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EFFECTS OF THE CONSECUTIVE SPEED HUMPS ON CHAOTIC VIBRATION OF A NONLINEAR VEHICLE MODEL

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Abstract. *The study is aimed at investigating the effects of the consecutive speed-control humps on the dynamic behaviour of the vehicle passing the consecutive speed-control humps. The consecutive speed-control humps were modeled by combination of sine and trapezoidal wave of constant amplitude and variable frequency. A two-DOF quarter vehicle model with nonlinear spring and damper is applied. Occurrence of chaotic vibration is analyzed by bifurcation diagram, time history, phase portrait, Poincar map and power spectrum. Furthermore, the exact range of excitation frequency that results in chaotic vibration is derived.*
Keywords: Chaotic vibration, Vehicle suspension system, Consecutive speed-control humps, Numerical simulation, Trapezoidal excitation

1. **Introduction.** Consecutive speed-control humps have become a popular tool for speed reducing devices at a particular site of highway [1, 2]. Speed-control hump is one of the most efficient equipment of speed limit preventing accidents and traffic fatalities. The ideal hump should have the function of controlling speed forcibly as well as have moderate influence of vibration on vehicle and comfort on passengers [3]. Unfortunately, the usage of speed-control hump inevitably increases the roughness of road surface [4]. Therefore, the problem of speed-control hump and its influence on vehicle and passengers unwanted vibration is still a subject of research [5, 6]. The chaotic response may appear when the vehicle moves over a bumpy road due to various nonlinearities in the vehicle [7, 8]. Therefore, chaotic vibration of a vehicle system model with nonlinear spring and nonlinear damper has been attracted much attention. The researcher mainly discussed the chaotic behaviour of vehicle excited by a sine wave of signal or multi-frequency with different amplitudes, or by random road excitation [9, 10]. As the road excitation amplitude increases, chaotic motion will occur in the nonlinear vehicle. However, for an actual road excitation generated by the consecutive speed-control humps, it is of constant low-amplitude and wide range frequency, and the higher the velocity of the vehicle, the higher of the excitation from road surface. The study on the effects of the consecutive speed-control humps on the dynamic behaviour of the vehicle passing the consecutive speed-control humps has not been almost considered by now. In this paper, main attention is given to the chaotic vibration of a nonlinear vehicle with low amplitude and high frequency of the consecutive speed-control humps. A mathematical model of road excitation coupling consecutive speed-control humps with velocity is built by combination of

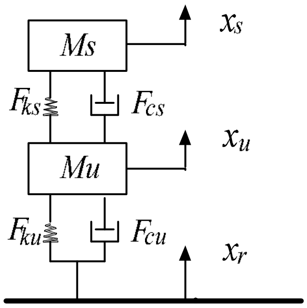
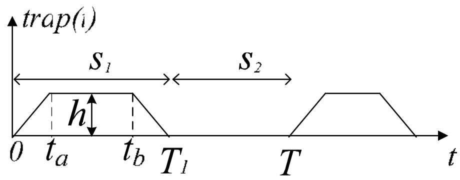


FIGURE 1. Two DOF nonlinear quarter-vehicle model

sine and trapezoidal wave, and a dynamic equation of a two-DOF quarter vehicle suspension system is derived firstly. Then the effect of the consecutive speed-control humps on the chaotic vibration of a vehicle is investigated by numerical simulation. The existence of chaotic vibration is verified via of bifurcation diagram, time history, phase portrait, Poincaré map and power spectrum.

2. **Model of Vehicle Suspension System and Road Excitation.** Figure 2 shows the model of two-DOF quarter vehicle suspension system simulated and discussed in this paper [11], where M_s and M_u are the vehicle body and tire mass respectively, k_s and k_t are respectively spring forces of suspension and tire, c_s and c_t are respectively damping forces of suspension and tire, x_s and x_u are vertical displacements of body and tire, respectively, x_r represents the road excitation in consecutive speed-control humps area.

(a)



(b)

FIGURE 2. Consecutive speed-control hump on highway and its model of trapezoidal excitation

The amplitude of hump excitation increases with certain slope when a vehicle enters the speed-control hump area, and decreases with certain slope when leaving this area, which is similar to a trapezoidal wave. Then the hump excitation is described as trapezoidal wave showed as $trap(t)$ in Figure 2. Furthermore, a sinusoidal wave is considered as the intrinsic excitation caused by highway surface. The combination of a sinusoidal wave and a trapezoidal wave is used do described the excitation generated by consecutive speed-control humps area on highway. Thus, the consecutive speed-control humps are approximated as

$$x_r = A\sin(2\pi t/T) + trap(t) \tag{1}$$

where

$$trap(t) = \begin{cases} \frac{h}{t_a} \times t, \ t < t_a \\ h, \ t_a < t < t_b \\ \frac{h}{t_a - t_b} \times (T_1 - t), \ t_b < t < T_1 \\ 0, \ T_1 < t < T \end{cases} \tag{2}$$

and A is the amplitude of sinusoidal excitation.

In Figure 2, T_1 is the time the vehicle passes a hump, $[T_1, T]$ is the time the vehicle passes the road between two adjacent humps, T is the period of road excitation, and h is the amplitude of trapezoidal excitation. Usually the value of h is between $0.003m$ and $0.015m$, and it is set to $0.008m$ here. When a vehicle passes a segment of speed-control hump with s_1 at speed v, the time passing adjacent humps will be $T - T_1$ or s_2/v, excitation frequency f is equal to $1/T$, thus the relation of speed v and frequency f can be described as

$$f = \frac{1}{T} = \frac{v}{s_1 + s_2} \tag{3}$$

In practice, the parameters of a hump are set to $s_1 = 0.5m$ and $s_2 = 0.8m$. The speed that the vehicle goes through consecutive speed-control humps area on highway is restricted to $25 - 90km/h$. Then, the range of frequency of road excitation on highway is bounded to $5 - 17Hz$, which is used in the following study.

According to Newton's second law, the equations of motion of vehicle suspension system showed in Figure 2 are given by

$$\begin{cases} M_s\ddot{x}_s &= -f_{ks} - f_{cs} - M_s g \\ M_u\ddot{x}_u &= f_{ks} + f_{cs} - f_{kt} - f_{ct} - M_u g \end{cases} \tag{4}$$

where f_{ks}, f_{ku}, f_{cs} and f_{cu} are spring forces of body and tire, damping forces of body and tire respectively, and have the following nonlinearity characteristics [11].

$$f_{ks} = k_{s1}(x_s - x_u - \delta_s) + k_{s2}(x_s - x_u - \delta_s)^2 + k_{s3}(x_s - x_u - \delta_s)^3 \tag{5}$$

$$f_{kt} = k_{t1}(x_u - x_u - \delta_u) + k_{t2}(x_u - x_r - \delta_u)^2 + k_{t3}(x_u - x_r - \delta_u)^3 \tag{6}$$

$$f_{cs} = c_s(\dot{x}_s - \dot{x}_u) \tag{7}$$

$$f_{ct} = c_t(\dot{x}_u - \dot{x}_r) \tag{8}$$

Taking δ_s and δ_u as the initial displacements of spring in a state of balance when vehicle body and tires are in loaded, the state equations of suspension system for equilibrium state can be expressed as

$$\begin{cases} M_s g &= k_{s1}\delta_s - k_{s2}\delta_s^2 + k_{s3}\delta_s^3 \\ (M_s + M_u)g &= k_{t1}\delta_u - k_{t2}\delta_u^2 + k_{t3}\delta_u^3 \end{cases} \tag{9}$$

Letting $(M_u + M_s)g = F$ and $M_s g = q$, from (9) we have

$$\begin{aligned} \delta_s = &\frac{k_{s2}}{3k_{s3}} - (2^{1/3}(-k_{s2}^2 + 3k_{s1}k_{s3}))/(3k_{s3}(2k_{s2}^2 - 9k_{s1}k_{s2}k_{s3} + 27qk_{s3}^2 \\ &+ \sqrt{(4(-k_{s2}^2 + 3k_{s1}k_{s3})^3 + (2k_{s2}^3 - 9k_{s1}k_{s2}k_{s3} + 27qk_{s3}^2))^2})^{1/3}) \\ &+ \frac{1}{3 * 2^{1/3} * k_{s3}}((2k_{s2}^3 - 9k_{s1}k_{s2}k_{s3} + 27qk_{s3}^2 \\ &+ \sqrt{(4(-k_{s2}^2 + 3k_{s1}k_{s3})^3 + (2k_{s2}^3 - 9k_{s1}k_{s2}k_{s3} + 27qk_{s3}^2))^2})^{1/3}) \end{aligned}$$

$$\begin{aligned} \delta_u = &\frac{k_{t2}}{3k_{t3}} - (2^{1/3}(-k_{t2}^2 + 3k_{t1}k_{t3}))/(3k_{t3}(2k_{t2}^2 - 9k_{t1}k_{t2}k_{t3} + 27Fk_{t3}^2 \\ &+ \sqrt{(4(-k_{t2}^2 + 3k_{t1}k_{t3})^3 + (2k_{t2}^3 - 9k_{t1}k_{t2}k_{t3} + 27Fk_{t3}^2))^2})^{1/3}) \\ &+ \frac{1}{3 * 2^{1/3} * k_{t3}}((2k_{t2}^2 - 9k_{t1}k_{t2}k_{t3} + 27Fk_{t3}^2 \\ &+ \sqrt{(4(-k_{t2}^2 + 3k_{t1}k_{t3})^3 + (2k_{t2}^3 - 9k_{t1}k_{t2}k_{t3} + 27Fk_{t3}^2))^2})^{1/3}) \end{aligned}$$

Parameters of system (9) for numerical simulation areshown in Table 1. Here, the values of above parameters are chosen from an experimental equipment of two-DOF quarter vehicle suspension system showed in Figure 2.

TABLE 1. Numerical values of the system parameters

Parameter	Value	Parameter	Value
M_s	$0.694kg$	k_{t2}	$19041.0N/m^2$
M_u	$0.353kg$	k_{t3}	$563307.0N/m^3$
k_{s1}	$517.8N/m$	c_s	$1.35N \cdot s/m$
k_{s2}	$26082.0N/m^2$	c_t	$1.8N \cdot s/m$
k_{s3}	$-718349.0N/m^3$	g	$9.8m/s^2$
k_{t1}	$380.8N/m$		

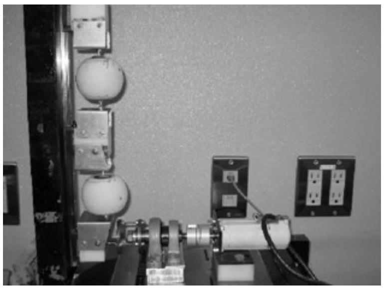
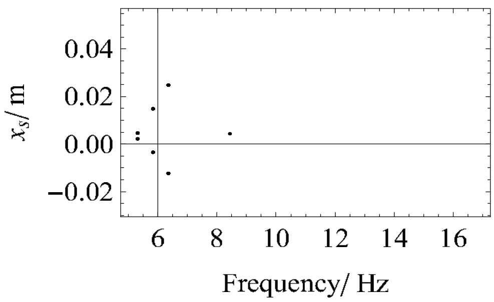


FIGURE 3. Experimental equipment of two-DOF one quarter vehicle suspension system

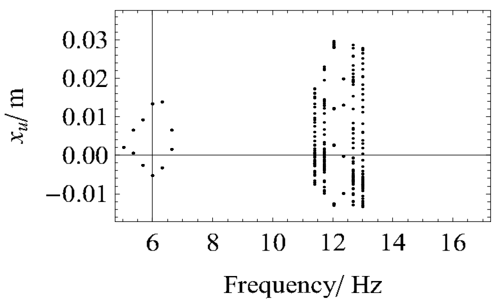
3. Numerical Simulation.

3.1. Analyzing with bifurcation diagram. It is known that the dynamics of a system may be analyzed through a bifurcation diagram, which is obtained by plotting the displacement of system versus the frequency of the excitation term. For studied system, the amplitudes of sinusoidal and trapezoidal excitation are $0.0015m$ and $0.008m$ respectively, and excitation frequency varies from $5Hz$ to $17Hz$.

Figure 4(a) and Figure 4(b) represent the bifurcation diagrams of x_s and x_u by evolving along with the varying excitation frequency. Figure 4(a) and Figure 4(b) show that x_s and x_u may move into chaotic vibration when the frequency of road excitation is in the ranges of $9.7-11.7Hz$ and $12.5-15Hz$. While there was no sign of chaotic vibration even if the error of the amplitude of response seems big, but there are signs of variation with multi-base frequencies between 11.7 and $12.5hz$. According to (3), the speed that will cause chaotic vibration is derived to be in the range of $45.4 - 54.8km/h$ and $58.5 - 70.2km/h$.



(a) Bifurcation diagram of x_s



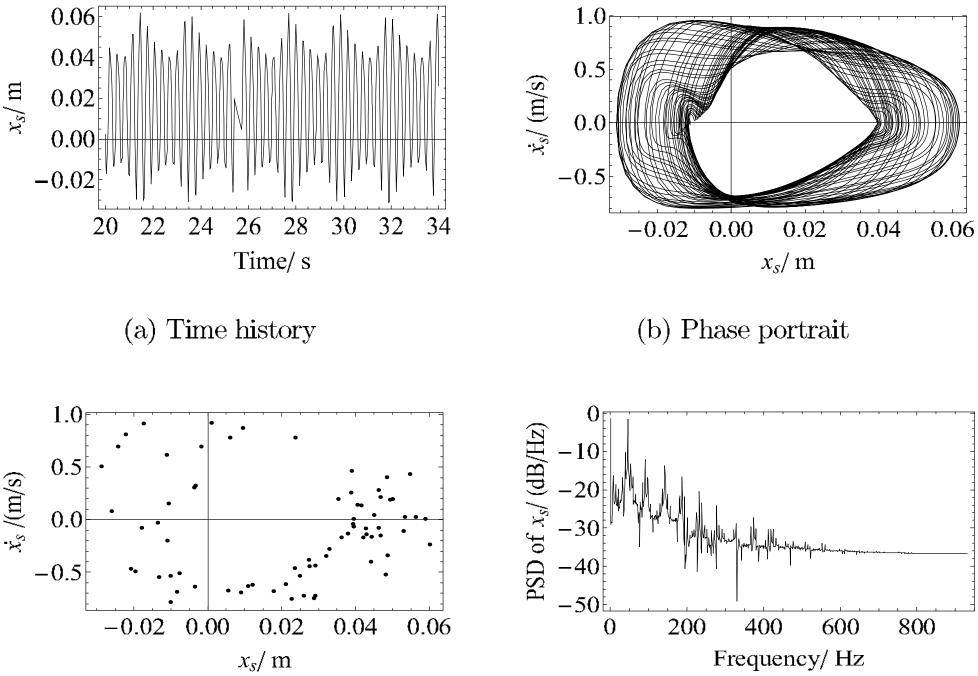
(b) Bifurcation diagram of x_u

FIGURE 4. Bifurcation diagram of x_s and x_u

3.2. Discussion of vibration performance. In order to further reveal the possible existence of chaotic in x_s and x_u, the chaotic vibration that x_s and x_u, $f = 13.3Hz$ and $f =$

$12.1Hz$ are respectively chosen as the frequency of road excitation. Then the vibration performance of the system is discussed using time history, phase portrait, Poincaré map and power spectrum.

3.2.1. *In the case of* $f = 13.3Hz$. Figure 5 shows the time history, phase portrait, Poincaré map and power spectrum of x_s when the excitation frequency is $13.3Hz$, and Figure 6 shows those of x_u.



(a) Time history (b) Phase portrait

(c) Poincaré map (d) Power spectrum

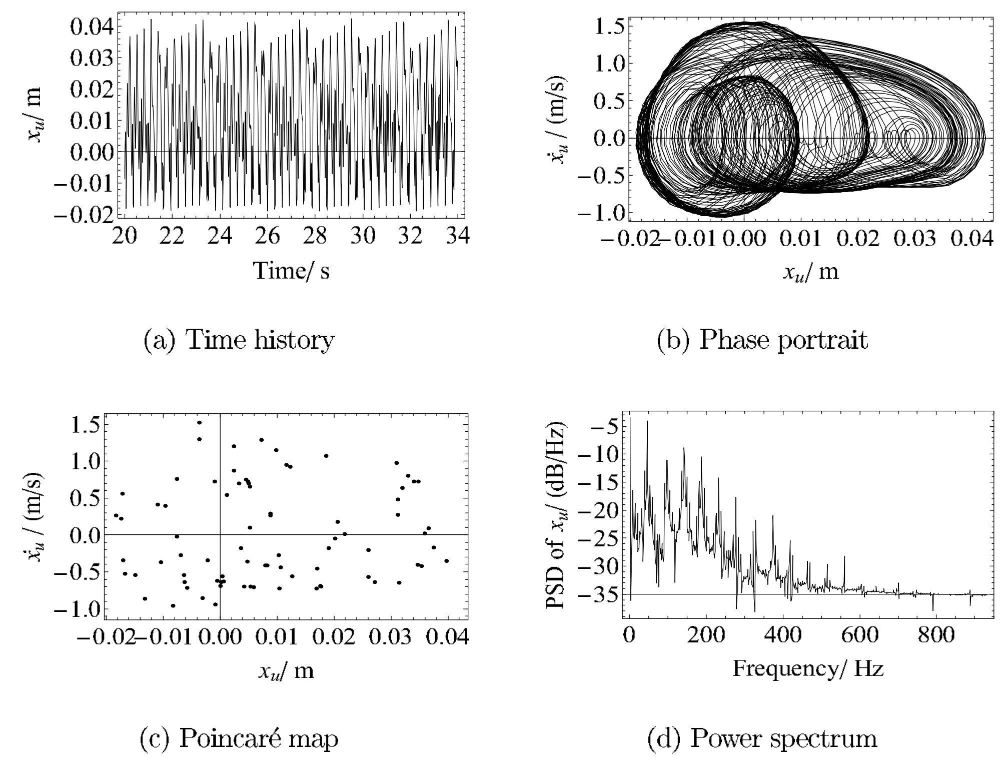
FIGURE 5. Time history, phase portrait, Poincaré map and power spectrum of x_s when $f = 13.3Hz$

Figure 5 shows that the time history of x_s appears erratically, the phase portrait and the Poincaré map are hazy, and power spectrum is continuous, which indicate that x_s has the character of chaotic vibration. Also in Figure 6, the time history of x_u has the feature of irregular, the phase portrait and Poincaré show hazily, power spectrum is continuous, which imply that x_s evolves chaotically. As a conclusion, the vehicle suspension system has a chaotic motion when excitation frequency is $13.3Hz$.

3.2.2. *In the case of* $f = 12.1Hz$. Figure 7 shows the time history, phase portrait, Poincaré map and power spectrum of x_s when the excitation frequency is $12.1Hz$, and Figure 8 shows those of x_u.

Figure 7 shows that the time history of x_s evolves periodically, the phase portrait is composed by circular lines, the Poincaré map is constituted by some discrete points, which indicate that evolves with multi-base frequencies. Further more, the power spectrum of x_s consists of few peak frequencies which hold that x_s evolves with multi-base frequencies as well. Likewise, Figure 8 shows x_u evolves with multi-base frequencies. Consequently, the vehicle suspension system posses the performance of multi-base frequencies motion when excitation frequency is $12.1Hz$.

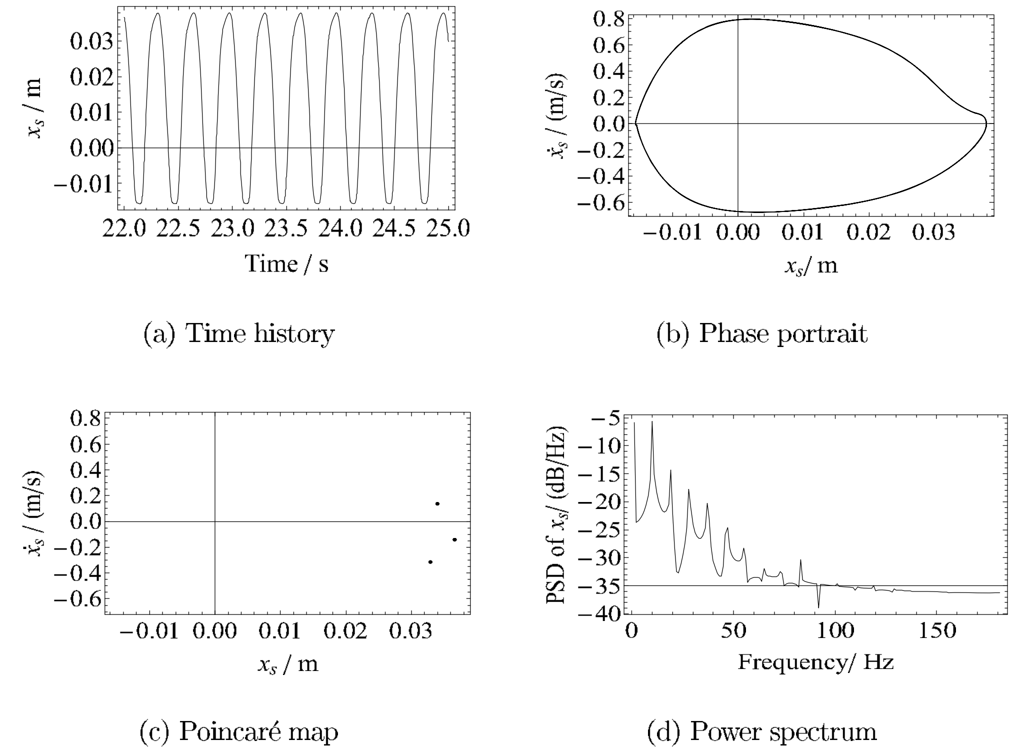
4. **Conclusions.** In the present paper, effect of consecutive speed-control humps on the chaotic vibration of a vehicle when passing consecutive speed-control humps area on



(a) Time history (b) Phase portrait

(c) Poincaré map (d) Power spectrum

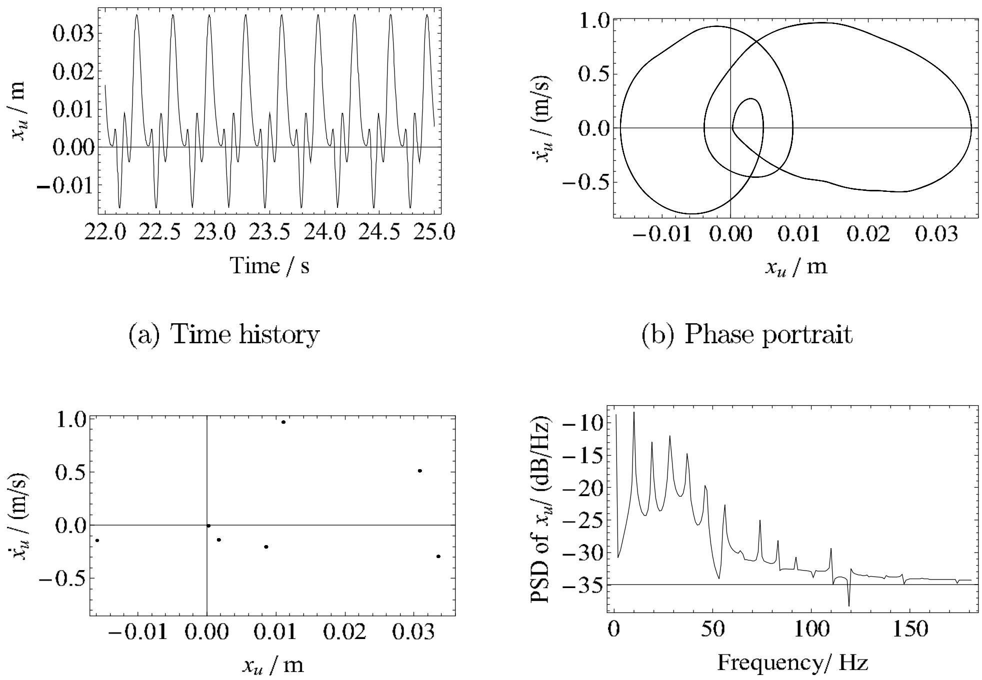
FIGURE 6. Time history, phase portrait, Poincaré map and power spectrum of x_u when $f = 13.3Hz$



(a) Time history (b) Phase portrait

(c) Poincaré map (d) Power spectrum

FIGURE 7. Time history, phase portrait, Poincaré map and power spectrum of x_s when $f = 12.1Hz$



(a) Time history (b) Phase portrait

(c) Poincaré map (d) Power spectrum

FIGURE 8. Time history, phase portrait, Poincaré map and power spectrum of x_u when $f = 12.1Hz$

highway is investigated. The excitation generated by consecutive speed-control humps is approximated by the combination of sinusoidal and trapezoidal wave. It is found that the chaotic vibration may appear in vehicle when the road excitation frequency is within the range of $9.7 - 11.7Hz$ and $12.5 - 15Hz$, which is corresponded to $45.4 - 54.8km/h$ and $58.5 - 70.2km/h$ of speed. The relation between excitation frequency and vehicle speed is derived in this paper, with which the vehicle speed can be controlled and speed-control humps can be better fixed to make the road excitation frequency appears out of the range above, further to keep the suspension system from chaotic motion. The study is mostly numerical simulation established on model of experiment equipment, and next experimental verification will be launched.

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