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Mathematics 1613: Trigonometry Exam #2

Problem 1: Using the cofunction properties

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta \qquad \sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

and either the cosine subtraction or addition formula,

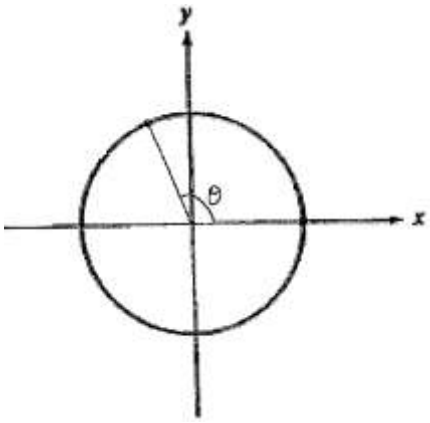
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

derive a formula for $\sin(\alpha-\beta)$.

Problem 2: In this problem, using a method similar to how we derived the cosine subtraction formula, you will work through another way to derive the formula for the sine of half of an angle.

(1) The diagram below shows the angle θ in standard position on the unit circle. Label the coordinates of the angle's initial and terminal sides.



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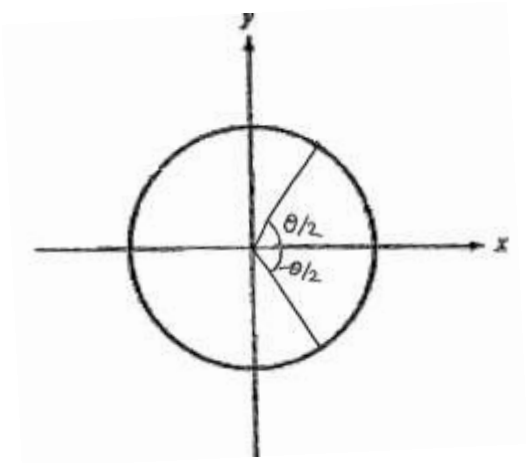
(2) Using the distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, show that the length of the line segment connecting the two coordinates just labeled is $\sqrt{2 - 2\cos\theta}$.

(3) In the diagram below, the angle θ has been rotated so that half of it lies above the *x* axis and half of it lies below, denoted as $\frac{\theta}{2}$ and $-\frac{\theta}{2}$. Label their corresponding coordinates on the unit circle.



(4) Using the distance formula $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, show that the length of the line segment connecting the two coordinates just labeled is $2\sin\frac{\theta}{2}$. (*Hint:* you will need to use the even/odd properties of sine and cosine to simplify the expression.)

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(5) Since the two lengths are equal, set them equal to each other to show that $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$. (In this case, we add in the $\pm$ sign in front of the square root afterwards because the angle could be in any quadrant.)

Problem 3: Use the above formula $\sin\frac{\theta}{2}$ to show that $\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$.

Problem 4: Use the subtraction (or addition) formulas to find expressions for the following:

(1) $\tan(\theta - \pi)$

(2) $\sin 2\theta$

(3) *Bonus*: $\sec(\theta + \frac{3\pi}{2})$

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Problem 5: Evaluate the following:

(1) $\sin 165°$

(2) $\cos \frac{7\pi}{8}$

(3) $\tan \frac{5\pi}{12}$

(4) $\csc \frac{\pi}{8}$

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Problem 6: Suppose that $\sin\alpha = \frac{4}{5}$ (with $0 \leq \alpha < \frac{\pi}{2}$) and $\cos\beta = -\frac{12}{13}$ (with $\frac{\pi}{2} \leq \beta < \pi$) . Find the following, simplifying as necessary:

(1) $\cos(\alpha - \beta)$

(2) $\sin\frac{\beta}{2}$

----------Scratch Work----------