreciate. Finally, all else being equal, a surge in Pareto weight signifies an increase in tradable consumption share of foreign country at all times and states (Lemma 2). Consequently, foreign nontradable consumption good increases in value, and so does foreign price index, or exchange rate drops. However, Pareto weight is an endogenous quantity in equilibrium. We next quantify the endogenous variations of the Pareto weight, which give rise to indirect effects of exogenous endowments on exchange rate.

5.1 Variations of the Pareto Weight

Because in complete-market equilibrium, λ characterizes the relative weight of foreign country to the world economy, the variation of Pareto weight closely reflects the foreign country's wealth dynamic across different economic conditions of the international and country-specific business cycles. Appendix (A.3) derives the variation of Pareto weights in terms of exogenous variations of endowments de's and expectation dp's in details.

We first observe that key to the variation of Pareto weight (60) is the following quantity,

$$\begin{aligned}\mathcal{V} \equiv \quad & \frac{\frac{u''_{hT0}}{u'_{hT0}}(c_{hT0}-e_{hT0})+1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}} + \beta_h \sum_s p(s) \frac{u'_{hT1}(s)}{u'_{hT0}} \frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}(c_{hT1}(s)-e_{hT1}(s))+1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}} \\ = \quad & \frac{\frac{u''_{fT0}}{u'_{fT0}}(c_{fT0}-e_{fT0})+1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}} + \beta_f \sum_s p(s) \frac{u'_{fT1}(s)}{u'_{fT0}} \frac{\frac{u''_{fT1}(s)}{u'_{fT1}(s)}(c_{fT1}(s)-e_{fT1}(s))+1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}. \end{aligned} \tag{41}$$

By virtue of (49), $\mathcal{V}$ is negative when either (i) home country has sufficiently low risk aversion at all times and states, (ii) foreign country has sufficiently low risk aversion at all times and states, or (iii) foreign country currently (at $t = 0$) lends[24] (to home country) and has sufficiently low risk aversion in all future states (at $t = 1$), or (iv) home country currently lends and has sufficiently low risk aversion in all future states,

$$\left.\begin{array}{rll} \text{either:} & \gamma_{ht} < 1, & \forall t \in \{0,1\}, \forall s \in \Omega, \\ \text{or:} & \gamma_{ft} < 1, & \forall t \in \{0,1\}, \forall s \in \Omega, \\ \text{or:} & e_{fT0} > c_{fT0}, & \text{and} \quad \gamma_{f1} < 1, \forall s \in \Omega, \\ \text{or:} & e_{hT0} > c_{hT0}, & \text{and} \quad \gamma_{h1} < 1, \forall s \in \Omega, \end{array}\right\} \implies \mathcal{V} < 0. \tag{42}$$

The above conditions are a strong reminiscence of the sufficient conditions (57)-(58) underlying the tatonnement stability (19) of the incomplete market setting. In fact, the similarity between $\mathcal{T}$ (19) and $\mathcal{V}$ (41) is far deeper in their respective effect on the variations of insurance expenditure (incomplete market setting) and Pareto weight (complete market setting), and therefore on the variations of the exchange rate.[25]

[24] In the difference with incomplete market seeting, in the current complete market setting, country f lends country h at $t = 0$ (i.e., $e_{i0} > c_{i0}$) does not necessarily mean that f buys insurance from h at $t = 0$ because many other (possibly, risky) types of assets are available for trades.

[25] Both $\mathcal{T}$ and $\mathcal{V}$ have representations that are symmetric to h and f. More importantly, in the incomplete market

The economic rationale underlying a negative sign of quantity $\mathcal{V}$ can be discerned from its effect on the variation of the Pareto weight. Let's consider an increase in home current tradable endowment at the expense of an exactly offsetting decrease in foreign current nontradable endowment.[26] The resulting variation of Pareto weight reads (we keep only term de_{hT0} in (60)),

$$\frac{d\lambda}{\lambda} = \mathcal{V}^{-1} \times de_{hT0}.$$

When $\mathcal{V}$ is negative as in (42), higher home current tradable endowment (and simultaneously lower foreign current tradable endowment) incurs lower Pareto weight λ. In turn, Lemma 2 implies an increase in home tradable consumptions in all times and states. In summary, an increase in home tradable endowment (and a decrease in foreign tradable endowment) will necessarily increase utility of home country when (42) holds.[27] Hereafter we assume $\mathcal{V} < 0$ by holding one of its sufficient conditions specified in (42).

We next investigate the dependence of Pareto weight on the nontradable endowments. The dependence is similar for $t = 0$ and $t = 1$. For ease of notation we consider the variations of current (at $t = 0$) nontradable endowments. The variation of λ in response to variation of home nontradable endowment reads (term de_{hN0} in (60)),

$$\frac{d\lambda}{\lambda} = \mathcal{V}^{-1} \times \frac{u''_{hT0,hN0}}{u'_{hT0}} \times \left(\frac{\frac{u''_{fT0}}{u'_{fT0}} \times [e_{hT0} - c_{hT0}] + 1}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \right) \times de_{hN0}$$

Assume that foreign country's risk aversion is sufficiently low in all times and states, so that indeed $\mathcal{V} < 0$ by virtue of (42), and the expression inside parentheses is negative by virtue of (49). Then, by virtue of (49), when home country has sufficiently low risk aversion at $t = 0$, the Pareto weight increases with current home nontradable endowment, and the dependence reverses when home

setting, sufficient conditions (57)-(58) can be identified and concern only period $t = 0$. Whereas in the complete market setting, sufficient conditions (42) involve both periods $t = 0$ and $t = 1$. This is because the equilibrium consumption plans follow endowment schedules rigidly in the incomplete market setting per our discussions concerning (12).

[26] In micro-economic literature, this is referred to as a "transfer" of tradable endowments from foreign country to home country.

[27] On the contrary, if (42) does not hold, $\mathcal{V} > 0$, and a transfer of tradable endowment from foreign to home country decreases home country's (and increases foreign country's) utility. This scenario is ruled out in regular economies under generic and mild conditions, see Balasko (2014).

country has sufficiently high risk aversion,[28]

$$\gamma_{h0} \equiv \frac{-c_{h0}u''_{h0}}{u'_{h0}} < 1 \Longrightarrow \frac{\partial \lambda}{\partial e_{hN0}} > 0, \qquad \gamma_{h0} \equiv \frac{-c_{h0}u''_{h0}}{u'_{h0}} > 1 \Longrightarrow \frac{\partial \lambda}{\partial e_{hN0}} < 0. \tag{43}$$

The intuitions underlying these relationships are as follows. When $\gamma_{h0} < 1$, home country's marginal utility of tradable consumption increases with its nontradable consumption. In complete-market setting, λ is the ratio of home and foreign marginal utilities (of tradable consumption). As a result, λ directly increases with e_{hN0}, and by virtue of Lemma 2, home tradable consumptions decreases in all times and states.[29]

Similarly, assume that home country's risk aversion is sufficiently low in all times and states, when foreign country has sufficiently low risk aversion at $t = 0$, the Pareto weight decreases with current foreign nontradable endowment (and the dependence reverses when foreign country has sufficiently high risk aversion),

$$\gamma_{f0} < 1 \Longrightarrow \frac{\partial \lambda}{\partial e_{fN0}} < 0, \qquad \gamma_{f0} > 1 \Longrightarrow \frac{\partial \lambda}{\partial e_{hN0}} > 0. \tag{44}$$

The Pareto weight also varies with expectations about future prospect of the economies. For simplicity, we consider the scenario of changes in the expectation specified in (22), which fixes initial state s_0 and concerns variations in transitions to only two states $\underline{s}$ and $\overline{s}$. In this premise, the variation of Pareto weight reads (term dp in (60)),

$$\frac{d\lambda}{\lambda} = \mathcal{V}^{-1} \times \beta_h \times \left(\frac{u'_{hT1}(\overline{s})}{u'_{hT0}} [e_{hT1}(\overline{s}) - c_{hT1}(\overline{s})] - \frac{u'_{hT1}(\underline{s})}{u'_{hT0}} [e_{hT1}(\underline{s}) - c_{hT1}(\underline{s})] \right) \times dp. \tag{45}$$

The above relationships, as well as (43), (44) indicate that time-varying marginal utilities (or substitutabilities, risk aversions, and expectations) of countries naturally induce variations in the Pareto weight the equilibrium. In turn, the pricing of currencies varies across different global economic conditions as we see next.

[28]This is not quite the risk aversion. As explained below equation (43), this is the marginal substitution of tradable and nontradable consumptions. But for additively separable preferences, γ_h confounds with h's risk aversion.

[29]There is also an indirect effect that as e_{hN0} increases, home marginal utility (of tradable consumption) increases so home country is tempted to increase its tradable consumption (dc_{hN0} increases with term de_{hN0} in the (39)). However this increase in c_{hN0} is dominated by an opposite drop induced by an increase in Pareto weight term.

5.2 Variations of the Exchange Rate

Endowment Effect on the Exchange Rate

Proposition 6 (Home nontradable endowment effect on the exchange rate) *Assume foereign country has sufficiently low risk aversion at all times and states ($\gamma_{ft} < 1$, $\forall t \in \{0, 1\}, s \in \Omega$).*

1. **Home recession state***: If home country has sufficiently low risk aversion in a future state $s \in \Omega$, then the current value of home currency decreases with home nontradable endowment in that future state s,*

$$\gamma_{h1}(s) < 1 \implies u''_{hT1,hN1}(s) > 0 \implies \frac{\partial S_0}{\partial e_{hN1}(s)} < 0.$$

2. **Home normal state***: If home country has sufficiently high risk aversion in a future state $s \in \Omega$, then the current value of home currency increases with home nontradable endowment in that future state s,*

$$\gamma_{h1}(s) > 1 \implies u''_{hT1,hN1}(s) < 0 \implies \frac{\partial S_0}{\partial e_{hN1}(s)} > 0.$$

We note that home country's future nontradable endowment affects the current exchange rate exclusively through the former's influence on the Pareto weight (40). When home nontradable endowment is higher in a future state s in which it is less risk averse ($\gamma_{h1}(s) < 1$), the same intuition underlying relationship (43) indicates that home country's marginal utility (of tradable consumption) at $t = 1$, state s, and Pareto weight are also higher. Lemma 2 then implies that home country's tradable consumptions are lower in all times and states. Tradable consumption good prices are higher, while nontradable consumption good prices are lower at home. As a result, home price index and exchange rate are lower. The opposite relationship holds when $\gamma_{h1}(s) > 1$, in which case the exchange rate increases with home nontradable endowment in state s at $t = 1$. Similarly, we have the following result concerning the exogenous variation of foreign country's nontradable endowments.

Proposition 7 (Foreign nontradable endowment effect on the exchange rate) *Assume home country has sufficiently low risk aversion at all times and states ($\gamma_{ht} < 1$, $\forall t \in \{0, 1\}, s \in \Omega$).*

1. **Foreign recession state***: If foreign country has sufficiently low risk aversion in a future*

state $s \in \Omega$, then the current value of home currency increases with foreign nontradable endowment in that future state s,

$$\gamma_{f1}(s) < 1 \implies u''_{fT1,fN1}(s) > 0 \implies \frac{\partial S_0}{\partial e_{fN1}(s)} > 0.$$

2. **Foreign normal state***: If foreign country has sufficiently high risk aversion in a future state $s \in \Omega$, then the current value of home currency decreases with foreign nontradable endowment in that future state s,*

$$\gamma_{f1}(s) > 1 \implies u''_{fT1,fN1}(s) < 0 \implies \frac{\partial S_0}{\partial e_{fN1}(s)} < 0.$$

The reversal in the relationship between current exchange rate variations and nontradable endowment variations when home (or foreign) country's risk aversion crosses the log-preference threshold ($\gamma = 1$) is key to explain the empirical pattern of USD exchange rates across the business cycle.

6 Conclusion

In this paper, we present evidences and construct a simple structural model to elucidate the unique role of the US Dollar as a safe-haven currency in international foreign exchange (FX) markets. This unique role of the US Dollar is revealed in four main features; (i) the US Dollar appreciates against foreign currencies when US economy has downturns, (ii) the US Dollar appreciates against foreign currencies when foreign economies have downturns, (iii) the US Dollar appreciates following bad US macro news in recessions, (iv) the US Dollar depreciates following bad US macro news in normal periods.

References

Balasko, Yves, 2014, The Transfer Problem: A Complete Characterization, *Theoretical Economics* 9, 435–444.

Barro, R., 2006, Rare Disasters and Asset Markets in the Twentieth Century, *Quarterly Journal of Economics* 121, 823–866.

Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, 2009, Understanding the forward premium puzzle: A microstructure approach, *American Economic Journal: Macroeconomics* 1, 127–154.

Emmanuel, Farhi, and Xavier Gabaix, 2008, Rare disasters and exchange rates, working paper no. 13805, NBER.

Fratzscher, Marcel, 2009, What Explains Global Exchange Rate Movements during the Financial Crisis, *Journal of International Money and Finance* 28, 1390–1407.

Kohler, Marion, 2010, Exchange rates during financial crises, *BIS Quarterly Review* March, 39–50.

Maggiori, Matteo, 2013, Financial intermediation, international risk sharing, and reserve currencies, Working paper, Harvard University.

Maggiori, Matteo, and Xavier Gabaix, 2015, International liquidity and exchange rate dynamics, *Quarterly Journal of Economics (forthcoming)* .

Mas-Colell, Andreu, Michael Whinston, and Jerry Green, 1995, Microeconomic Theory, *Oxford University Press,* 1008 pp.

Obstfeld, Maurice, and Kenneth Rogoff, 1996, Foundations of International Macroeconomics, *MIT Press,* 832 pp.

Rietz, T. A., 1988, The Equity Risk Premium: A Solution, *Journal of Monetary Economics* 22, 117–131.

Appendices

A Technical Derivations

A.1 Identities Concerning Utility Partial Derivatives

In what follows and throughout the paper, $u'_{it} \equiv \frac{\partial u}{\partial c_{it}}$, and $u''_{it} \equiv \frac{\partial^2 u}{\partial^2 c_{it}}$ denote respectively first-order and second-order partial derivative of utility function $u(c_{it})$, and $\gamma_{it} \equiv \frac{-c_{it}u''_{it}}{u'_{it}}$ denotes country i's (time-varying) relative risk aversion. For any country $i \in \{h, f\}$, time t and state s, the identities below follow from preference (4) (we omit state s for ease of notation),

$$u'_{iTt} \equiv \frac{\partial u}{\partial c_{it}} \frac{\partial c_{it}}{\partial c_{iTt}} = u'_{it} \times \epsilon_i \times \frac{c_{it}}{c_{iTt}}, \qquad u'_{iNt} \equiv \frac{\partial u}{\partial c_{it}} \frac{\partial c_{it}}{\partial c_{iNt}} = u'_{it} \times (1 - \epsilon_i) \times \frac{c_{it}}{c_{iNt}},$$

$$u''_{iTt} \equiv \frac{\partial^2 u}{\partial c^2_{iTt}} = \epsilon_i \times [\epsilon_i \times (1 - \gamma_{it}) - 1] \times u'_{it} \times \frac{c_{it}}{c^2_{iTt}},$$

$$u''_{iNt} \equiv \frac{\partial^2 u}{\partial c^2_{iNt}} = (1 - \epsilon_i) \times [(1 - \epsilon_i) \times (1 - \gamma_{it}) - 1] \times u'_{it} \times \frac{c_{it}}{c^2_{iNt}},$$

$$u''_{iTt,iNt} \equiv \frac{\partial^2 u}{\partial c_{iTt} \partial c_{iNt}} = \epsilon_i \times (1 - \epsilon_i) \times (1 - \gamma_{it}) \times u'_{it} \times \frac{c_{it}}{c_{iTt} c_{iNt}},$$

which imply

$$\frac{u''_{iTt}}{u'_{iTt}} = \frac{(\epsilon_i - 1) - \epsilon_i \times \gamma_{it}}{c_{iTt}} < 0, \qquad \frac{u''_{iNt}}{u'_{iNt}} = \frac{(1 - \epsilon_i) \times (1 - \gamma_{it}) - 1}{e_{iNt}} < 0, \tag{46}$$

$$\frac{u''_{iTt,iNt}}{u'_{iTt}} = \frac{(1 - \epsilon_i) \times (1 - \gamma_{it})}{e_{iNt}} > 0 \quad \text{if } \gamma_{it} < 1; \qquad \frac{u''_{iTt,iNt}}{u'_{iNt}} = \frac{\epsilon_i \times (1 - \gamma_{it})}{c_{iTt}} > 0 \quad \text{if } \gamma_{it} < 1,$$

$$\frac{u''_{iTt,iNt}}{u'_{iNt}} - \frac{u''_{iTt}}{u'_{iTt}} = \frac{1}{c_{iTt}} > 0, \qquad \frac{u''_{iNt}}{u'_{iNt}} - \frac{u''_{iTt,iNt}}{u'_{iTt}} = \frac{-1}{e_{iNt}} < 0. \tag{47}$$

Identity (46) implies useful relationships, for each time $t \in \{0, 1\}$ and each state $s \in \Omega$

$$\frac{u'_{iTt}}{u''_{iTt}} - (e_{iTt} - c_{iTt}) = \frac{c_{iTt}}{\epsilon_i - 1 - \epsilon_i \gamma_{it}} - (e_{iTt} - c_{iTt}) = \frac{\epsilon_i \times (1 - \gamma_{it})}{(\epsilon_i - 1) - \epsilon_i \times \gamma_{it}} \times c_{iTt} - e_{iTt}. \tag{48}$$

Given that country i is risk averse at time t (and state s), its risk aversion $\gamma_{it} > 0$,[30] and the relative taste $\epsilon_i \in [0,1]$, the expression (48) is negative when country i either (i) has sufficiently low risk aversion at t (and state s), or (ii) lends to the other country at t,

$$\left.\begin{array}{rl} \text{either:} & \gamma_{it} < 1 \\ \text{or:} & e_{iTt} > c_{iTt} \end{array}\right\} \Longrightarrow \frac{u'_{iTt}}{u''_{iTt}} - (e_{iTt} - c_{iTt}) < 0, \quad i \in \{h, f\}. \tag{49}$$

When market is incomplete and countries can only trade insurance contracts (Section 4), using budget constraint (10), relationships (48) can also be written more explicitly (more stringently) and separately for each period. For $t = 0$, (48) is negative when i either has sufficiently low risk aversion (at $t = 0$) or currently (at $t = 0$) lends to (or borrows limited amount from) the other country. Vice versa, (48) is positive when i currently borrows sufficiently large amount from the other country.

$$\text{At } t = 0, i \in \{h, f\}: \left\{\begin{array}{lll} \left\{\text{either } \gamma_{i0} < 1, \quad \text{or} \quad \alpha_i > -\frac{c_{iT0}}{I_0}\frac{1}{1-\epsilon_i+\gamma_{i0}\epsilon_i}\right\} & \Longrightarrow & \frac{u'_{iT0}}{u''_{iT0}} - \alpha_i \times I_0 < 0, \\ & & \\ \alpha_i < -\frac{c_{iT0}}{I_0}\frac{1}{1-\epsilon_i+\gamma_{i0}\epsilon_i} & \Longrightarrow & \frac{u'_{iT0}}{u''_{iT0}} - \alpha_i \times I_0 > 0, \end{array}\right. \tag{50}$$

For $t = 1$, (48) is negative when i either has sufficiently low risk aversion (at $t = 1$ and state s) or initially (at $t = 0$) borrowed from (or lent limited amount to) the other country. Vice versa, (48) is positive when i initially (at $t = 0$) lent sufficiently large amount to the other country.

$$\text{At } t = 1, i \in \{h, f\}: \left\{\begin{array}{lll} \left\{\text{either } \gamma_{it} < 1, \quad \text{or} \quad \alpha_i < \frac{c_{iT1}}{1-\epsilon_i+\gamma_{i1}\epsilon_i}\right\} & \Longrightarrow & \frac{u'_{iT1}}{u''_{iT1}} + \alpha_i < 0, \\ & & \\ \alpha_i > \frac{c_{iT1}}{1-\epsilon_i+\gamma_{i1}\epsilon_i} & \Longrightarrow & \frac{u'_{iT1}}{u''_{iT1}} + \alpha_i > 0, \end{array}\right. \tag{51}$$

A.2 Tatonnement Stability

The tatonnement stability is that an increase in insurance price leads to a drop in aggregate demand of insurance, keeping endowment distributions (probabilities and supports) at $t = 1$ intact (see e.g., Mas-Colell et al. (1995), section 17.H). Keeping unchanged the aggregate tradable and nontradable endowments and their probabilities, $de_{T1}(s)$=0, $de_{iN1}(s)$=0, $dp(s)$=0, $\forall i \in \{h, f\}$,

[30] We recall from (4) that risk aversion $\gamma_{it} \equiv \frac{-c_{it}u''_{it}}{u'_{it}} > 0$ in general depends on both time t and state s at that time.

$\forall s \in \Omega$, tatonnement stability is characterized by the following relationship,

$$\left(\frac{\partial \alpha_h}{\partial I_0} + \frac{\partial \alpha_f}{\partial I_0}\right)\Big|_{de=0;dp=0} < 0. \tag{52}$$

We remark that the above tatonnement stability is an endogenous assumption that is stated given the equilibrium price I and demand α_h. Whereas the partial derivatives in (52) are evaluated for any perturbations in price dI_0 (including off-equilibrium paths). Therefore market clearing conditions (9) need not to hold in the computation of (52).[31]

We now show that the tatonnement stability condition (52) for the incomplete financial market follows from the condition $\mathcal{T} < 0$ (19). Rewriting equation (11) as,

$$u'_{hT0} I_0 = \beta_h \sum_{s\in\Omega} p(s) u'_{hT1}(s), \qquad u'_{fT0} I_0 = \beta_f \sum_{s\in\Omega} p(s) u'_{fT1}(s).$$

Totally differentiating both sides of each equation above and using budget constraints (10) yield[32]

$$\begin{aligned} & \left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0\right) dI_0 - \left(I_0^2 + \beta_h \sum_{s\in\Omega} p(s) \frac{u''_{hT1}(s)}{u''_{hT0}}\right) d\alpha_h \\ = \quad & -I_0 de_{hT0} - I_0 \frac{u''_{hT0,hN0}}{u''_{hT0}} de_{hN0} + \beta_h \sum_{s\in\Omega} p(s) \frac{u''_{hT1}(s)}{u''_{hT0}} de_{hT1}(s) \\ & + \beta_h \sum_{s\in\Omega} p(s) \frac{u''_{hT1,hN1}(s)}{u''_{hT0}} de_{hN1}(s) + \beta_h \sum_{s\in\Omega} \frac{u'_{hT1}(s)}{u''_{hT0}} dp(s). \end{aligned} \tag{53}$$

and

$$\begin{aligned} & \left(\frac{u'_{fT0}}{u''_{fT0}} - \alpha_f I_0\right) dI_0 - \left(I_0^2 + \beta_f \sum_{s\in\Omega} p(s) \frac{u''_{fT1}(s)}{u''_{fT0}}\right) d\alpha_f \\ = \quad & -I_0 de_{fT0} - I_0 \frac{u''_{fT0,fN0}}{u''_{fT0}} de_{fN0} + \beta_f \sum_{s\in\Omega} p(s) \frac{u''_{fT1}(s)}{u''_{fT0}} de_{fT1}(s) \\ & + \beta_f \sum_{s\in\Omega} p(s) \frac{u''_{fT1,fN1}(s)}{u''_{fT0}} de_{fN1}(s) + \beta_f \sum_{s\in\Omega} \frac{u'_{fT1}(s)}{u''_{fT0}} dp(s). \end{aligned} \tag{54}$$

[31] That is, the tatonnement stability per ser does not require market clearing (9) $d\alpha_h = -d\alpha_f$. When we apply this tatonnement stability to the changes in equilibrium (endogenous) quantities, we can enforce the market clearing.

[32] We place all endogenous variations of insurance price dI_0 and holdings $d\alpha$'s in one side, and exogenous variations of endowments de's and expectations $\{dp(s)\}$ in the other.

Using the system (53)–(54), the tatonnement stability (52) can be expressed as follows,

$$\frac{\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h \times I_0}{I_0^2 + \beta_h \sum_s p(s) \frac{u''_{hT1}(s)}{u''_{hT0}}} + \frac{\frac{u'_{fT0}}{u''_{fT0}} - \alpha_f \times I_0}{I_0^2 + \beta_f \sum_s p(s) \frac{u''_{fT1}(s)}{u''_{fT0}}} < 0, \tag{55}$$

or equivalently,

$$\left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h \times I_0\right) \times \left(I_0^2 + \beta_f \sum_s p(s) \frac{u''_{fT1}(s)}{u''_{fT0}}\right) + \left(\frac{u'_{fT0}}{u''_{fT0}} - \alpha_f \times I_0\right) \times \left(I_0^2 + \beta_h \sum_s p(s) \frac{u''_{hT1}(s)}{u''_{hT0}}\right) < 0.$$

After straightforward simplifications, this inequality is $\mathcal{T} < 0$ (19).

Note that using the insurance price (11), the tatonnement $\mathcal{T}$ above can also be written as,

$$\begin{aligned} \mathcal{T} &= \beta_h \frac{I_0}{u''_{hT0}} \sum_s p(s) \left[u'_{hT1}(s) + \alpha_h u''_{hT1}(s)\right] + \beta_f \frac{I_0}{u''_{fT0}} \sum_s p(s) \left[u'_{fT1}(s) + \alpha_f u''_{fT1}(s)\right] \\ &+ \frac{u'_{hT0}}{u''_{hT0}} \frac{u'_{fT0}}{u''_{fT0}} \sum_s p(s) \left[\beta_h \frac{u''_{hT1}(s)}{u'_{hT0}} + \beta_f \frac{u''_{fT1}(s)}{u'_{fT0}}\right]. \end{aligned} \tag{56}$$

As an implication of the above expression,[33] the tatonnement stability arises in the following premises. (Below, $\gamma_{i0} \equiv \frac{-c_{i0} u''_{i0}}{u'_{i0}}$ denotes country i's current (at $t = 0$) relative risk aversion, following our convention (4).)

(i) When countries have sufficiently low risk aversions,

$$\{\gamma_{h0} < 1 \quad \text{and} \quad \gamma_{f0} < 1\} \Longrightarrow \mathcal{T} < 0. \tag{57}$$

(ii) When foreign country currently buys insurance, and home country has sufficiently low risk aversion (or vice versa),

$$\left.\begin{array}{rl} \text{either:} & \{\gamma_{h0} < 1 \quad \text{and} \quad \alpha_f > 0\}, \\ \text{or:} & \{\gamma_{f0} < 1 \quad \text{and} \quad \alpha_h > 0\}, \end{array}\right\} \Longrightarrow \mathcal{T} < 0. \tag{58}$$

[33]See also equation (50).

A.3 Complete-market Variational Analysis

Derivation of Equation (39)

Totally differentiating the FOC (36) at $t = 0$ yields

$$u''_{hT0} dc_{hT0} + u''_{hT0,hN0} de_{hN0} = u'_{fT0} d\lambda + \lambda u''_{fT0} dc_{fT0} + \lambda u''_{fT0,fN0} de_{fN0}.$$

Dividing both sides by FOC at $t = 0$,

$$\frac{u''_{hT0}}{u'_{hT0}} dc_{hT0} + \frac{u''_{hT0,hN0}}{u'_{hT0}} de_{hN0} = \frac{d\lambda}{\lambda} + \frac{u''_{fT0}}{u'_{fT0}} dc_{fT0} + \frac{u''_{fT0,fN0}}{u'_{fT0}} de_{fN0}.$$

We take λ as given, and solve for the linear system (of two equation, two unknowns) dc_{hT0}, dc_{fT0} (in term of the exogenous variations de_{T0}, $de_{T0}(s)$, de_{hN0}, de_{fN0}) to obtain (39) for time $t = 0$.

Totally differentiating the FOC (36) at $t = 1$ and state s yields,

$$\beta_h \left[u''_{hT1}(s) dc_{hT1}(s) + u''_{hT1,hN1}(s) de_{hN1}(s) \right] = \beta_f \left[u'_{fT1}(s) d\lambda + \lambda u''_{fT1}(s) dc_{fT1}(s) + \lambda u''_{fT1,fN1}(s) de_{fN1}(s) \right].$$

Dividing both sides by FOC at $t = 1$ and state s,

$$\frac{u''_{hT1}(s)}{u'_{hT1}(s)} dc_{hT1}(s) + \frac{u''_{hT1,hN1}(s)}{u'_{hT1}(s)} de_{hN1}(s) = \frac{d\lambda}{\lambda} + \frac{u''_{fT1}(s)}{u'_{fT1}(s)} dc_{fT1}(s) + \frac{u''_{fT1,fN1}(s)}{u'_{fT1}(s)} de_{fN1}(s).$$

Again, we take $d\lambda$ as given, and solve the above linear system for $dc_{hT1}(s)$, $dc_{fT1}(s)$, in term of the contemporaneous exogenous endowments $de_{T1}(s)$, $de_{hN1}(s)$, $de_{fN1}(s)$ to obtain (39) for time $t = 1$ and (any) state s.

Derivation of Pareto Weight's Variations

First, we substitute AD price $q(s)$ from (38) into home country's budget constraint (37), then totally differentiating (37), and finally diving both sides by u'_{hT0} yield (we place the endogenous consumption variations dc's on one side, and exogenous endowment variations de's and expectation variations dp's on the other side),

$$
\begin{aligned}
& \left[\frac{u''_{hT0}}{u'_{hT0}}(c_{hT0}-e_{hT0})+1\right]dc_{hT0}+\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\left(\frac{u''_{hT1}(s)}{u'_{hT1}(s)}[c_{hT1}(s)-e_{hT1}(s)]+1\right)dc_{hT1}(s)\\
= \quad & de_{hT0}+\frac{u''_{hT0,hN0}}{u'_{hT0}}(e_{hT0}-c_{hT0})de_{hN0} \qquad (59)\\
& +\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}de_{hT1}(s)+\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\frac{u''_{hT1,hN1}(s)}{u'_{hT1}(s)}[e_{hT1}(s)-c_{hT1}(s)]de_{hN1}(s)\\
& +\beta_h\sum_s\frac{u'_{hT1}(s)}{u'_{hT0}}[e_{hT1}(s)-c_{hT1}(s)]dp(s).
\end{aligned}
$$

Note that once we used the budget constraint for home country, and resource constraints (2), then the budget constraint for foreign country is redundant. Substituting consumption variations dc's from (39) into budget constraint (59) yields an equation for (endogenous) variation $d\lambda$ in term of exogenous variations de's and dp's, (whete $\mathcal{V}$ is analyzed in (41))

$$
\begin{aligned}
& \frac{d\lambda}{\lambda}\times\underbrace{\left(\frac{\frac{u''_{hT0}}{u'_{hT0}}(c_{hT0}-e_{hT0})+1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}+\beta_h\sum_s p(s)\times\frac{u'_{hT1}(s)}{u'_{hT0}}\times\frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}\times[c_{hT1}(s)-e_{hT1}(s)]+1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}\right)}_{\equiv\mathcal{V}}\\
= \quad & \frac{u''_{hT0,hN0}}{u'_{hT0}}\times\frac{\frac{u''_{fT0}}{u'_{fT0}}\times[e_{hT0}-c_{hT0}]+1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}\times de_{hN0} \qquad (60)\\
& -\frac{u''_{fT0,fN0}}{u'_{fT0}}\times\frac{\frac{u''_{hT0}}{u'_{hT0}}\times[c_{hT0}-e_{hT0}]+1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}\times de_{fN0}\\
& +\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\times\frac{u''_{hT1,hN1}(s)}{u'_{hT1}(s)}\times\frac{\frac{u''_{fT1}(s)}{u'_{fT1}(s)}\times[e_{hT1}(s)-c_{hT1}(s)]+1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}\times de_{hN1}(s)\\
& -\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\times\frac{u''_{fT1,fN1}(s)}{u'_{fT1}(s)}\times\frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}\times[c_{hT1}(s)-e_{hT1}(s)]+1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}\times de_{fN1}(s)\\
& +de_{hT0}-\frac{u''_{fT0}}{u'_{fT0}}\times\frac{\frac{u''_{hT0}}{u'_{hT0}}\times[c_{hT0}-e_{hT0}]+1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}\times de_{T0}\\
& +\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\times\left(de_{hT1}(s)-\frac{u''_{fT1}(s)}{u'_{fT1}(s)}\times\frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}\times[c_{hT1}(s)-e_{hT1}(s)]+1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}\times de_{T1}(s)\right)\\
& +\beta_h\sum_s\frac{u'_{hT1}(s)}{u'_{hT0}}[e_{hT1}(s)-c_{hT1}(s)]\times dp(s)
\end{aligned}
$$

From this follows the variation of Pareto weight with respect to home current tradable endowment (keeping all else constant),

$$\frac{d\lambda}{\lambda} = \mathcal{V}^{-1} \times \frac{u''_{hT0}}{u'_{hT0}} \times \frac{\frac{u''_{fT0}}{u'_{fT0}} \times [c_{fT0} - e_{fT0}] + 1}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \times de_{hT0},$$

and the variation of Pareto weight with respect to foreign current tradable endowment (keeping all else constant),

$$\frac{d\lambda}{\lambda} = -\mathcal{V}^{-1} \times \frac{u''_{fT0}}{u'_{fT0}} \times \frac{\frac{u''_{hT0}}{u'_{hT0}} \times [c_{hT0} - e_{hT0}] + 1}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} \times de_{fT0},$$

and the variation of Pareto weight with respect to home future tradable endowment (keeping all else constant),

$$\frac{d\lambda}{\lambda} = \mathcal{V}^{-1} \times \beta_h \times \sum_s p(s) \frac{u'_{hT1}(s)}{u'_{hT0}} \times \frac{u''_{hT1}(s)}{u'_{hT1}(s)} \times \frac{\frac{u''_{fT1}(s)}{u'_{fT1}(s)} \times [c_{fT1}(s) - e_{fT1}(s)] + 1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)} + \frac{u''_{fT1}(s)}{u'_{fT1}(s)}} \times de_{hT1}(s),$$

and the variation of Pareto weight with respect to foreign future tradable endowment (keeping all else constant),

$$\frac{d\lambda}{\lambda} = -\mathcal{V}^{-1} \times \beta_h \times \sum_s p(s) \frac{u'_{hT1}(s)}{u'_{hT0}} \times \frac{u''_{fT1}(s)}{u'_{fT1}(s)} \times \frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)} \times [c_{hT1}(s) - e_{hT1}(s)] + 1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)} + \frac{u''_{fT1}(s)}{u'_{fT1}(s)}} \times de_{fT1}(s).$$

Derivation of Equivalent Representations (41) for $\mathcal{V}$

We begin with the budget constraint (37) (and using (38) for AD prices) for country h,

$$(c_{hT0} - e_{hT0}) + \beta_h \sum_s p(s) \times \frac{u'_{hT1}(s)}{u'_{hT0}} (c_{hT1}(s) - e_{hT1}(s)) = 0$$

which is equivalent to

$$\frac{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}}{\frac{u''_{hT0}}{u'_{hT0}} + \frac{u''_{fT0}}{u'_{fT0}}} (c_{hT0} - e_{hT0}) + \beta_h \sum_s p(s) \frac{u'_{hT1}(s)}{u'_{hT0}} \frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)} + \frac{u''_{fT1}(s)}{u'_{fT1}(s)}}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)} + \frac{u''_{fT1}(s)}{u'_{fT1}(s)}} (c_{hT1}(s) - e_{hT1}(s)) = 0,$$

or

$$\frac{\frac{u''_{hT0}}{u'_{hT0}}}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}(c_{hT0}-e_{hT0})+\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}(c_{hT1}(s)-e_{hT1}(s)) \tag{61}$$
$$= \frac{\frac{u''_{fT0}}{u'_{fT0}}}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}(e_{hT0}-c_{hT0})+\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\frac{\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}(e_{hT1}(s)-c_{hT1}(s))$$

Now using the resource constraints, $e_{hT0}-c_{hT0}=c_{fT0}-e_{fT0}$, $e_{hT1}(s)-c_{hT1}(s)=c_{fT1}(s)-e_{fT1}(s)$, $\forall s$, and the FOC (36) $\beta_h\frac{u'_{hT1}(s)}{u'_{hT0}}=\beta_f\frac{u'_{fT1}(s)}{u'_{fT0}}$, $\forall s$, we can rewrite the right-hand side of (61), and obtain,

$$\frac{\frac{u''_{hT0}}{u'_{hT0}}}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}(c_{hT0}-e_{hT0})+\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\frac{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}(c_{hT1}(s)-e_{hT1}(s))$$
$$= \frac{\frac{u''_{fT0}}{u'_{fT0}}}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}(c_{fT0}-e_{fT0})+\beta_f\sum_s p(s)\frac{u'_{fT1}(s)}{u'_{fT0}}\frac{\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}(c_{fT1}(s)-e_{fT1}(s))$$

Adding both above equalities by the following, (which arises again from FOC (36)),

$$\frac{1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}+\beta_h\sum_s p(s)\frac{u'_{hT1}(s)}{u'_{hT0}}\frac{1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}=\frac{1}{\frac{u''_{hT0}}{u'_{hT0}}+\frac{u''_{fT0}}{u'_{fT0}}}+\beta_f\sum_s p(s)\frac{u'_{fT1}(s)}{u'_{fT0}}\frac{1}{\frac{u''_{hT1}(s)}{u'_{hT1}(s)}+\frac{u''_{fT1}(s)}{u'_{fT1}(s)}}$$

we have obtain the equality (41).

A.4 Proofs

Proof of Lemma 1

Given $\gamma_h<1$, (57) indicates that Lemma 1's first sufficient condition (i) $\gamma_f<1$ implies the third sufficient condition (iii) $\mathcal{T}<0$. Whereas, (58) indicates that Lemma 1's second sufficient condition (ii) $\alpha_h<0$ also implies the third sufficient condition. Therefore, Lemma 1's third sufficient condition (iii) $\mathcal{T}<0$ is the weakest. It suffices to prove this lemma under the third sufficient condition of tatonnement stability (and $\gamma_h<1$).

Using (9) and solving the system (53)–(54) of two equations and two unknowns yield the following variation of insurance price (where $\mathcal{T}$ is defined in (19)),

$$dI_0 = \frac{1}{\mathcal{T}} \times \left\{ -I_0 \left(I_0^2 + \beta_f \sum_s p(s) \frac{u''_{fT1}(s)}{u''_{fT0}} \right) de_{hT0} - I_0 \left(I_0^2 + \beta_h \sum_s p(s) \frac{u''_{hT1}(s)}{u''_{hT0}} \right) de_{fT0} \right. \tag{62}$$

$$-I_0 \left(I_0^2 + \beta_f \sum_s p(s) \frac{u''_{fT1}(s)}{u''_{fT0}} \right) \frac{u''_{hT0,hN0}}{u''_{hT0}} de_{hN0} - I_0 \left(I_0^2 + \beta_h \sum_s p(s) \frac{u''_{hT1}(s)}{u''_{hT0}} \right) \frac{u''_{fT0,hN0}}{u''_{fT0}} de_{fN0}$$

$$+\beta_h \left(I_0^2 + \beta_f \sum_s p(s) \frac{u''_{fT1}(s)}{u''_{fT0}} \right) \sum_s p(s) \left[\frac{u''_{hT1}(s)}{u''_{hT0}} de_{hT1}(s) + \frac{u''_{hT1,hN1}(s)}{u''_{hT0}} de_{hN1}(s) \right]$$

$$+\beta_f \left(I_0^2 + \beta_h \sum_s p(s) \frac{u''_{hT1}(s)}{u''_{hT0}} \right) \sum_s p(s) \left[\frac{u''_{fT1}(s)}{u''_{fT0}} de_{fT1}(s) + \frac{u''_{fT1,fN1}(s)}{u''_{fT0}} de_{fN1}(s) \right]$$

$$\left. + \sum_s \left[\beta_h \left(I_0^2 + \beta_f \sum_s p(s) \frac{u''_{fT1}(s)}{u''_{fT0}} \right) \frac{u'_{hT1}(s)}{u''_{hT0}} + \beta_f \left(I_0^2 + \beta_h \sum_s p(s) \frac{u''_{hT1}(s)}{u''_{hT0}} \right) \frac{u'_{fT1}(s)}{u''_{fT0}} \right] dp(s) \right\}.$$

and the variation of insurance demand by home country,

$$d\alpha_h = \frac{1}{\mathcal{T}} \times \left\{ I_0 \left(\frac{u'_{fT0}}{u''_{fT0}} + \alpha_h I_0 \right) de_{hT0} - I_0 \left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0 \right) de_{fT0} \right. \tag{63}$$

$$+I_0 \left(\frac{u'_{fT0}}{u''_{fT0}} + \alpha_h I_0 \right) \frac{u''_{hT0,hN0}}{u''_{hT0}} de_{hN0} - I_0 \left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0 \right) \frac{u''_{fT0,hN0}}{u''_{fT0}} de_{fN0}$$

$$-\beta_h \sum_s p(s) \left(\frac{u'_{fT0}}{u''_{fT0}} + \alpha_h I_0 \right) \left[\frac{u''_{hT1}(s)}{u''_{hT0}} de_{hT1}(s) + \frac{u''_{hT1,hN1}(s)}{u''_{hT0}} de_{hN1}(s) \right]$$

$$+\beta_f \sum_s p(s) \left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0 \right) \left[\frac{u''_{fT1}(s)}{u''_{fT0}} de_{fT1}(s) + \frac{u''_{fT1,fN1}(s)}{u''_{fT0}} de_{fN1}(s) \right]$$

$$\left. + \sum_s \left[-\beta_h \frac{u'_{hT1}(s)}{u''_{hT0}} \left(\frac{u'_{fT0}}{u''_{fT0}} + \alpha_h I_0 \right) + \beta_f \frac{u'_{fT1}(s)}{u''_{fT0}} \left(\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0 \right) \right] dp(s) \right\},$$

where $\{de_{iTt}(s), de_{iNt}(s)\}$ are exogenous variations of endowments, and $\{dp(s)\}$ are exogenous variations of expectations.

When $\mathcal{T} < 0$, (62) implies that a surge in current (at $t = 0$) endowments, either tradable or nontradable, in either countries, unambiguously increases the current price of insurance in equilibrium,

$$\text{either } \; de_{hT0} > 0, \;\; \text{or } de_{fT0} > 0, \;\; \text{or } \; de_{hN0} > 0, \;\; \text{or } \; de_{fN0} > 0 \implies dI_0 > 0. \tag{64}$$

Similarly, when $\mathcal{T} < 0$, a drop in future (at $t = 1$) endowments, either tradable or nontradable, in any state and in either countries, also unambiguously increases the current price of insurance in equilibrium,

$$\text{either } \; de_{hT1}(s) < 0, \;\; \text{or } \; de_{fT1}(s) < 0, \;\; \text{or } \; de_{hN1}(s) < 0, \;\; \text{or } \; de_{fN1}(s) < 0 \implies dI_0 > 0, \quad \forall s. \tag{65}$$

When h's current risk aversion $\gamma_{h0} \equiv \frac{-c_{h0}u''_{h0}}{u'_{h0}} < 1$, (50) implies that $\frac{u'_{hT0}}{u''_{hT0}} - \alpha_h I_0 < 0$. Additionally, when $\mathcal{T} < 0$, (63) implies that a surge in country f's current (at $t = 0$) endowments, either tradable or nontradable, unambiguously increases the current demand for insurance by country f in equilibrium (note that because insurance market clears, $\alpha_h = -\alpha_f$, $d\alpha_f$ is given by the inverse of (63)),

$$\text{either } \; de_{fT0} > 0, \;\; \text{or } \; de_{fN0} > 0 \implies d\alpha_f > 0. \tag{66}$$

Similarly (when $\gamma_{h0} < 1$ and $\mathcal{T} < 0$), a drop in country f's future (at $t = 1$) endowments, either tradable or nontradable, in any state, also unambiguously increases the current demand for insurance by country f in equilibrium,

$$\text{either } \; de_{fT1}(s) < 0, \;\; \text{or } \; de_{fN1}(s) < 0 \implies d\alpha_f > 0, \quad \forall s \in \Omega. \tag{67}$$

Combining (64) with (66) yields the first result of Lemma 1. Combining (65) with (67) yields the second result ■